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Pursuit game problem of an infinite system of differential equations with geometric and integral constraints

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Abstract. The present paper considers a pursuit game problem of an infinite system of 1st-order differential equations in the space l_2. In the game, the player's control functions satisfy both geometric and integral constraints. The pursuer's action is to force the state of the system to coincides with another state for a finite time and the evader's action is to retrieve this. In each case, we obtain a condition for the completion of the game for which the pursuer's strategy is explicitly constructed. In addition, we study a control problem.

1. Introduction

In view of the excellent papers devoted to differential games with the player's control functions subjected to either integral, geometric or both constraint. For example, in the works of [1], [2], [3], [4], [5], [6], [7], [8], [9], [10] , [12], [13] and [11] have attracted the attention of many authors.

Many research works are dedicated to controls problem and Differential games of transferring the state of a system with a distributed parameter from one stage to another particularly to the origin (for instance, see [14], [15], [16], [17], [18], [19] and [20]).

In the paper [20], proposed pursuit game problem in control system of distributed parameters for transferring the state of the system to zeroth state (i.e., the origin) in the presence of a player attempts to stops this. The game occurs by partial equations for which the right part that contains the control of the player is in additive form.

$$\frac{\partial z}{\partial t} + Az(t) = u(t) - v(t),\ z(0) = z_0,\ 0 < t < T, \tag{1}$$

where

$$Az = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial z}{\partial x_i}\right),$$

is a linear operator independent of t. Decomposition method is also applied to reduced the problem to an infinite system of ordinary differential equations

$$\dot{z_k}(t) + \lambda_k z_k(t) = -u_k(t) + v_k(t),\ z_k(0) = z_{0k},\ 0 < t < T,\ k = 1, 2, ... \tag{2}$$

where $\lambda_k, k = 1, 2, ...$ are eigenvalues of the operator A satisfying $0 \le \lambda_1, \lambda_2, ... \to \infty$. The game involved the integral, geometric and both constraints on the player's control functions. Conditions which are sufficient for completion of the game is also presented.

The paper [11], studied a game of pursuit problem for an infinite system of 2-systems of 1st-order differential equations in different approaches to that of partial differential equations. In the game, pursuer attempts to bring the state of the system to the origin, and the evader actions in the opposite. Geometric constraints are imposed on the player's control functions where they proved that the game is completed.

Therefore, the present paper proposes to study differential game of pursuit for an infinite system of (2) with the case of integral and geometric constraints in Hilbert space l_2, where

$$l_2 = \left\{ \xi = (\xi_1, \xi_2, ...) |\ \sum_{k=1}^{\infty} |\xi_k|^2 < \infty \right\},$$

with

$$\langle \xi, \eta \rangle = \sum_{k=1}^{\infty} \xi_k \eta_k,\ \xi, \eta \in l_2,\quad ||\xi|| = \left(\sum_{k=1}^{\infty} |\xi_k|^2 \right)^{1/2},$$

to be inner product and norm respectively. The pursuer attempts to forward the state of the system to coincides with another state and the evader actions in the opposite. In contrast of previous studies of differential games (e.g., [4], [5], [6], [11], [20], [21] and [22]), the pursuer actions to coincides the state of the system to the origin. In each case, we obtain a condition for the completion of the game for a finite time.

2. Statement of Problem

We study the following infinite system of of 1st-order differential equations

$$\dot{z}_k + \lambda_k z_k = -u_k + v_k,\ z_k(0) = z_k^0,\quad k = 1, 2, \dots\ , \tag{3}$$

where $z_k, u_k, v_k \in \mathbb{R}^1,\ k = 1, 2, ...,\ z^0 = (z_1^0, z_2^0, ...,) \in l_2,\ u_1, u_2, ...,$ is the pursuer's control parameters and $v_1, v_2, ...,$ is that of the evader, $\lambda_k,\ k = 1, 2, ...$ is non negative real numbers. Let

$$L_2(0, T, l_2) = \left\{ w(t) = (w_1, w_2, ...) |\ \int_0^T \|w(t)\|^2\, dt < \infty,\ w_k(\cdot) \in L_2(0, T) \right\},$$

where

$$\|w(t)\|^2 = \sum_{k=1}^{\infty} |w_k(t)|^2,\ t \in [0,\ T].$$

Let $z^1 = (z_1^1, z_2^1, ...) \in l_2$ be a given another state.

Definition 2.1 *A function $w(\cdot) = (w_1(\cdot), w_2(\cdot), ...)$ such that $w : [0,\ T] \longrightarrow l_2$, with measurable coordinates $w_k(t),\ k = 1, 2, \dots,\ 0 \le t \le T$, satisfy the condition*

$$\|w(t)\| = \left(\sum_{k=1}^{\infty} |w_k(t)|^2 \right)^{1/2} \le \rho, \left[\left(\int_0^T \|w(t)\|^2\, dt \right)^{1/2} \le \rho, \right]$$

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Then the function $w(\cdot)$ is called an admissible control subject to geometric and integral constraints respectively, where ρ is a non negative number.

Let $S(\rho)$ be the set of all admissible controls with respect to geometric constraint and $S^*(\rho)$ be that of integral constraint.

Definition 2.2 *The control $u(\cdot)$ of pursuer and the control $v(\cdot)$ of the evader are said to be admissible if it's satisfy one of the following conditions*

$$\|u(t)\| = \left(\sum_{k=1}^{\infty} |u_k(t)|^2\right)^{1/2} \leq \rho, \left[\|v(t)\| = \left(\sum_{k=1}^{\infty} |v_k(t)|^2\right)^{1/2} \leq \sigma\right], \tag{4}$$

$$\left(\int_0^T \|u(t)\|^2 \, dt\right)^{1/2} \leq \rho, \left[\left(\int_0^T \|v(t)\|^2 \, dt\right)^{1/2} \leq \sigma\right], \tag{5}$$

where ρ and σ are non negative numbers and $t \in [0, T]$.

The Infinite system (3) in which $u(\cdot) = (u_1(\cdot), u_2(\cdot), ...)$ and $v(\cdot) = (v_1(\cdot), v_2(\cdot), ...)$ satisfy inequalities (4) and (5) respectively, are called differential games Q_1 and Q_2 respectively.

Definition 2.3 *Let $w(\cdot)$ belongs to $S(\rho)$ and $w(\cdot)$ belongs to $S^*(\rho)$, then one will say that a function $z(t) = (z_1(t), z_2(t), \dots)$, $0 \leq t \leq T$, with $z_k(0) = z_k^0$, $k = 1, 2, \dots$, is called solution of the initial value problem*

$$\dot{z}_k(t) + \lambda_k z_k(t) = w_k(t), \; z_k(0) = z_k^0, \; k = 1, 2, \dots, \tag{6}$$

if $z_k(t)$, $k = 1, 2, ...$, are absolutely continuous and almost everywhere on $[0, T]$ satisfy equation (6).

Assume that the space $C(0, T; l_2)$ of the continuous functions $z(\cdot) = (z_1(\cdot), z_2(\cdot), ...)$ such that for each t, $0 \leq t \leq T$, $z(t)$ belongs to l_2.

Definition 2.4 *Suppose that the function $u(t, v)$, such that $u : [0, \; T] \times l_2 \to l_2$ of the form*

$$u_k(t, \; v) = v_k(t) + w_k(t),$$

is called the pursuer's strategy with respect to geometric constraint if:

i. For any control $v(\cdot)$ of the evader (i.e., the control $v(\cdot)$ is admissible), the system (3) has the only solution at $u(t) = u(t, v)$, where $w(\cdot) = (w_1(\cdot), w_2(\cdot), ...) \in S(\rho - \sigma)$.

ii. $u(\cdot, v(\cdot)) \in S(\rho)$.

In similar way, one will define the pursuer's strategy with respect to integral constraint and $u(\cdot, v(\cdot)) \in S^*(\rho)$.

Definition 2.5 *If there exists a pursuer's strategy $u(t, v)$, such that for any control $v(\cdot)$ of the evader (i.e., the control $v(\cdot)$ is admissible) to guaranteed $z(t) = z^1$ at some t, $0 \leq t \leq T$, then one will say that pursuit can be completed for some $t \in [0, \; T]$, in differential game Q_1 and Q_2 respectively, where $z(t)$ is the solution of the system (3).*

The problem is to find a condition:
For which pursuit can be completed in differential games Q_1 and Q_2 respectively.

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3. Control Problem with Geometric and Integral Constraints

In this section, we study a control problem of an infinite systems (6).
It has been proven in [21] that if $w(\cdot)$ belongs to $S(\rho)$ and $\beta_k > 0$, then for any given $T > 0$ in the space $C(0, T; l_2)$, the infinite system (6) has a unique solution $z(t) = (z_1(t), z_2(t), ...),\ 0 \le t \le T$, where $z_k(t),\ t \in [0, T],\ k = 1, 2, ..$ defined by

$$z_k(t) = e^{-\beta_k t}\delta_k(t),\ k = 1, 2, ... \tag{7}$$

with $\delta_k(t) = z_k^0 + \int_0^t e^{\beta_k s} w_k(s)ds$ and $\lambda_k = \beta_k \ge 0$.
Now, we define the following sets

$$\gamma_1(T) = \left\{ (z^0,\ z^1) | \sum_{k=1}^{\infty} \left(2|z_k^0|^2 q_k(T) + 2|z_k^1|^2 q_k^*(T) \right) \le \rho^2 \right\},$$

where

$$q_k(T) = \frac{\beta_k^2}{(e^{\beta_k T} - 1)^2},\ q_k^*(T) = -q_k(-T),\ k = 1, 2, ...,$$

and

$$\gamma_2(T) = \left\{ (z^0,\ z^1) | \sum_{k=1}^{\infty} \left(|z_k^0|^2 p_k(T) + |z_k^1|^2 p_k^*(T) \right) \le \rho^2 \right\},$$

where

$$p_k(T) = \frac{2\beta_k}{e^{2\beta_k T} - 1},\ p_k^*(T) = -p_k(-T),\ k = 1, 2, ...$$

The problem is to find controls $w(\cdot) = (w_1(\cdot), w_2(\cdot), ...) \in S(\rho)$ and $w(\cdot) = (w_1(\cdot), w_2(\cdot), ...) \in S^*(\rho)$ respectively to forwards the state of (6) from the initial position z^0 to another state z^1 for a finite time.

Theorem 3.1 *Let $z^0, z^1 \in l_2$. If $z^0, z^1 \in \gamma_1(T)$ and $z^0, z^1 \in \gamma_2(T)$ respectively, then there exist a control $w(\cdot) \in S(\rho)$ and $w(\cdot) \in S^*(\rho)$ respectively, to forwards the state z^0 of (6) to coincides with z^1.*

Proof: We proof the theorem in two different cases.
Firstly, consider a control problem for which control functions satisfying the geometric constraint.
We construct the control as follows

$$w_k(t) = \left[e^{\beta_k T} z_k^1 - z_k^0 \right] \sqrt{q_k(T)},\ k = 1, 2, ...,\ 0 \le t \le T. \tag{8}$$

Show that (8) is admissible. Using (8), we have

$$\begin{aligned} \|w(t)\|^2 = \sum_{k=1}^{\infty} |w_k(t)|^2 &= \sum_{k=1}^{\infty} \left| \left[e^{\beta_k T} z_k^1 - z_k^0 \right] \sqrt{q_k(T)} \right|^2 \\ &= \sum_{k=1}^{\infty} \left| e^{\beta_k T} z_k^1 - z_k^0 \right|^2 q_k(T). \end{aligned}$$

Using the obvious inequality $|a-b|^2 \le 2|a|^2 + 2|b|^2$. It's not difficult to see that $\left|e^{\beta_k T} z_k^1 - z_k^0\right|^2 \le 2\left|z_k^1\right|^2 e^{2\beta_k T} + 2\left|z_k^0\right|^2$, we have

$$\|w(t)\|^2 = \sum_{k=1}^{\infty} |w_k(t)|^2 \le \sum_{k=1}^{\infty} \left(2e^{2\beta_k T}|z_k^1|^2 + 2|z_k^0|^2\right) q_k(T)$$
$$= 2\sum_{k=1}^{\infty} \left(|z_k^0|^2 q_k(T) + |z_k^1|^2 q_k^*(T)\right) \le \rho^2.$$

Consequently, (8) is admissible.
Proof that $z(T) = z^1$. Using (7) and admissible control (8), we obtain

$$\delta_k(T) = z_k^0 + \int_0^T e^{\beta_k s}\left(\left[e^{\beta_k T} z_k^1 - z_k^0\right]\sqrt{q_k(T)}\right) ds$$
$$= z_k^0 + \left(e^{\beta_k T} z_k^1 - z_k^0\right)\sqrt{q_k(T)}\int_0^T e^{\beta_k s} ds$$
$$= z_k^0 + \left(e^{\beta_k T} z_k^1 - z_k^0\right) = e^{\beta_k T} z_k^1.$$

Indeed,

$$z_k(T) = e^{-\beta_k T}\delta_k(T) = z_k^1.$$

Consequently, the system (6) is transferred from z^0 to z^1.
Secondly, we turn to the case for which control $w(\cdot)$ satisfying the integral constraint. Define the control

$$w_k(t) = e^{\beta_k T}\left[e^{\beta_k T} z_k^1 - z_k^0\right] p_k(T),\ k = 1, 2, ...,\ 0 \le t \le T. \tag{9}$$

Show that (9) is admissible. Using (9), we have

$$\int_0^T \|w(s)\|^2 ds = \sum_{k=1}^{\infty}\int_0^T \left|e^{\beta_k s}\left[e^{\beta_k T} z_k^1 - z_k^0\right] p_k(T)\right|^2 ds$$
$$= \sum_{k=1}^{\infty}\left|e^{\beta_k T} z_k^1 - z_k^0\right|^2 (p_k(T))^2 \int_0^T e^{2\beta_k s} ds$$

Using the obvious inequality $\left|e^{\beta_k T} z_k^1 - z_k^0\right|^2 \le 2\left|z_k^1\right|^2 e^{2\beta_k T} + 2\left|z_k^0\right|^2$, we have

$$\int_0^T \|w(s)\|^2 ds \le \sum_{k=1}^{\infty}\left(2e^{2\beta_k T}|z_k^1|^2 + 2|z_k^0|^2\right)(p_k(T))^2 \int_0^T e^{2\beta_k s} ds$$
$$= \sum_{k=1}^{\infty}\left(|z_k^0|^2 p_k(T) + |z_k^1|^2 p_k^*(T)\right) \le \rho^2.$$

Therefore, strategy (9) is admissible.
Forwarding the state of the system to another. Using (7) and constructed control (9), it's easy

to see that

$$\begin{aligned}\delta_k(T) &= z_k^0 + \int_0^T e^{\beta_k s}\left(e^{\beta_k s}\left[e^{\beta_k T} z_k^1 - z_k^0\right] p_k(T)\right) ds \\ &= z_k^0 + \left(e^{\beta_k T} z_k^1 - z_k^0\right) p_k(T) \int_0^T e^{2\beta_k s} ds \\ &= z_k^0 + e^{\beta_k T} z_k^1 - z_k^0 = e^{\beta_k T} z_k^1.\end{aligned}$$

Hence,

$$z_k(T) = e^{-\beta_k T}\delta_k(T) = z_k^1.$$

Thus, the system (6) is transferred from z^0 to z^1. The proof of the Theorem 3.1 is complete

4. Pursuit Problem with Geometric and Integral Constraints

The present part proposes a differential game of pursuit with a case of integral and geometric constraints imposed on the player's control functions.
Let,

$$\chi_1(T) = \left\{(z^0,\ z^1)|\ \sum_{k=1}^{\infty}\left(2|z_k^0|^2 q_k(T) + 2|z_k^1|^2 q_k^*(T)\right) \le (\rho-\sigma)^2,\ \rho > \sigma\right\},$$

and

$$\chi_2(T) = \left\{(z^0,\ z^1)|\ \sum_{k=1}^{\infty}\left(|z_k^0|^2 p_k(T) + |z_k^1|^2 p_k^*(T)\right) \le (\rho-\sigma)^2,\ \rho > \sigma\right\}.$$

Now, we proof the main result.

Theorem 4.1 *Let $\rho > \sigma$ and $z^0, z^1 \in l_2$. If in general, $z^0, z^1 \in \chi_1(T)$ and $z^0, z^1 \in \chi_2(T)$ respectively, then pursuit can be completed in differential game Q_1 and Q_2 respectively, for the time T.*

Proof: Also, we proof the theorem in two different cases.
Case1. We consider a pursuit problem for which control functions satisfying the geometric constraint.
We construct the pursuer's strategy on $[0, T]$ as follows

$$u_k(t, v) = v_k(t) - \left[e^{\beta_k T} z_k^1 - z_k^0\right]\sqrt{q_k(T)},\ 0 \le t \le T,\ k = 1, 2, ... \tag{10}$$

Proof that strategy (10) is admissible. From the fact that $v(\cdot)$ belongs to $S(\rho)$, we have

$$\begin{aligned}\|u(t)\| &= \left(\sum_{k=1}^{\infty}|u_k(t)|^2\, ds\right)^{1/2} = \left(\sum_{k=1}^{\infty}\left|v(t) - \left[e^{\beta_k T} z_k^1 - z_k^0\right]\sqrt{q_k(T)}\right|^2\right)^{1/2} \\ &\le \|v(t)\| + \left(\sum_{k=1}^{\infty}\left|\left[e^{\beta_k T} z_k^1 - z_k^0\right]\sqrt{q_k(T)}\right|^2\right)^{1/2} \\ &\le \sigma + \left(\sum_{k=1}^{\infty}\left|e^{\beta_k T} z_k^1 - z_k^0\right|^2 q_k(T)\right)^{1/2},\end{aligned} \tag{11}$$

(Minkowski inequality is imposed here). Using the obvious inequality $\left|e^{\beta_k T} z_k^1 - z_k^0\right|^2 \leq 2\left|z_k^1\right|^2 e^{2\beta_k T} + 2\left|z_k^0\right|^2$, we have

$$\begin{aligned} \|u(t)\| = \left(\sum_{k=1}^{\infty} |u_k(t)|^2\right)^{1/2} &\leq \sigma + \left(\sum_{k=1}^{\infty}\left(2e^{2\beta_k T}|z_k^1|^2 + 2|z_k^0|^2\right) q_k(T)\right)^{1/2} \\ &= \sigma + \left(\sum_{k=1}^{\infty}\left(2|z_k^0|^2 q_k(T) + 2|z_k^1|^2 q_k^*(T)\right)\right)^{1/2} \\ &= \sigma + \rho - \sigma = \rho. \end{aligned}$$

Consequently, (11) is admissible.

Show that pursuit is completed at time T: From fact that the infinite number of 1st-order differential equations (3) has the solution

$$z_k(t) = e^{-\beta_k t}\delta_k(t),\ k = 1, 2, ..., \tag{12}$$

where

$$\delta_k(t) = z_k^0 - \int_0^t e^{\beta_k s} u_k(s) ds + \int_0^t e^{\beta_k s} v_k(s) ds.$$

Then using (12) and and strategy (10), we obtain

$$\begin{aligned} \delta_k(T) &= z_k^0 + \int_0^T e^{\beta_k s} v_k(s) ds - \int_0^T e^{\beta_k s}\left(v_k(s) - \left(\left[e^{\beta_k T} z_k^1 - z_k^0\right]\sqrt{q_k(T)}\right)\right) ds \\ &= z_k^0 + \left(e^{\beta_k T} z_k^1 - z_k^0\right)\sqrt{q_k(T)}\int_0^T e^{\beta_k s} ds \\ &= z_k^0 + \left(e^{\beta_k T} z_k^1 - z_k^0\right) = e^{\beta_k T} z_k^1,\ k = 1, 2, ... \end{aligned}$$

Consequently,

$$z_k(T) = e^{-\beta_k T}\delta_k(T) = z_k^1,\ k = 1, 2, ...$$

Hence, pursuit is completed in differential game Q_1.

Case2. We now turn to the case for which control $u(\cdot)$ satisfying the integral constraint. *We set the pursuer's strategy on* $[0, T]$ *as*

$$u_k(t, v) = v_k(t) - e^{\beta_k T}\left[e^{\beta_k T} z_k^1 - z_k^0\right] p_k(T),\ 0 \leq t \leq T,\ k = 1, 2, ... \tag{13}$$

Show that strategy (13) is admissible. Using Minkowski inequality and the fact that $v(\cdot)$ belongs to $S^*(\rho)$, we have

$$\begin{aligned} \left(\int_0^T \|u(s)\|^2 ds\right)^{1/2} &\leq \left(\int_0^T \|v(s)\|^2 ds\right)^{1/2} + \left(\sum_{k=1}^{\infty}\int_0^T \left|e^{\beta_k s}\left[e^{\beta_k T} z_k^1 - z_k^0\right] p_k(T)\right|^2 ds\right)^{1/2} \\ &\leq \sigma + \left(\sum_{k=1}^{\infty}\left|e^{\beta_k T} z_k^1 - z_k^0\right|^2 (p_k(T))^2 \int_0^T e^{2\beta_k s} ds\right)^{1/2}. \end{aligned} \tag{14}$$

Applying the obvious inequality $\left|e^{\beta_k T} z_k^1 - z_k^0\right|^2 \le 2\left|z_k^1\right|^2 e^{2\beta_k T} + 2\left|z_k^0\right|^2$, we obtain

$$\left(\int_0^T \|u(s)\|^2\, ds\right)^{1/2} \le \sigma + \left(\sum_{k=1}^{\infty}\left(2|z_k^0|^2 + 2|z_k^1|^2 e^{2\beta_k T}\right)(p_k(T))^2 \int_0^T e^{2\beta_k s} ds\right)^{1/2}$$
$$= \sigma + \left(\sum_{k=1}^{\infty}\left(|z_k^0|^2 p_k(T) + |z_k^1|^2 p_k^*(T)\right)\right)^{1/2}$$
$$= \sigma + \rho - \sigma = \rho.$$

Hence, the strategy (13) is admissible.

Proof that pursuit is completed: Using (12) and (13), in view of the previous cases, it is not difficult to show that

$$\delta_k(T) = z_k^0 + \int_0^T e^{\beta_k s} v_k(s) ds - \int_0^T e^{\beta_k s}\left(v_k(s) - \left(e^{\beta_k s}\left[z_k^1 e^{\beta_k T} - z_k^0\right] p_k(T)\right)\right) ds$$
$$= z_k^0 + \left[z_k^1 e^{\beta_k T} - z_k^0\right] p_k(T) \int_0^T e^{2\beta_k s} ds$$
$$= z_k^0 + \left[z_k^1 e^{\beta_k T} - z_k^0\right] = z_k^1 e^{\beta_k T}.$$

Indeed,

$$z_k(T) = e^{-\beta_k T}\delta_k(T) = z_k^1,\ k = 1, 2, ...$$

Hence, in differential game Q_2 pursuit can be completed. Theorem 4.1 is proved.

5. Conclusion

Pursuit game problem of an infinite system of 1st-order differential equations in l_2 space with geometric and Integral constraints has been considered. The player's control functions satisfied both the constraints.

In each case, we have solved a control problem of forwarding the state z^0 of (6) to z^1 for a finite time. Moreover, we give conditions of completion of pursuit for both the cases of constraints in differential game Q_1 and Q_2 respectively. In addition to this, the of the pursuer's strategy is explicitly constructed for both cases considered.

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