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COMPLEX-VALUED NONLINEAR ADAPTIVE FILTERS FOR NONCIRCULAR SIGNALS

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By

AMADI CHUKWUEMENA CYPRIAN

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of Requirements for the Degree of Master of Science

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DEDICATION

This project is dedicated to Mr. & Mrs., Cyprian Amadi.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fullfilment of the requirement for the degree of Master of Science

COMPLEX-VALUED NONLINEAR ADAPTIVE FILTERS FOR NONCIRCULAR SIGNALS

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AMADI CHUKWUEMENA CYPRIAN

May 2017

Chairman : Fazirulhisyam Bin Hashim, PhD
Faculty : Engineering

Complex signal has been the backbone of large class of signals encountered in many modern applications as biomedical engineering, power system, radar, communication system, renewable energy and military technologies. However, statistical signal processing in complex domain are suited to only the conventional complex-valued signal processing technique for subset of complex signal known as circular (proper), which is inadequate for the generality of complex signals, as they do not rigorously exploit the statistical information available in the signal. This is because of the under-modelling of the underlying system or due to the inherent blindness of the algorithm (for example, the CNGD algorithm) to capture the full second-order statistical information available in the signal. With the limitation of the CNGD algorithm toward signal generality, an improved CNGD algorithm known as the ACNGD which is derived based on the concept of augmented complex statistic which gives optimal algorithm for the generality of signals in complex domain is introduced. The augmented CNGD has shown low Means Square Error (MSE) capabilities and have optimal performance than the conventional algorithm. To this end, a supervised complex adaptive algorithm convex combination complex nonlinear gradient descent (CC-CNGD) is developed to address the capabilities of processing the generality of complex signals (both circular and non-circular) and systems in either a noisy or a noise-free environment. Their importance in real-world application is showed through case studies. The CC-CNGD algorithm rigorously takes advantage of the fast convergence rate of the CNGD algorithm and as well exploit the low Means Square Error (MSE) of the ACNGD algorithm in order to circumvent the problem of slow convergence rate and high Mean Square Error (MSE) seen in the family of complex signal. The introduced approach is capable of facilitating real-time application, supported by numerous case studies, such as those in renewable energy. This class of algorithm performs well in either noisy or noise-free environments, the introduced approached has achieved a 20% better modelling. Fast convergence and low Mean

Square Error (MSE) performance over the conventional and existing methods in the literature review. A rigorous mathematic analysis for the understanding of the proposed algorithm is shown, with ranges of simulations on both synthetic and real-world data; support the approach taken in this thesis.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

TURAS ADAPTIF TAK LINEAR BERNILAI KOMPLEKS BAGI SIGNAL TAK MEMBULAT

Oleh

AMADI CHUKWUEMENA CYPRIAN

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Signal kompleks merupakan tulang belakang bagi kelas signal besar yang didapati dalam banyak aplikasi moden sebagai kejuruteraan biomedikal, sistem kuasa, radar, sistem komunikasi, tenaga boleh diperbaharui, dan teknologi ketenteraan. Walau bagaimanapun, pemprosesan signal secara statistik dalam domain kompleks telah disesuaikan kepada hanya teknik pemprosesan signal bernilai kompleks konvensional bagi subset signal kompleks dikenali sebagai bulat (wajar), yang tidak cukup bagi generaliti signal kompleks, disebabkan mereka secara tidak teliti mengeksploitasi maklumat statistikal yang terdapat dalam signal tersebut. Perkara tersebut akibat sistem pemodelan dasar bagi sistem mendasari atau disebabkan kelemahan algoritma inheren bagi mencapai maklumat statistikal susunan kedua penuh yang terdapat dalam signal. Pada akhir-akhir ini, algoritma adaptif kompleks yang diselia telah dibangunkan bagi menerangkan kapabiliti pemprosesan generaliti signal kompleks (kedua-duanya, membulat dan tidak membulat) dan sistem sama ada dalam persekitaran yang bising atau tanpa bising. Kepentingannya dalam aplikasi dunia sebenar dapat diperlihatkan melalui kajian kes.

Tesis ini memfokus pada penggunaan anjakan terkini dalam statistik terimbuh dan dalam pemodelan tidak linear yang telah melebihi jangkauan dan memperlihatkan limitasi pemprosesan signal kompleks secara statistik dan konvensional (standard). Melalui cara yang teliti, pengeksplotasian manfaat yang kenali sebagai statistik terimbuh, suatu kelas algoritma turasan adaptif dengan kerangka berpadu dan menggalakkan prestasi bagi generaliti signal kompleks, berbanding dengan teknik konvensional telah dicadangkan. Pendekatan yang dicadangkan berkebolehan untuk memudahkan aplikasi masa sebenar, disokong oleh pelbagai kajian kes, seperti yang terdapat pada tenaga yang boleh diperbaharui semula dan pemodelan trafik jaringan. Kelas algoritma tersebut dapat dilaksanakan dengan berkesan sama ada dalam

persekitaran yang bising ataupun persekitaran tidak bising, pendekatan yang dicadangkan telah memperoleh pemodelan 20% lebih baik, konvergens dan prestasi. Min Kuasa Ralat Dua (MSE) ke atas kaedah konvensional dan kini dalam kaji semula literatur. Analisis matematik yang teliti bagi pemahaman mengenai algoritma yang dicadangkan telah ditunjukkan,dengan julat simulasi ke atas kedua-dua data sintetik dan dunia sebenar; menyokong pendekatan yang diambil dalam tesis ini.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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Declaration by Members of Supervisory Committee

This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) were adhered to.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

ADALINE	Adaptive Linear
ACLMS	Augmented Complex Least Mean Square
CLMS	Complex Least Mean Square
BPSK	Binary Phase Shift Key
CCA	Canonical Correlation Analysis
CDMA	Code Division Multiple-Access
DS-CDMA	Direct Sequence Code Division Multiple-Access
DCRLMS	Dual Channel Real Least Mean Square
FFT	Fast Fourier Transform
GNGD	Generalised Normalised Gradient Descent
MSE	Mean Square Error
NMA (3)	Nonlinear Moving Average (3)
PDF	Probability Density Function
pPSD	Pseudo Power Spectral Density
PSD	Power Spectral Density
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying
RSL	Recursive Least Square
FCRTRL	Fully Complex Real Time Recurrent Learning Algorithm
TSE	Taylor Series Expansion
LMS	Least Mean Square
NLMS	Normalized Least Mean Square
NN	Neural Network
RNN	Recurrent Neural Network
BP	Backpropagation
CBP	Complex Backpropagation
CRTRL	Complex Real Time Recurrent Learning
RTRL	Real Time Recurrent Learning
FIR	Finite Impulse Response

NGD	Nonlinear Gradient Descent
CNGD	Complex Nonlinear Gradient Descent
ACNGD	Augmented Complex Nonlinear Gradient Descent
CR	Cauchy-Riemann
GCR	Generalized Cauchy Riemann
CRF	Cauchy-Riemann-Fueter
ETF	Elementary Transcendental Function
LAC	Local Analyticity Condition
WGN	White Gaussian Noise
WNMA (3)	Widely Nonlinear Moving Average (3)
IID	Independent Identical Distribution
KF	Kalman Filter
ASP	Adaptive Signal Processing
CD	Circularity Difference
CC-CNGD	Collaborative Combination Complex Nonlinear Gradient Descent

MATHEMATICAL NOTATION

$\vert\cdot\vert$	Modulus operator
$(\cdot)^*$	Complex conjugate operator
$(\cdot)^{-1}$	Matrix inverse operator
$(\cdot)^H$	Matrix pseudo-inverse operator
$(\cdot)^T$	Vector or matrix transpose operator
$(\cdot)^H$	Conjugate Transpose (Hermitian) operator
$\triangleq$	Defined as
∇	Gradient operator
∂	Partial derivative operator
0	Vector or matrix with all zero elements
$\mathbb{C}$	Field of complex numbers
C_{xx}	Covariance matrix of random vector x
C_{xx}^a	Augmented covariance matrix of random vector x
C_{xx}^R	Bivariate covariance matrix
$E\{\cdot\}$	Expectation operator
$E\{y\vert x\}$	Conditional expectation of y given x
$\mathcal{F}(\cdot)$	Fourier transform operator
g	Filter coefficient vector
h	Filter coefficient vector
i	$\sqrt{-1}$
I	Identity matrix
$\Im\{\cdot\}$	Imaginary part of a complex number
j	$\sqrt{-1}$
J_N	Real to Complex mapping matrix of size 2N × 2N
$\mathcal{J}(\cdot)$	Cost function
k	Discrete time index
$PX(x)$	Probability density of a random vector x
P_{xx}	Pseudo-covariance matrix of a random vector x

r	Degree of noncircularity
$\mathbb{R}$	Field of real numbers
$\Re(\cdot)$	Real part of a complex number
$x(k)$	Input vector at a discrete time k, observed mixture at a discrete time k
x	Complex random vector
x^a	Augmented complex random vector
x_r, x_i	Vector of real/imaginary parts of x
δ	Discrete time delay
δ_0	Delta function
λ	Mixing parameter of a hybrid filter
$\rho(x)$	Circularity quotient of random variable x
σ_x^2	Variance of a random variable x
τ_x^2	Pseudo-variance of a random variable x
Φ	Nonlinear activation function
$\boldsymbol{R}$	Correlation matrix
$\boldsymbol{P}$	Cross correlation matrix
$(\cdot)'$	First order derivative
μ	Learning rate
$\in$	Is an element of
$\approx$	Approximation
$\rightarrow$	Approaches

CHAPTER 1

INTRODUCTION

1.1 Background

We live in an information age, where computing technologies has vastly increase the availability of data set. This data set need the capability to robustly deal with it in achieving enhance performance of the system and evolving technology.

This is where signal processing plays an important role offering a mathematical framework for processing large data set acquired from image and audio processing to data compression and weather forecasting. Recently, the need to develop next generation signal processing solution, efficient enough to meet the challenging requirement of low-cost, fast and accurate data processing for more advanced data is in demand.

This thesis focuses on signal processing, more detailed on adaptive filters, compare to other technique employed in signal processing. The adaptive filters operates in real-time with their performance optimize on arrival at new signal sample. Application such as electrical smart-grids, navigational satellite, brain-computer-interface and wireless communication had made the adaptive filters ubiquitous due to their time-constrain.

One of the most common feature of adaptive signal processing is that the input signal and a desired signal are always used in order to get an output error signal, this error signal are then used to optimize the filter weight set. The input signal and output signal are configured in a certain way to allow the adaptive signal processing to be possible in different applications.

However, depending on the used configuration setting, adaptive signal processing has four possible adaptive application setting, these application settings are:

i. System Identification

This setting aims at finding a linear or nonlinear model that is best describe as an unknown system. This is achieved by configuring the adaptive filter such that, the unknown system works parallel with the adaptive filter at which the same signal is fed into the unknown system and the adaptive system.

ii. **Future Sample Prediction**

This configuration aim at predicting future signal sample, the input signal is always the delayed version of the desired signal. However, this type of model is widely used in wind prediction and speech processing application.

iii. Inverse Modeling

The inverse modelling aim at obtaining the inverse of an unknown noisy system. This is mostly used in channel equalization [1], where it is applied to reduce channel distortion in modem as a result of data transmitted via telephone channel. The unknown system and adaptive filters are configured in series.

iv. Noise Cancellation

This class of adaptive filter aim at subtracting an unknown noise source from a primary (main) signal (e.g. electrocardiography [1, 2]). The desired signal is configured to correlate with the interference/noise signal responsible for corrupting/damaging the primary (main) signal.

With the recent rapid development of adaptive filters which can be dated back to 1950s, since the first merging of the Recursive Least Square (RLS), attributed to placket [3], many authors since then has derived variant RLS. However, in 1959, Widrow and Holf [4], while studying the adaptive pattern classification machine known as adaptive linear (Adaline), developed the Least Mean Square (LMS), using the stochastic gradient instead of using the least square solution in [3]. The LMS has since been the workhorse of adaptive signal processing due to its simplicity.

1.2 Signal Processing $\mathbb{R}$

In adaptive signal processing, supervised and unsupervised (blind) are the two distinctive categories under the algorithms. In supervised algorithms, the signals are trained, resulting in straight forward technique for adaptive filtering. However, in unsupervised (blind) algorithms, the output is processed without the knowledge of the system. Since the system is unknown, this scenario makes it more challenging, where certain assumption on the input signal or system is required.

Supervised adaptive algorithm had been extensively studied in real domain $\mathbb{R}$ based on Wiener and Kalman filters. The LMS algorithm in [4] is the most used and well-known practice of supervised adaptive algorithm in $\mathbb{R}$, with much noticeable research effort being put into enhancing the performance and analyzing of the LMS. This includes "the class of variable step-size LMS algorithms" proposed by Benvensite, et.al [5] which adopts to the LMS step-size in a linear fashion, making it more suitable for non-stationary and time varying conditions. While "Generalized Normalized Gradient Descent (GNGD) algorithm [6, 7, 8] adapt the learning rate in a nonlinear fashion. The GNGD is based on Normalized LMS (NLMS). This avoids spurious solution due to tiny signal magnitudes by adapting the regularized parameter. With both algorithms based on LMS with an adaptive step-size, the GNGD algorithm has shown to be more powerful in performance and improved stability due to it nonlinear step-size update [6].

With this in mind, this research aim at developing a novel theoretical framework to enhanced the practical solution for adaptive signal processing of complex-valued signals, with real and imaginary components.

1.3 Signal Processing in $\mathbb{C}$

Signal encountered in complex domain $\mathbb{C}$ can be broadly categorize into two groups, those that are made complex by convenience of representation and those complex by design. Example of signal considered to be complex by design are found in communication field, sonar and radar. While complex wind signal are represented by convenience of representation when the speed and direction are combine into complex vector.

Widrow et.al extended the LMS algorithm [4] to the complex domain in 1975 [10]. Therefore making two-dimensional signal processing using the complex LMS (CLMS) algorithm possible. Complex algorithm such as the Widrow's complex LMS was derived by extending already existing algorithm that reside in the real domain to complex domain, where the transpose operator represented as $(\cdot)^T$ becomes $(\cdot)^H$ Hermitical operator in complex domain. This causes the covariance matrix $E[xx^T]$ in $\mathbb{R}$ to be transmitted to $E[zz^H]$ in $\mathbb{C}$ which is considered necessary.

The implicit assumption of using $E[zz^H]$ to describe the second order statistic of the vector is that the distribution of the vector is circular, this assumption implies the independence of the real and imaginary signal component. However, this assumption is not correct for majority of complex-valued signal. Using this approach would not be optimal for generality of complex-valued signals derived on any complex-valued algorithm.

Due to lack of rigorous mathematical standard for describing the complex gradient, derivatives and statistics, the pace for developing a complex-valued adaptive filtering algorithms was slow. However, Brandwood [12] remedy this challenge in 1983, by introducing the Cauchy-Riemann Calculus, which was develop by Wirtinger [13] in 1927, translated in English [14] in 1992. The $\mathbb{CR}$-Calculus was exploited by German Speaking Scientific Community and was not popularly known in the English Speaking Scientific Community until the translation of the $\mathbb{CR}$-Calculus, which allows the treatment of functions of complex variable directly in complex domain and permits the consideration of both analytic and non-analytic function in a unified manner, this simplifies the differentiation and analysis of complex functions. Brandwood also developed the complex gradient known as conjugate gradient with necessary condition for a point to be stationary, which was the problem of minimizing a real function of complex variables. Then making these result a very important approach for developing a complex-valued adaptive filtering algorithms.

Furthermore, with the breakthrough in complex statistic in 1900 by Massey and Neeser [15] which address the concept of complex random variable circularity. The widely linear model (augmented) and second-order complex random variable statistic. A complex-valued random variable is consider circular if it has a rotation invariant distribution. If it does not, it is otherwise known as noncircular [16]. The second order statistic of a complex-valued random vector is showed in [17] and [18]. Depicting that the covariance matrix is incapable of modeling the full statistic and pseudo-covariance matric is needed to fully capture the real and imaginary component of the vector. Hence, both the covariance and pseudo-covariance are important to model/capture the full second order information available within the signal. Massey and Neeser also showed that only circular (or proper) signals has vanishing pseudo-covariance, which coincide with the assumption of traditional algorithm in $\mathbb{C}$. However, for noncircular (improper) signals, the pseudo-covariance matrix is non-zero.

From this understanding, the augmented statics is introduced in [17] which incorporates the information of both covariance and pseudo-covariance. This was followed by Pincinbono [18, 19] who depicts on taking advantage of the full second order statistics of complex signals that the complex conjugate must be included to form the augmented linear model given by,

$$y = hx + gx \tag{1.1}$$

where y, h, g and x are the outputs of the augmented linear model, coefficient and input signal respectively.

Adaptive filtering algorithm suitable for processing both proper and improper signals were made possible, based on the work carried out on complex statistic, gradients and widely linear models in the 1980s and early 1990s.

Unlike linear adaptive filters, the nonlinear adaptive filtering algorithm faced a serious challenge in finding the best and suitable analytic complex-valued nonlinear activation functions. This is as a result of the Liouville theorem, which states that a bounded entire function must be constant in $\mathbb{C}$, this limit the scope of the nonlinear activation function that were once suitable in $\mathbb{R}$, to solve this direct consequences, Kim and Adali proved a class of complex Element Transcendental Functions (ETFs) based on the entire adaptive filtering application [20].

The ETFs depict to satisfy the Cauchy-Riemann ($\mathbb{CR}$) criteria's condition proving to be analytic in $\mathbb{C}$, which was implemented in Fully Complex real Time Recurrent Learning algorithm (FCRTRL) [21]. The FCRTRL takes advantage of the correlation between the real and imaginary parts resulting in an improved performance of the algorithm. Hence, the introduction of the (ETFs) has pathways for more nonlinear adaptive filtering algorithm which will be discussed in detail in the next chapter.

1.4 Problem Statement

In the past five decades, adaptive signal processing has been the center for statistic signal processing, attracting more researchers, due to increase in demand for high power digital processing processors, with low power consumption rate and costing. This demand has placed more investigation for more complex computational and ambitious problems. However, in adaptive filtering and change detection problems, linear models affected by Gaussian noise are usually used to solve the challenge, noise approximation and linearization are often used when not the challenge of estimation and detection to create approximate Gaussian and linear system. The performance effect of this technique to nonlinear system has a low attention (sometimes ignored) [11]. Although there are existing techniques to process non-Gaussian and nonlinear system such as statistical signal processing and others. The computational complexity of the algorithm itself suffers from slow convergence and high MSE, which is the same problem encountered by the family of complex-valued algorithms [22].

However, statistical signal processing in complex domain are suited to only the conventional complex-valued signal processing technique for subset of complex signal known as circular (proper), which is inadequate for the generality of complex signals, as they do not rigorously exploit the statistical information available in the signal. This is because of the under-modelling of the underlying system or due to the inherent blindness of the algorithm (for example, the CNGD algorithm) to capture the full second-order statistical information available in the signal, the CNGD algorithm is equipped with fast convergence rate and high MSE capabilities. With the limitation of the CNGD algorithm toward signal generality. An improved CNGD algorithm known as the ACNGD which is derived based on the concept of augmented complex statistic which gives optimal algorithm for the generality of signals in complex domain is introduced. The augmented CNGD has shown low Means Square Error (MSE) capabilities with slow convergence rate and have optimal performance than the conventional algorithm.

To this end, an approach in the field of real-valued nonlinear adaptive filtering will be explored by evaluating the performance of linear algorithm for the modeling of nonlinear systems. The linear algorithm will be extended to nonlinear algorithm for the evaluating performance of the nonlinear system.

An approach to combine the nonlinear algorithm is proposed, the proposed algorithm will benefit from the fast convergence of one of the individual algorithm and as well the low Means Square Error (MSE) of the other algorithm.

1.5 Aim and Objectives

The main aim of this thesis is to design a nonlinear adaptive filter with less computational complexity.

The objective of this thesis introduces contribution to supervised adaptive signal processing of noncircular signals:

- Evaluating the performance of linear algorithm for the modeling of nonlinear systems
- The linear algorithm will be extended to nonlinear algorithm for the evaluating performance of the nonlinear system
- Develop algorithm that will benefit from the fast convergence of one of the individual algorithm and as well the low Means Square Error (MSE) of the other algorithm

1.6 Research Activities Flowchart

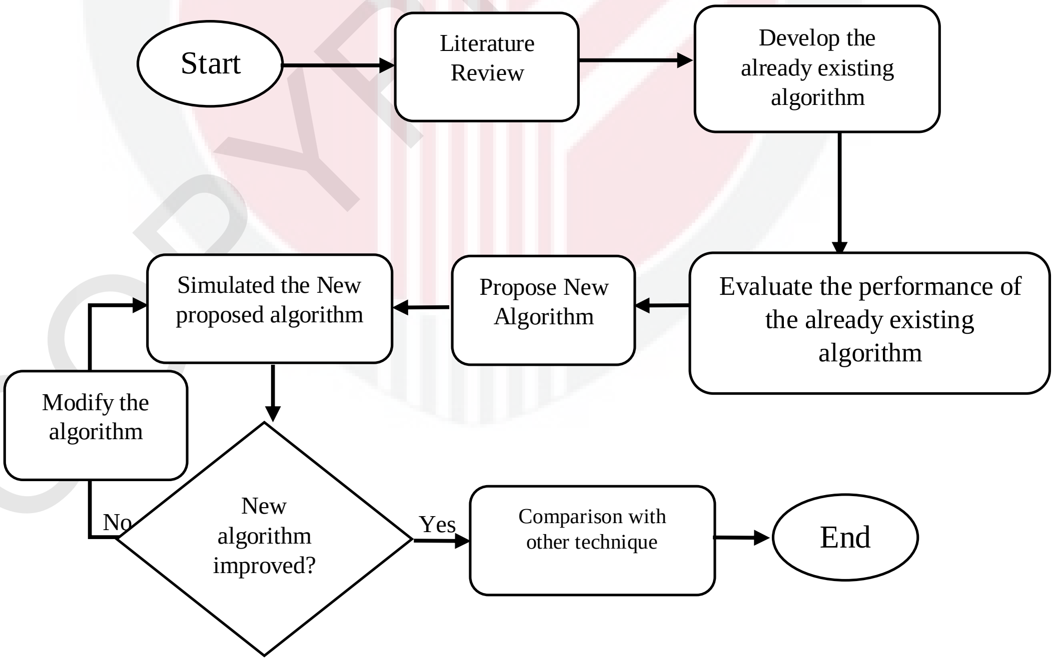

Figure 1.1 : Research activities flowchart

1.7 Research Scope

The research approached employed to successfully complete this thesis is shown in Figure 1.2. The solid line represent the followed direction to implement and achieve the goals of this research while the dot dashed line denotes other research area related to this work.

In this thesis, the adaptive signal processing is categorized into two, namely supervised and unsupervised. The unsupervised signal processing refers to blind signal processing where the system is unknown, whereas supervised signal processing refers to trained signal processing where the system is known. This thesis adopts supervised signal processing with algorithm such as Least Mean Square (LMS) and Nonlinear Gradient Descent as the workforces of signal processing. Type of signals domain processed by this algorithm are real and complex domain. Since this thesis is about complex signal processing, the complex domain direction is adopted under nonlinear signals processing used in applications such as system identification and modelling.

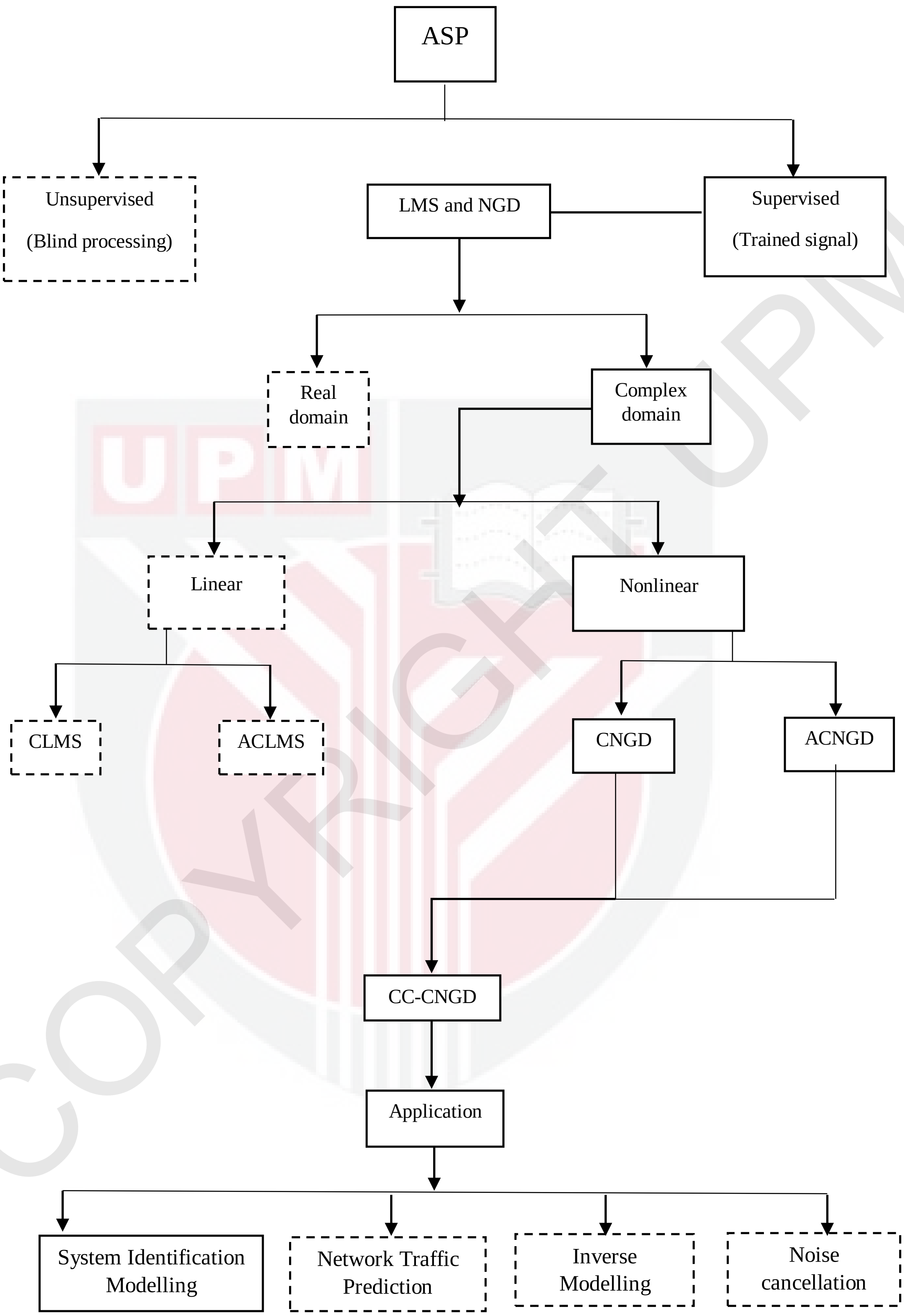

Figure 1.2 : Study module

1.8 Organization of Thesis

This thesis is organized as follows:

Chapter 2

Introduces the theoretical background and fundamental concept used in the development of the work presented in this thesis, this include background theory in respect to complex signals using duality of both real and complex domain. Next, the measurement of circular models with comparison to the standard linear and nonlinear model are discussed.

Chapter 3

Deals with the methodology approach taken towards the proposed algorithm, detailing the mathematical framework of the proposed algorithm.

Chapter 4

Introduces the nonlinear system modeling utilizing second order augmented statistic complex-valued algorithm as well with the collaborative adaptive filtering approach for the identification of complex-valued improper signals. This chapter will show the result and discussion of the thesis.

Chapter 5

This thesis is concluded in this chapter, where the overall conclusion is drawn and suggestion for future work is recommended.

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