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The impact of slip conditions on magnetohydrodynamics radiating fluid beyond an exponentially extended sheet

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Abstract. A mathematical model is performed to analyse the magnetohydrodynamics (MHD) fluid flow and transmission of heat in the case of an exponentially extended sheet. The system is supressed by thermal radiation, which is constituted in the energy equation. The boundary contains velocity, thermal and mass slips. Similarity transformation is applied, to convert the governing equations (continuity, momentum, energy and mass diffusion equations) into ordinary differential equations (ODE). Then, the purpose of numerical shooting method is to perform the solutions in the ODE part. Finally, results are presented in the forms of table and graphs (skin friction coefficient, heat transfer rate, concentration gradient at the plate, and the profiles of velocity, temperature, concentration and concentration gradient) for various values of controlling parameters. The consequences of the controlling parameters on the system are described in details.

1. Introduction

The theoretical reports of fluid flow and transmission of heat past a continuous extending surface contribute numerous applications in cooling of metallic plate in a bath, glass fibre and paper production, extrusion of polymer sheet from a die, and continuous casting [1]. This is due to the desired characteristics of final product in industrial manufacturing processes mostly depend on the rate of extending and heat transfer. In addition, the radiation effects and the study of magnetohydrodynamics (MHD) are also included in a variety of extending sheet problems. The thermal radiation impact is very important in gas turbines and devices for satellites, nuclear plants, space vehicles and aircraft [1]. Meanwhile, the study of MHD is applied in crystals growing, electromagnetic field pumps, plasma jets, MHD couples and bearing, and chemical synthesis [2]. Therefore, many researchers [3–16] described the radiating MHD fluid flow caused by a extending surface for non-slip boundary conditions.

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In engineering industry, the requirement of selecting the condition regarding to the partial slip as the boundary conditions, instead of non-slip is due to the reason that partial slip boundary condition is more consistent with all physical features in real situations. The presence of boundary slip in the flow field has applications in a technology, namely polishing of artificial heart valves and internal cavities [17]. Slip for velocity aspect arises when the fluid is not sticking to a solid boundary. Beavers and Joseph [18] described the fluid flow due to the condition of slipped boundary over a permeable wall. Fluid flow bounded by a partial slip, in the case of an extending sheet were investigated by Andersson [19], Wang [20], Ariel [21] and Sajid et al. [22]. The transmission of heat in MHD fluid flow, and also has a radiative characteristics are reported by Ibrahim and Shankar [23], Mukhopadhyay [24], Loganathan and Vimala [25] and Abdul Hakeem et al. [26] with the exerted slip conditions at the boundary. Ibrahim and Shankar [23] studied the impact of partial slips at boundary conditions (thermal, velocity and solutal slip) on MHD radiating nanofluid flow. On the other hand, Mukhopadhyay [24] analysed the impact of slips (velocity and thermal) at boundary conditions on flow of MHD fluid and transmission of heat due to a extending sheet with thermal radiation. This sheet is assumed to extend with exponential variation in velocity. However, Loganathan, and Vimala [25] developed the model of unsteady state in MHD nanofluid with thermal radiation. They also add another effect in the system related to the exponentially extending sheet, which is heat generation/absorption. The internal heat generation or absorption, velocity slip impact and thermal radiation on MHD porous medium flow regarding to a linear velocity of the extending surface with wall mass transfer are analysed by Abdul Hakeem et al. [26]. In addition, many researchers [27–32] investigated the different flow problems in the case of extending sheet. The system developed by these investigators is bounded by the partial slip and suppressed by magnetic field field and thermal radiation. Furthermore, some of the researchers [16, 33-38] are interested to present the illustrations of concentration distribution, as an addition to the profiles of temperature and velocity. To produce the profile of fluid concentration, they need to incorporate mass diffusion equation, together with momentum and energy equations as governing equations.

Motivated by the work by previous investigator (Mukhopadhyay [24]) and others, we study the effect of partial slips boundary conditions (velocity, thermal and mass slips) on MHD radiating viscous fluid. The transmission of heat in this flow are caused by sheet which is extending in exponential distributions. We introduce similarity variables to convert momentum, energy, and mass diffusion equations into nonlinear ordinary differential equations. Shooting technique is used to solve the ODE section. Computed numerical results are performed through table and graphs. Characteristics of transferring heat and mass in fluid flow is reported, subjected to certain controlling parameters.

2. Mathematical formulation

Consider the steady boundary layer flow fluid over an isothermal extending surface, which illustrated in Figure 1. The fluid is assumed to be a viscous incompressible electrically conducting and radiating, and modelled in two-dimensional Cartesian coordinates. The direction of the extending surface is propagates into x-axis, and the y-axis is directed to this x-axis. The presence of an externally applied magnetic field field B(x) is directed to the y-axis. The governing equations, to develop mathematical problem equations are written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho}u$$
(2)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y}$$
(3)

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}$$
(4)

where *u* and *v* are the *x* and *y* component of velocity in Cartesian coordinate, $v = \mu/\rho$ is the kinematic viscosity, μ is the viscosity, ρ is the fluid density, σ is the electrical conductivity, q_r is the radiative heat flux, c_p is the specific heat at constant temperature, κ is the thermal conductivity of the fluid, *C* is the concentration of the fluid, *D* is the coefficient of mass diffusivity and *T* is the temperature of the fluid. The induced magnetic field field in Equation (2) is disregarded since the magnetic field Reynolds number for the flow is assumed to be very small. In our design, the magnetic field field is formed as $B(x) = B_0 e^{(x/2L)}$, where B_0 being a constant.

Using Rosseland approximation for radiation [39], radiative heat flux q_r is written as

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \tag{5}$$

where σ^* is the Stefan-Boltzman constant and k^* is the absorption coefficient. The higher orders in Equation (5) is neglected, and Equation (3) becomes

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_{\infty}^3}{3\rho c_p k^*}\frac{\partial^2 T}{\partial y^2}$$
(6)

where T_{∞} is the temperature of the fluid at infinity.



Figure 1. Sketch of physical problem.

The boundary conditions are

$$u = U + N \frac{\partial u}{\partial y}, \quad v = -V(x),$$

$$T = T_w + D \frac{\partial T}{\partial y}, \quad C = C_w + F \frac{\partial C}{\partial y} \quad \text{at } y = 0,$$

$$u \to 0, \quad T \to T_{\infty}, \quad C \to C_{\infty} \quad \text{as } y \to \infty$$
(7)

where $U = U_0 e^{(x/L)}$ is the extending velocity, $T_w(x) = T_\infty + T_0 e^{(x/2L)}$ is temperature at the sheet, $C_w(x) = C_\infty + C_0 e^{(x/2L)}$ is the concentration of the plate, C_∞ is the concentration in the free stream, U_0 , T_0 and C_0 are the reference velocity, temperature and concentration respectively, $N = N_1 e^{(-x/2L)}$ is the velocity slip factor, N_1 is the initial value of velocity slip factor, $D = D_1 e^{(-x/2L)}$ is the thermal slip factor, D_1 is the initial value of thermal slip factor, $F = F_1 e^{(-x/2L)}$ is the mass slip factor, and F_1 is the initial value of mass slip factor. The condition of the no-slip case is attained when N = D = F = 0. The velocity at the wall is defined as $V(x) = V_0 e^{(x/2L)}$, where V_0 is the initial strength of suction. The velocity of suction is V(x) > 0, and the velocity of blowing is denoted by V(x) < 0.

Introducing new similarity variables by Bala Anki Reddy [27]:

$$\eta = y \left(\frac{U_0}{2\nu L}\right)^{1/2} e^{(x/2L)}, \quad u = U_0 e^{(x/L)} f'(\eta), \qquad v = -\left(\frac{\nu U_0}{2L}\right)^{1/2} e^{(x/2L)} \left[f(\eta) + \eta f'(\eta)\right], \quad (8)$$
$$T = T_{\infty} + T_0 e^{(x/2L)} \theta(\eta), \qquad C = C_{\infty} + C_0 e^{(x/2L)} \phi(\eta)$$

where prime denotes differentiation with respect to η .

$$f''' + ff'' - 2(f')^2 - M^2 f' = 0$$
(9)

$$\left(1 + \frac{4}{3}R\right)\theta'' + \Pr\left(f\theta' - f'\theta\right) = 0$$
⁽¹⁰⁾

$$f'\phi - f\phi' = \frac{1}{Sc}\phi'' \tag{11}$$

and the boundary conditions reduce to

$$f' = 1 + \lambda f'', \quad f = s, \quad \theta = 1 + \delta \theta', \quad \phi = 1 + \gamma \phi' \quad \text{at } \eta = 0,$$

$$f' \to 0, \quad \theta \to 0, \quad \phi \to 0 \qquad \text{as } \eta \to \infty$$
(12)

where $M = (2\sigma B_0^2 L/\rho U_0)^{1/2}$ is the magnetic field parameter, $\lambda = N_1 (U_0/2L\nu)^{1/2}$ is the velocity slip parameter, $\delta = D_1 (U_0/2\nu L)^{1/2}$ is the thermal slip parameter, $\gamma = F_1 (U_0/2\nu L)^{1/2}$ is the mass slip parameter, $R = 4\sigma^* T_{\infty}^3/\kappa k^*$ is the radiation parameter, $\Pr = \mu c_p/\kappa$ is the Prandtl number, $Sc = \nu/D$ is the Schmidt number and $s = V_0/(U_0\nu/2L)^{1/2} > 0$ is the suction parameter.

The skin friction coefficient C_f , Nusselt number Nu_x and local Sherwood number Sh_x are stated as below:

$$C_f = \frac{\tau_w}{\rho u_w^2(x)}, \qquad N u_x = \frac{L q_w}{k(T_w - T_\infty)}, \qquad S h_x = \frac{L m_w}{D(C_w - C_\infty)}$$
(13)

where the shear stress at the surface τ_w , the heat transfer rate at the surface flux at the wall q_w and the mass transfer rate at the surface flux at the wall m_w are given by

$$\tau_w = \mu \left(\frac{du}{dy}\right)_{y=0}, \quad q_w = -k \left(\frac{dT}{dy}\right)_{y=0}, \quad m_w = -D \left(\frac{dC}{dy}\right)_{y=0}$$
(14)

Substituting Equation (8) into Equations (13)-(14), we get

$$C_f (2 \operatorname{Re}_x)^{1/2} = f''(0), \quad Nu_x \left(\frac{2}{\operatorname{Re}_x}\right)^{1/2} = -\theta'(0), \quad Sh_x \left(\frac{2}{\operatorname{Re}_x}\right)^{1/2} = -\phi'(0) \quad (15)$$

where local Reynolds number is defined as $\operatorname{Re}_{x} = Lu_{w}(x)/\upsilon$.

2.1. Numerical method for solution

In this section, shooting method is performed in Maple programming to solve the boundary value problems in equations (9)–(12). These equations are transformed into an initial value problem. The programming has applied fourth-order Runge–Kutta integration scheme to solve the initial value problem. The following equations are fixed:

$$f' = fp, \qquad fp' = fpp, \qquad fpp' = -f(fpp) + 2(fp)^2 + M^2 fp$$
 (16)

$$\theta' = \theta p, \qquad \theta p' = \frac{-\Pr(f\theta p) + \Pr(fp\theta)}{\left(1 + \frac{4}{3}R\right)}$$
(17)

$$\phi' = \phi p, \qquad \phi p' = Sc \left(fp \phi - f \phi p \right) \tag{18}$$

with the boundary conditions

$$\begin{aligned} fp = 1 + \lambda fpp, & f = s, & \theta = 1 + \delta \theta p, & \phi = 1 + \gamma \phi p & \text{at } \eta = 0, \\ fp \to 0, & \theta \to 0, & \phi \to 0 & \text{as } \eta \to \infty \end{aligned}$$
 (19)

The first step in this shooting technique is to set predicted values for fpp(0), $\theta p(0)$ and $\phi p(0)$ $(f''(0), \theta'(0))$ and $\phi'(0)$. A highest finite value for η is also predicted, denoted as η_{∞} . These predicted values $(\eta_{\infty}, f''(0), \theta'(0))$ and $\phi'(0)$ must satisfy the boundary conditions in Equation (19) at $\eta = 0$ and as $\eta \to \infty$. In addition, the predicted values are fixed when two consecutive values of $f''(0), \theta'(0)$ and $\phi'(0)$ vary significantly by a specified value.

3. Results and discussion

The numerical calculations, namely as shooting method is used to solve equations (9)–(12). The graphs of skin friction coefficient f''(0), local Nusselt number $-\theta'(0)$, local Sherwood number $-\phi'(0)$, velocity profile $f'(\eta)$, temperature profile $\theta(\eta)$, concentration profile $\phi(\eta)$ and concentration gradient profile $\phi'(\eta)$ are illustrated in this section. These illustrations are controlled by parameters of velocity slip λ , thermal slip δ , mass slip γ , magnetic field field M, radiation R and Schmidt number *Sc*. When the comparison of the presents findings with the previous results (Ishak [7] and Mukhopadhyay [24]) is performed, then the accuracy of current numerical method in this problem is achieved. This comparison is tabulated in Table 1. As a conclusion, the present values of $-\theta'(0)$ for some special cases are in good agreement with those reported by the previous investigators. Therefore, the good comparison proves that our numerical shooting method is applicable to present our theoretical study and numerical computation.

magnetic field field in the absence of suction, velocity, thermal and mass slips							
parameter and Schmidt number.							
Pr	R	Μ	Ishak	Mukhopadhyay	Present		
			[7]	[24]			
1	0	0	0.9548	0.9547	0.9548		
2			1.4715	1.4714	1.4715		
3			1.8691	1.8691	1.8691		
5			2.5001	2.5001	2.5001		
10			3.6604	3.6603	3.6604		
1		1	0.8611	0.8610	0.8615		
	0.5	0	-	-	0.6765		
	1		0.5312	0.5311	0.5313		
		1	0.4505	0.4503	0.4506		
2	0.5	0	-	1.0734	1.0735		
	1		-	0.8626	0.8629		
3	0.5		-	1.3807	1.3807		
	1		-	1.1213	1.1214		

Table 1.

Values of $-\theta'(0)$ for several values of Prandtl number, radiation parameter and

In Figure 2, the representation of velocity distribution is shown for different values of velocity slip parameter λ . In this figure, the velocity $f'(\eta)$ at a point diminishes at large distance from the sheet. This figure also displays that the fluid velocity declines with an enhancement in parameter λ , which happens because fluid experiences less drag with increase in velocity slip. The occurrence of slip will affect the value of the fluid velocity close to the sheet. As a result, the velocity of the fluid at this point is slightly differ to the velocity of the extending sheet. It is observed that the velocity of the fluid reduces due to the enhancement of velocity slip. This result due to the pulling of the extending sheet can be only transmitted partially to the fluid. Figure 3 represents that fluid velocity reduces with the enhancement in parameter M. This happens due to setting up of Lorentz force, caused the fluid motion is opposed by this force.



Figure 2. Variation of velocity for various λ.



Figure 3. Variation of velocity for various M.

The temperature profiles $\theta(\eta)$ with the impact of various values of related parameters (λ , δ , R and M) has been showed in Figures 4–7, respectively. Figures 4, 6 and 7 show that the fluid temperature increase with an increment in the values of λ , R and M. Otherwise, the effect of thermal slip in Figure 5 is to minimize the fluid temperature, together with the thickness of thermal boundary layer.



Figure 4. Variation of temperature for various λ .



Figure 5. Variation of temperature for various δ .



Figure 6. Variation of temperature for various *R*.



Figure 7. Variation of temperature for various *M*.

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Figures 8–11 show the effect of related parameters (velocity slip λ , mass slip γ , magnetic field M and Schmidt number Sc) on the concentration profile $\phi(\eta)$. From these figures, the concentration is decreasing until it reaches zero at a point far from the extending sheet. Refer to Figures 8 and 10, the concentration enhance with the increase of λ and M. Figure 9 reveals that the values of concentration of the fluid reduces as we increase the values of mass slip parameter. The concentration is reduced when the mass transfer from the plate to the fluid reduced, due to the effect of mass slip. Refer to the Figure 11, the increment in the value of Sc will lead to the decrement in the rate of concentration and the thickness of concentration boundary layer. This is because due to an enhancement in Schmidt number, the diffusion coefficient diminishes. Consequently, the effect of Sc causes the decrement in the concentration boundary layer.

Figures 12–15 illustrate the concentration gradient profile $\phi'(\eta)$, with the various value of λ , γ , M and Sc. The increment of the concentration gradient profile at the point near to the wall can be seen by adding the effect of λ and M. These can be observed in Figures 12 and 14. From Figure 13, it is showed that the concentration gradient at a fixed point increase with increase of mass slip parameter γ . In Figure 15, the values of $\phi'(\eta)$ increase at the point far from the extending sheet with the impact of parameter Sc. However, this profile shows an opposite trend due to the various values of Sc at the point near to the extending sheet.

The illustrations of skin friction coefficient f''(0) versus velocity slip parameter λ by including the effect of magnetic field are figured in Figure 16. From Figure 16, it is certain that the values of wall drag is minimized caused by the impact of magnetic field whereas it is enhanced when velocity slip is included. The variation of $-\theta'(0)$ with thermal slip parameter δ for various values of λ is displayed in Figure 17. The purpose of the velocity slip parameter is to lessen the transmission of heat, for a large value of thermal slip δ . However, the opposite nature can be observed for a small rate of thermal slip. The decrement on heat transfer rate is presented by adding the impact of magnetic field and radiation, which are illustrated in Figures 18 and 19. It is also noted that the values of $-\theta'(0)$ reduce when the thermal slip increases. The plots of local Sherwood number $-\phi'(0)$ against mass slip parameter γ for the various values of λ , M and Sc are showed in Figures 20–22. It is observed from Figures 20 and 21 that the magnitude of $-\phi'(0)$ decreases with larger value of λ and M, while Figure 22 shows that the values of $-\phi'(0)$ increase by increasing the values of Sc. Moreover, it can be seen that an increment in the values of γ lead to a decrement of the local Sherwood number $-\phi'(0)$.



Figure 8. Variation of concentration for various λ .



Figure 9. Variation of concentration for various *y*.



Figure 10. Variation of concentration for various *M*.



Figure 11. Variation of concentration for various *Sc*.



Figure 12. Variation of concentration gradient for various λ .



Figure 13. Variation of concentration gradient for various γ .



Figure 14. Variation of concentration gradient for various *M*.



Figure 15. Variation of concentration gradient for various *Sc*.



Figure 16. Variation of f''(0) versus λ for various M.



Figure 17. Variation of $-\theta'(0)$ versus δ for various λ .



Figure 18. Variation of $-\theta'(0)$ versus δ for various *M*.



Figure 19. Variation of $-\theta'(0)$ versus δ for various *R*.



Figure 20. Variation of $-\phi'(0)$ versus γ for various λ .



Figure 21. Variation of $-\phi'(0)$ versus γ for various *M*.



Figure 22. Variation of $-\phi'(0)$ versus γ for various *Sc*.

4. Conclusion

The numerical solution for the problem of MHD fluid flow towards a permeable exponentially extending sheet is developed. The system of the radiating fluid flow is bounded by slip condition. The following observations can be made from the present findings:

- a) The drag experienced by the fluid reduces with the increment in magnetic parameter while it shows a reverse trend with the impact of velocity slip parameter.
- b) The heat transfer rate is decreased when the parameters of velocity and thermal slip, magnetic and radiation increase.
- c) The concentration gradient at the plate increase with increase of Schmidt number, whereas it drops when parameters of velocity slip, magnetic and mass slip enhance.
- d) Parameter of magnetic field and velocity slip lessen the fluid velocity.
- e) The fluid temperature rises with the addition in the parameters of velocity slip, radiation or magnetic field. On the other hand, the rate of the temperature reduces with the addition in thermal slip rate.
- f) The raise of the value in mass slip parameter and Schmidt number lead to the decrement in the concentration of the fluid. However, by considering the impact of magnetic field and velocity slip, the temperature of the fluid shows an increment.
- g) The concentration gradient profile enhances when the mass slip parameter occurs in the system. In addition, the impact of velocity slip and magnetic field lead to the decrement of concentration gradient of adjacent fluid. The increment of concentration gradient of adjacent fluid is due to the increasing Schmidt number.

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