UNIVERSITI PUTRA MALAYSIA

FINITE ELEMENT AND DIFFERENTIAL QUADRATURE METHODS FOR HEAT DISTRIBUTION IN RECTANGULAR FINS

MD MOSLEMUDDIN FAKIR

FINITE ELEMENT AND DIFFERENTIAL QUADRATURE METHODS FOR HEAT DISTRIBUTION IN RECTANGULAR FINS

By

MD MOSLEMUDDIN FAKIR

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Partial Fulfillment of the Requirement for the Degree of Doctor of Philosophy

April 2009

DEDICATION

This Thesis is Dedicated to My

Parents

Children:

Nusrat Jahan Shoumy
Rifatul Bari

&

Wife:

Sabira Khatun

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy.

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FINITE ELEMENT AND DIFFERENTIAL QUADRATURE METHODS FOR HEAT DISTRIBUTION IN RECTANGULAR FINS

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April 2009

Chairman: Professor ShahNor Basri, PhD

Faculty: Engineering

Presently there are many numerical solution techniques such as finite element method (FEM), differential quadrature method (DQM), finite difference method (FDM), boundary element method (BEM), Raleigh-Ritz method (RRM), etc. These methods have their respective drawbacks. However, FEM and DQM are important techniques among those.

The conventional FEM (CFEM) provides flexibility to model complex geometries than FDM and conventional DQM (CDQM) do in spite some of its own drawbacks. It has been widely used in solving structural, mechanical, heat transfer, and fluid dynamics problems as well as problems of other disciplines. It has the characteristic that the solution must be calculated with a large number of mesh points (uniformly distributed) in order to obtain moderately accurate results at the points of interest. Consequently, both the computing time and storage required often prohibit the calculation. Therefore, focus is given to optimize the CFEM.

The Optimum FEM (OFEM) has been presented in this thesis to solve heat conduction problems in rectangular thin fins. This method is a simple and direct technique, which can be applied in a large number of cases to circumvent the computational time and complexity. The accuracy of the method depends mainly on the accuracy of the mesh generation (non-uniformly distributed) and stiffness matrix calculation, which is a key of the method. In this thesis, the algorithm for OFEM solution and the optimum mesh generation formula have been developed and presented. The technique has been illustrated with the solution of four heat conduction problems in fins for two types of mesh size distribution (uniformly distributed and non-uniformly distributed). The obtained OFEM results are of good accuracy with the exact solutions. It is also shown that the obtained OFEM results are at least 90% and 7% improved than those of similar published CFEM and ODQM results respectively. This method is a vital alternative to the conventional numerical methods, such as FDM, CFEM and DQM.

On the other hand, DQM is suitable for simple geometry and not suitable for practical large-scale problems or on complex geometries. DQM is used efficiently to solve various one-dimensional heat transfer problems. For two-dimensional case, this technique is so far used to solve Poisson's equation and some fluid flow problems but not the heat conduction problems in fins. Hence, in this thesis, a two-dimensional heat conduction problem in a thin rectangular fin is solved using DQM by means of the accurate discretization (for uniformly distributed (CDQM) and non-uniformly distributed (ODQM) mesh size.

DQM optimum discretization rule and mesh generation formula have been presented. The governing equations have been discretized according to DQM rule.

The technique has been illustrated with the solution of two two-dimentional heat conduction problems in fins. The obtained results show that the DQM results are of good accuracy with the FEM results. Optimum DQM (ODQM) shows better accuracy and stability than CDQM and CFEM. But in some cases, OFEM shows better efficiency than ODQM.

Abstrak tesis dipersembahkan kepada Senat Universiti Putra Malaysia sebagai memenuhi syarat keperluan untuk ijazah Doktor Falasafah.

KAEDAH UNSUR TERHINGGA DAN PEMBEZAAN KUADRATIK UNTUK PENGAGIHAN HABA DALAM SIRIP SEGIEMPAT TEPAT

Oleh

MD. MOSLEMUDDIN FAKIR

April 2009

Pengerusi: **Profesor ShahNor Basri, PhD**

Fakulti: **Kejuruteraan**

Kini terdapat banyak kaedah penyelesaian berangka iaitu seperti kaedah unsur terhingga (FEM), kaedah pembezaan kuadratik (DQM), kaedah pembezaan terhingga (FDM), kaedah unsur sempadan (BEM), kaedah Raleigh-Ritz (RRM), dll. Kesemua kaedah tersebut mempunyai kelemahan masing-masing. Namun demikian, FDM, FEM dan DQM adalah antara teknik-teknik yang penting.

Kaedah konvensional FEM (CFEM) memberi lebih kelonggaran dalam permodelan geometri yang komplek berbanding dengan FDM dan konvensional DQM (CDQM) walaupun ianya mempunyai kelemahan tersendiri. Ia telah digunakan dengan meluas dalam menyelesaikan masalah stuktur, mekanikal, pemindahan haba dan dinamik bendalir termasuk juga masalah dari disiplin yang lain. Ia mempunyai ciri dimana penyelesaiannya mesti dihitung dengan jumlah titik jaringan yang besar (diagihkan dengan seragam) bagi mendapatkan keputusan yang sederhana tepat pada titik yang dikehendaki. Dengan itu, kedua-dua masa pengiraan dan penyimpanan komputer yang diperlukan biasanya akan menghalang pengiraan. Oleh itu fokus akan diberikan kepada mengoptimumkan CFEM.

Pengoptimuman FEM (OFEM) telah dibentangkan dalam tesis ini untuk menyelesaikan masalah pengaliran haba dalam sirip nipis empat segi tepat. Kaedah ini mudah dan terus dimana ia boleh digunakan dalam kebanyakan kes untuk mengatasi tempoh dan kesukaran pengaturcaraan. Ketepatan kaedah ini terutamanya bergantung kepada ketepatan penjanaan jaringan (agihan tidak seragam) dan pengiraan matrik kekenyalan, dimana ia adalah kunci kepada kaedah ini. Dalam tesis ini, penyelesaian algoritma OFEM dan formula pengoptimuman penjanaan jaringan telah dibangunkan dan dibentangkan. Teknik ini telah diilustrasikan dengan menyelesaikan empat masalah pemindahan haba dalam sirip untuk dua jenis agihan saiz jaringan (agihan seragam dan agihan tidak seragam). Keputusan OFEM yang diperolehi mempunyai ketepatan yang baik berbanding dengan penyelesaian sebenar. Dapat ditunjukkan juga bahawa keputusan OFEM yang diperolehi telah dipertingkatkan sekurang-kurangnya 90% dan 7% berbanding dengan keputusan yang sama yang diterbitkan dengan CFEM dan DQM. Kaedah ini merupakan pilihan penting kepada kaedah berangka konvesional seperti FDM, CFEM dan DQM.

Namun demikian, DQM adalah sesuai untuk geometri mudah dan tidak sesuai untuk masalah praktikal berskala besar serta bergeometri kompleks. DQM digunakan dengan cekap untuk menyelesaikan pelbagai masalah pemindahan haba satu dimensi. Untuk kes dua dimensi, teknik ini masih digunakan untuk menyelesaikan persamaan Poisson dan sedikit masalah aliran bendalir tetapi tidak untuk masalah pengaliran haba dalam sirip. Oleh itu, dalam tesis ini, masalah dua dimensi pengaliran haba untuk sirip nipis empat segi tepat telah diselesaikan menggunakan DQM dengan cara pengagihan tepat (untuk agihan titik jaringan seragam dan agihan titik jaringan tidak seragam).

Hukum pendiskretan optimum DQM dan rumus penjanaan jaringan telah dibentangkan. Persamaan-persamaan menakluk telah didiskretkan mengikut hukum DQM. Teknik ini telah diilustrasikan dengan penyelesaian dua masalah dua-dimensi pengaliran haba dalam sirip. Keputusan yang diperolehi menunjukkan keputusan DQM mempunyai ketepatan yang baik dengan keputusan FEM. DQM optimum (ODQM) menunjukkan ketepatan dan kestabilan yang lebih baik dari CDQM dan CFEM. Tetapi untuk sesetengah kes, OFEM menunjukan kecekapan yang lebih baik berbanding ODQM.

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I certify that an Examination Committee met on 2nd of April 2008 to conduct the final examination of Md. Moslem Uddin Fakir on his Doctor of Philosophy Thesis entitled "Finite Element and Differential Quadrature Methods for Heat Distribution in Rectangular Fins" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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Date: 8 June 2009

DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

(MD. MOSLEMUDDIN FAKIR)

Date:

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