UNIVERSITI PUTRA MALAYSIA

WIDELY LINEAR DYNAMIC QUATERNION VALUED LEAST MEAN SQUARE ALGORITHM FOR LINEAR FILTERING

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By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

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The Recent developments in sensor technology; human centered computing and robotics have brought to light new classes of multidimensional data which are naturally represented as quaternion three and four-dimensional vector-valued processes. Such signals are readily modeled as real normal vectors in R3 and R4; however; it has become obvious that there are advantages in processing multidimensional data in division algebras (quaternion domain). The progress in the statistics of quaternion variable, particularly augmented statistics and widely linear modeling; has opened up a new front of research in channel equalization, vector sensor modeling and system identification. However, prediction gain, tracking ability and convergence speed of quaternion adaptive filters still need to be improved due to the fixed step size of those types of algorithms. Choosing the right value of step size is very important for the adaptation process of the algorithm. There is a tradeoff between the convergence speed and the missadjustment of the system. Using large step size value will produce high convergence speed and high missadjustment while using small step size value will produce slow convergence speed and low missadjustment. since in real scenario the input signal power does not remain constant, that will change the step-size according the changes of the input signal of the algorithm which increase the tradeoff between the convergence speed and the missadjustment. This changing will cause noise amplification and affects the convergence speed. In this thesis, a new quaternion gradient based adaptive algorithm for FIR adaptive filter is developed. The proposed algorithm is capable of processing the generality of quaternion and complex data signals in both noisy and noise-free environments. The new adaptive algorithm is called dynamic quaternion least mean square algorithm (DQLMS) because of the normalization process of the filter input and the variable step-size. Those techniques proved to be very useful to enhance the trade-off between the convergence speed and the steady-state MSE and achieve small misadjustment and fast convergence speed. The sign function has been implemented in the process of filter coefficients adjustments in order to get faster adaptation processes, for high speed communication. The DQLMS algorithm is extended to the widely linear model forming the WL-DQLMS algorithm in order for the algorithm to be able to capture the full second order statistics. Prediction gain, tracking ability and convergence speed of the proposed algorithms are analyzed and validated experimentally by various simulations on

both synthetic and real world multidimensional data. The performance of the proposed algorithms are compared with quaternion least mean square QLMS, zero-attract quaternion least mean square ZA-QLMS, and widely linear quaternion least mean square WL-QLMS algorithms. In noise cancellation, the DQLMS and WL-DQLMS algorithms were able to recover the input signal in 30 and 50 samples respectively while the QLMS and ZA-QLMS needed 250 and 200 samples respectively in order to recover the same data. A superior performance is achieved by the proposed algorithms in system modeling where the DQLMS was able to track the correct weights values of the different modeled systems 430 sample faster than the QLMS and ZA-QLMS algorithms while the WL-DQLMS was faster than the WLQLMS algorithm by 950 samples. In prediction setting the proposed algorithms showed 4dp to 8dp higher prediction gain than other algorithms. Thus, the proposed algorithms proved to be superior over the other algorithms in all aspects.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

ALGORITMA KUASA DUA MIN TERKECIL DINILAI OLEH KUATERNION DINAMIK LINEAR MELUAS UNTUK PENAPISAN LINEAR

Oleh

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Perkembangan Perkembangan baru di bidang teknologi sensor, pengkomputeran berpusatkan manusia dan robotik telah menunjukkan kelas-kelas baru data multidimensi yang secara semula jadinya diwakili sebagai proses dinilai-vektor kuaternion tiga- dan empat-dimensi. Isyarat sebegitu mudah dimodelkan sebagai vektor normal sebenar di R3 dan R4; walau bagaimanapun telah menjadi jelas bahawa ada terdapat kelebihan dalam pemprosesan data multidimensi melalui algebra pembahagian (domain kuaternion). Kemajuan dalam statistik pembolehubah kuaternion, terutamanya statistik diperkukuhkan dan pemodelan linear secara meluas, telah membuka satu barisan baru penyelidikan dalam penyamaan saluran, pemodelan sensor vektor dan pengenalan sistem. Walau bagaimanapun gandaan ramalan, keupayaan mengesan dan kelajuan penumpuan penapis penyesuaian kuaternion masih perlu diperbaiki. Di dalam tesis ini, suatu algoritma penyesuaian kuaternion baru berdasarkan kecerunan untuk penapis penyesuaian FIR yang mampu memproses sifat umum isyarat data kuaternion dan rumit di dalam kedua-duanya persekitaran berhingar dan tanpa-hingar dibangunkan. Algoritma penyesuaian baru itu dipanggil algoritma kuaternion dinamik kuasa dua min terkecil (DQLMS) kerana proses penormalan input penapis dan saiz-langkah yang berubah-ubah. Teknik-teknik tersebut terbukti sangat berguna untuk meningkatkan keseimbangan antara kelajuan penumpuan dan MSE keadaan-mantap dan mencapai salah larasan yang kecil serta kelajuan penumpuan yang cepat. Fungsi tanda telah dilaksanakan di dalam proses pelarasan pekali penapis untuk mendapatkan proses penyesuaian yang lebih cepat, untuk komunikasi berkelajuan tinggi. Algoritma DQLMS dilanjutkan kepada model linear secara meluas yang membentuk algoritma WL-DQLMS agar algoritma tersebut dapat menangkap statistik order kedua yang penuh. Gandaan ramalan, keupayaan mengesan dan kelajuan penumpuan algoritma yang dicadangkan dianalisis dan disahkan secara eksperimen oleh pelbagai simulasi ke atas kedua-duanya data multidimensi dunia nyata dan sintetik. Prestasi algoritma yang dicadangkan dibandingkan dengan algoritma-algoritma kuasa dua min terkecil kuaternion QLMS, kuasa dua min terkecil kuaternion tarikan-sifar ZA-QLMS, dan kuasa dua min terkecil linear meluas kuaternion WL-QLMS. Dari segi penghapusan hingar algoritma-algoritma DQLMS dan WL-DQLMS berupaya untuk mendapatkan semula isyarat input

bagi 30 dan 50 sampel masing-masing manakala QLMS dan ZA-QLMS masing-masing memerlukan 250 dan 200 sampel untuk mendapatkan semula data yang sama. Prestasi yang lebih baik dicapai oleh algoritma yang dicadangkan dari segi pemodelan sistem di mana DQLMS dapat mengesan nilai pemberat yang betul bagi sistem berbeza yang dimodelkan dengan 430 sampel lebih cepat daripada algoritma QLMS dan ZA-QLMS manakala WL-DQLMS adalah lebih cepat daripada algoritma WLQLMS dengan 950 sampel. Dari segi menetapkan ramalan algoritma yang dicadangkan menunjukkan gandaan ramalan 4dp hingga 8dp lebih tinggi daripada algoritma lain. Oleh itu algoritma yang dicadangkan terbukti lebih baik berbanding dengan algoritma lain dari segala aspek.

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Declaration by Members of Supervisory Committee

This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) are adhered to.

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LIST OF ABBREVIATIONS

R	Real domain
R3	Three-dimensional real domain
R4	Four-dimensional real domain
APA	Affine projection algorithm
AQLMS	Augmented Complex Least Mean Square algorithm
CLMS	Complex Least Mean Square algorithm
CR	Cauchy-Riemann
FIR	Finite Impulse Response
FX-LMS	Filtered Least Mean Square algorithm
IIR	Infinite Impulse Response
KF	Kalman Filter
LMS	Least Mean Square algorithm
LPC	Linear predictive coding
MA (3)	Moving Average Third Order
MLMS	Multichannel Least Mean Square algorithm
MSE	Mean Square Error
NCLMS	Normalized Complex Least Mean Square algorithm
NLMS	Normalized Least Mean Square algorithm
PLMS	Proportionate Least Mean Square algorithm
PNLMS	Normalized Proportionate LMS
QLMS	Quaternion Least Mean Square algorithm
QMA (3)	Quaternion Moving Average Third Order
RP	Prediction Gain
RPVS-LMS	Ramadan and Poularikas LMS
VS-LMS	Variable Step Size Least Mean Square algorithm
WL	Widely Linear

WLCLMS	Widely Linear Complex Least Mean Square algorithm
WL-QLMS	Widely Linear Quaternion LMS
WLQMA (3)	Widely Linear Quaternion Third Moving Average
YWVS-LMS	Yue Wang Least Mean Square algorithm
ZA-QLMS	Zero Attract Quaternion Least Mean Square algorithm

LIST OF NOTATIONS

R	Real field
C	Complex field
H	Quaternion field
Rn	Vector field
$[\cdot]^T$	Transpose operation
$[\cdot]$	H Hermitian operation
$[\cdot]^*$	Conjugate operation
$[\cdot]^{j,j,k}$	j,jand k involution
Rp	Prediction gain
E	White gaussian noise
ϕ	Locally Analytic Quaternion function
μ	Learning rate
R	Correlation matrix
P	Cross correlation matrix
r	Gradient
$Q^{a,b,c,d}$	Real, i, j, k part of the quaternion vector
R	Real part of the variable
I	Imaginary parts of the variable
C_{QQ}	Covariance matrix
P_{QQ}	Pseudocovariance matrix
C_{Qi}	ı-covariance matrix
CQ_j	j-covariance matrix
CQ_k	k-covariance matrix

CHAPTER 1

INTRODUCTION

1.1 General Background

For the last five decades, the adaptive filters have stand out and attracted the attention of many researchers owing to their characteristic of self-designing. Different filters have been elaborated and applied in order to meet the demand for better tracking and faster convergence properties than earlier methods could offer.

An optimal linear filter for a specific application can be designed in advance when prior information about the statistics of the signal is available such as the Wiener filter which has the ability to minimize the mean squared error (MSE) between the desired signal and the output of the filter. When the prior information is unavailable, the solution is to use adaptive filters which have the ability to alter their coefficients according to the statistics of the signals involved in a process known as the weight update. As a result, the adaptive filters and algorithms have been successfully adopted and implemented in a wide variety of devices for various application fields such as biomedical engineering, control, radar, and communications.

The adaptive filters began practically with the efforts of research and development in the late fifties of the 20th century, while the field of adaptive signal processing has been established as a different discipline in its own right in the 1980's. There are two basic operations involved in adaptive filtering process; the filtering process followed by the adaptation process. An output signal is generated by the filtering process from an input signal data using a digital filter, while in the adaptation process an algorithm handles the weight update process, the adjustments of the coefficients of the filter in order to minimize the desired cost function. The above mentioned capability of adaptive filters has attracted many researchers to this field.

In adaptive filtering, there is a large variety of filter structures and algorithms used, each of them is more suitable for a specific application. The adaptive filters can be classified into two main categories; the infinite impulse response (IIR) and the finite impulse response (FIR) filters. In IIR filters, the existence of the internal feedback makes the impulse response in the system does not settle to zero while the impulse response of the FIR filters is of finite time duration, thus settles to zero after some finite duration of time.

Moreover, in the class of FIR filters, there are three different filter structures, namely: the transversal filter, the lattice predictor and the systolic array [1]. There are other FIR structures such as sub-band FIR adaptive filters and frequency-domain adaptive filters.

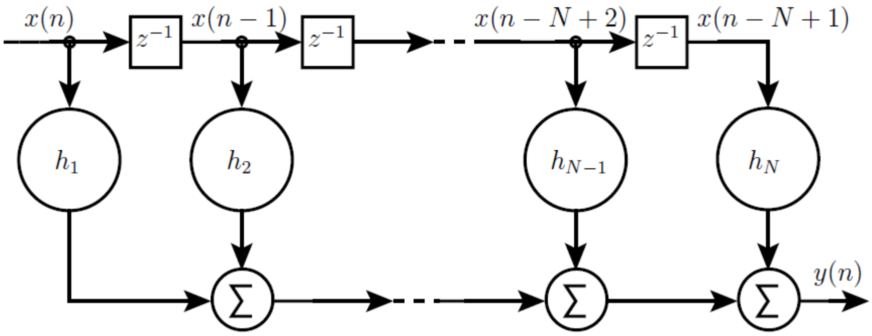

Figure 1.1: The block diagram of the transversal filter [1]

The structure of transversal adaptive filter is illustrated in Figure 1.1, where x(n) is the input of the adaptive filter, h_N is the coefficients of the filter, and y(n) is the filter output. Many adaptive algorithms can be addressed to the transversal adaptive FIR filters due to the large number of adaptive filtering algorithms, some algorithms may be preferred over the others. This preferability is due to some performance criteria. These criteria may include the following:

1. Rate of convergence: This can be defined as the time required by an algorithm to reach near optimum Wiener solution in the mean and mean-square sense. Usually, a fast convergence rate is preferable.

2. Tracking ability: It is the ability of an adaptive filtering algorithm to track (change in response to) the statistical variations in a stationary or a non-stationary environment.
3. Misadjustment: This is a quantitative parameter that measures the difference between the final value of the mean-square-error (MSE) reached by the algorithm and the MSE produced by Wiener filter.

4. Structure: This is concerned with the hardware implementation. Structure means the flow of information in the algorithm which determines the nature in which the algorithm was implemented.

Due to its simplicity and robustness, the least mean square (LMS) algorithm has been at the core of adaptive filtering applications [3], [4], and its online adaptive mode of operation makes it suited for the processing of non-stationary real world signals. In many fields, the simultaneous processing of the two dimensions of a signal (radar, sonar) can lead to a more efficient signal processing algorithm than processing each dimension separately. As the quaternion domain represents an extension of the complex field, the class of LMS algorithms has been extended to quaternion least mean square (QLMS) to cater for adaptive filtering of three- and four- dimensional (hyper-complex) signals. Quaternions have been used for more than 150 years (conceived by Hamilton in 1843) and have found applications in computer graphics, for the modeling of three-dimensional (3-D) rotations [5], in robotics [6], and molecular modeling [7]. Although the quaternion least mean square algorithm has addressed many slandered least mean square algorithm problems but it is necessary to further improve the performance of QLMS algorithm for the processing of quaternion valued signals in different aspects.

1.2 Problem Statement

Adaptive filters play an important role in the fields related to digital signal processing and communication, such as system identification, wind prediction, noise cancellation, channel equalization, and beamforming. In practical applications, the stability of an adaptive filter is an important consideration. The four channels LMS and QLMS algorithms are widely used due to their stability. However, it is well known that the performance of LMS and QLMS still need to be improved in terms of prediction gain, tracking ability and convergence speed especially for non-stationary input signals.

Adaptive filtering is applied commonly to prediction because of its ability to track and converge upon the stochastic characteristics of a signal. The performance of a predictor is measured using a quantity known as prediction gain. In prediction, a filter is used to estimate future values of a signal from prior observations. However, the convergence speed of the QLMS and widely linear quaternion least mean square (WLQLMS) are slowed by input signals with high non-stationary nature.

The tracking ability of an adaptive algorithm is the ability to track (change in response to) the statistical variations in a stationary or a non-stationary environment. The tracking performance of the algorithm is influenced by convergence rate and steady-state fluctuation contradictory features.

The convergence rate is the number of iterations required for the algorithm to converge to the steady state solution. A fast convergence rate allows the algorithm to adapt rapidly to a non-stationary environment of unknown statistics. In QLMS and WLQLMS algorithms choosing the right value of step size is very important for the adaptation process of the algorithm. There is a tradeoff between the convergence speed and the missadjustment of the system. Using large step size value will produce high convergence speed and high missadjustment while using small step size value will produce slow convergence speed and low missadjustment. since in real scenario the input signal power does not remain constant, that will change the step-size according the changes of the input signal of the algorithm which increase the tradeoff between the convergence speed and the missadjustment. This changing will cause noise amplification and affects the convergence speed.

1.3 Objectives

The primary objectives of this research are:

1- To develop quaternion valued gradient based hyper-complex algorithm with variable step size. The new algorithm should be tolerant to three and four-dimensional reference signal while having better prediction gain, tracking ability and convergence speed than the QLMS and ZAQLMS.

2- To extend the new algorithm to the widely linear model in order to account rigorously for the second-order statistics of the quaternion system.

3- To compare the performance of the proposed algorithm with existing algorithms under non-stationary environment. Evaluating the performance of the proposed algorithm operating under various conditions will be conducted using a Matlab baseband simulation platform.

1.4 Scope of the Thesis

To increase the efficiency of gradient based adaptive filters for the modeling of three and four dimensional synthetic and real-world signals, many researchers introduced techniques such as multichannel adaptive filters, multiple univariate LMS, a pair of complex LMS (CLMS) and QLMS adaptive filters. Some signals exhibit nonlinear complex dynamics, together with the coupling between their components such as 3D Lorenz attractor, Saito, and 4D wind data which makes the quaternion domain is the optimal solution due to its unique properties. An efficient quaternion based hyper-complex algorithm is introduced for the modeling of three and four dimensional synthetic and real-world data. The new designed algorithms have been tested in system modeling and system identification modes and have been compared with some of the early designed available systems. All the algorithms are fed with four dimensional Saito synthetic signal and three and four real world wind data as an input.

1.5 Study Module

The summary of chosen approach in this thesis is illustrated in Figure 1.2, where the gray colored boxes with solid lines refer to the followed direction to reach desired goals and the white boxes with dotted lines show the very close research areas which were beneficial in a way or another to the developing process of the proposed algorithm.

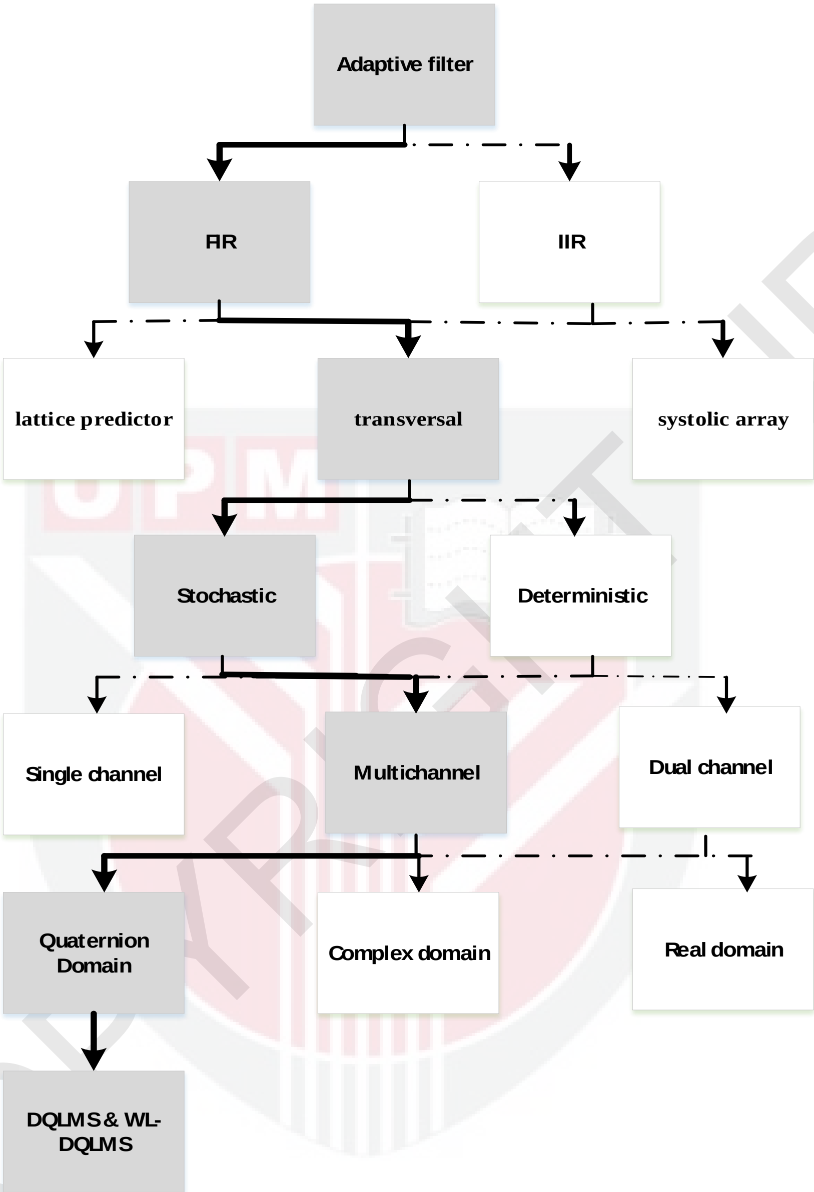

Figure 1.2: Study Module

1.6 Thesis Organization

The thesis is organized into five chapters as follows: Chapter 1 provides a brief introduction to adaptive filters and their applications and issues in the modeling of three and four-dimensional signals and the required improvements to the currently available gradient based adaptive filters in term of prediction gain, tracking ability and convergence speed. Problem statement, objectives, and scope of the thesis are highlighted as well.

Chapter 2 presents literature review on the important types of adaptive filters focusing on families of the LMS and FIR algorithms.

Chapter 3 considers the main body of this thesis where it provides the basics of quaternion algebra and introduces the concept of quaternion augmented statistics. This was followed by introducing the new algorithm (DQLMS) and its widely linear extension WLDQLMS.

Chapter 4 delivers the obtained results from the simulation model for the proposed algorithm and the results were delineated using respective diagrams and graphs. The results of the proposed algorithms are compared with two of the most stable algorithms in the field.

Chapter 5 provides the conclusion and thesis contribution in addition to some recommendation and suggestions for future works.

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