UNIVERSITI PUTRA MALAYSIA

CONVECTION BOUNDARY LAYER FLOWS OVER NEEDLES AND CYLINDERS IN VISCOUS FLUIDS

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CONVECTION BOUNDARY LAYER FLOWS OVER NEEDLES AND CYLINDERS IN VISCOUS FLUIDS

By

SYAKILA BINTI AHMAD

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

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To My Beloved Family and Friends.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

CONVECTION BOUNDARY LAYER FLOWS OVER NEEDLES AND CYLINDERS IN VISCOUS FLUIDS

By

SYAKILA BINTI AHMAD

May 2009

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Convection is the heat transfer process which is frequently encountered in environmental and engineering applications. In this study, the problems of steady laminar convection boundary layer flows over needles and cylinders immersed in an incompressible and viscous fluid are theoretically considered. The dimensional partial differential equations governing the boundary layer flows are first transformed into non-dimensional equations. These equations are then transformed using non-similar transformation. Then, these transformed nonlinear systems of equations are solved using an implicit finite difference scheme known as the Keller-box method, which has been found to be very suitable in dealing with nonlinear and parabolic equations. The complete numerical method used in this study is programmed in Fortran. Numerical computations are carried out for various values of the dimensionless parameters of the problems, which include the Prandtl number Pr, the ratio of the major and minor axes of the cylinder b_c/a_c, the mixed convection parameter λ, the modified mixed convection parameter $\hat{\lambda}$, the transverse curvature parameter Λ, the parameter a representing the needle size and the viscosity/temperature parameter θ_r. Numerical results

presented in this study are the skin friction coefficient, the heat transfer coefficient, the local Nusselt number, the cylinder temperature as well as the velocity and temperature profiles. The obtained results show that the flow and the thermal characteristics are significantly influenced by these parameters.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

ALIRAN LAPISAN SEMPADAN OLAKAN PADA JARUM DAN SILINDER DALAM BENDALIR LIKAT

Oleh

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Olakan adalah suatu proses pemindahan haba yang sering berlaku dalam persekitaran dan juga dalam kebanyakan aplikasi kejuruteraan. Dalam kajian ini, masalah aliran lapisan sempadan olakan mantap dan berlamina terhadap jarum dan silinder dalam bendalir likat dan tak termampatkan telah dipertimbangkan secara teori. Persamaan pembezaan separa berdimensi yang menakluk aliran lapisan sempadan terlebih dahulu dijelmakan kepada persamaan tak berdimensi. Seterusnya, persamaan tersebut akan dijelma menggunakan penjelmaan tak serupa. Sistem persamaan terjelma tak linear yang diperoleh diselesaikan secara berangka menggunakan skim beza terhingga tersirat iaitu kaedah kotak Keller yang merupakan satu kaedah yang sangat sesuai untuk menyelesaikan persamaan tak linear dan parabolik. Kaedah berangka yang digunakan dalam kajian ini telah dibangunkan dalam bentuk pengaturcaraan komputer dengan menggunakan Fortran. Pengiraan berangka dilakukan untuk pelbagai nilai parameter tak berdimensi seperti nombor Prandtl Pr, nisbah paksi major dan minor silinder b_c/a_c, parameter olakan campuran λ, parameter olakan campuran ubahan $\hat{\lambda}$, parameter kelengkungan melintang Λ, parameter a yang mewakili saiz jarum dan parameter

kelikatan/suhu θ_r. Keputusan berangka yang dipersembahkan dalam kajian ini adalah pekali geseran kulit, pekali pemindahan haba, nombor Nusselt setempat, suhu silinder beserta profil halaju dan suhu. Keputusan yang diperoleh menunjukkan bahawa ciri-ciri aliran dan terma adalah sangat dipengaruhi oleh parameter-parameter yang dipertimbangkan di atas.

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I certify that a Thesis Examination Committee has met on 27 May 2009 to conduct the final examination of Syakila binti Ahmad on her thesis entitled "Convection Boundary Layer Flows Over Needles and Cylinders in Viscous Fluids" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations, which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

SYAKILA BINTI AHMAD

Date: 10 July 2009

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LIST OF ABBREVIATIONS

a	dimensionless needle size
a_c	length of semi-major axis for a cylinder of elliptic cross section
a_{cc}	radius of the circular cylinder
b_c	length of semi-minor axis for a cylinder of elliptic cross section
C_f	skin friction coefficient
f	non-dimensional stream function
g	acceleration due to gravity
Gr	Grashof number
k	thermal conductivity of the fluid
m	power index
Nu	Nusselt number
Nu_x	local Nusselt number
Pr	Prandtl number
q_w	heat flux from the cylinder
$R(x)$	non-dimensional needle radius
Re	Reynolds number
Re_x	local Reynolds number
T	non-dimensional fluid temperature
T_r	reference temperature
T_w	needle or cylinder temperature
T_∞	ambient temperature
u, v	non-dimensional velocity components along the $x-$ and $y-$ directions, respectively, for a cylinder of elliptic cross section and a circular cylinder

u, v — non-dimensional velocity components along the $x-$ and $r-$ directions, respectively, for a thin needle and a slender cylinder

$u_e(x)$ — non-dimensional velocity outside boundary layer

U_∞ — free stream velocity

x, y — non-dimensional Cartesian coordinates along the surface of the cylinder and normal to it, respectively, for a cylinder of elliptic cross section and a circular cylinder

x, r — non-dimensional axial and radial coordinates, respectively, for a thin needle and a slender cylinder

x_s — boundary layer separation point

Greek symbols

α — thermal diffusivity

β — thermal expansion coefficient

δ_h — velocity boundary layer thickness

δ_T — thermal boundary layer thickness

ΔT — characteristic temperature

η — similarity variable

γ — thermal property of the fluid

θ — non-dimensional temperature

θ_r — viscosity/temperature parameter

λ — mixed convection parameter

$\hat{\lambda}$ — modified mixed convection parameter

Λ — transverse curvature parameter

ν — kinematic viscosity

ν_∞ — constant kinematic viscosity of the ambient fluid

μ	dynamic viscosity
μ_∞	constant dynamic viscosity of the ambient fluid
ξ	non-dimensional coordinate
ρ	fluid density
τ_w	wall shear stress
ψ	stream function
ζ	eccentric angle of a cylinder of elliptic cross section

Subscripts

c	refers to a cylinder of elliptic cross section
cc	refers to a circular cylinder
w	condition at the surface of the cylinder
∞	ambient/free stream condition

Superscripts

$'$	differentiation with respect to y
$\bar{}$	dimensional variables