UNIVERSITI PUTRA MALAYSIA

SIMULATION OF CRACKS PROPAGATION OF REINFORCED CONCRETE BEAM-TO-COLUMN JOINT WITH FRP STRENGTHENING IN FLEXURAL AND SHEAR REGION

SHAHRIAR SHAHBAZPANAHI

FK 2015 127

SIMULATION OF CRACKS PROPAGATION OF REINFORCED CONCRETE BEAM-TO-COLUMN JOINT WITH FRP STRENGTHENING IN FLEXURAL AND SHEAR REGION

By

SHAHRIAR SHAHBAZPANAHI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

November 2015

DEDICATION

This research is dedicated to my lovely wife, Alaleh. I also want to dedicate this thesis to my father, mother and brothers for their supports.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

SIMULATION OF CRACKS PROPAGATION OF REINFORCED CONCRETE BEAM-TO-COLUMN JOINT WITH FRP STRENGTHENING IN FLEXURAL AND SHEAR REGION

By

SHAHRIAR SHAHBAZPANAHI

November 2015

Chairman : Professor Abang Abdullah Abang Ali, PhD
Faculty : Engineering

Nowadays, rehabilitation of structural members is a challenging issue for structural engineers, and much effort has been made to predict crack propagation in structural members. In the present study, a stiffness matrix is formulated for the Fracture Process Zone (FPZ). Based on the derived formulation, a new element was developed in order to model crack propagation using finite element analysis. Size effects such as depth, thickness of the beam and effective crack length were considered in the calculation of FPZ length and crack extension. Based on the new element, the Griffith differential energy method was developed to predict the crack propagation criterion with high accuracy.

Therefore, in the present investigation a numerical model was developed to model crack propagation in concrete beams flexural or shear strengthened with FRP. To validate the present model, experimental testing on reinforced concrete beams and beam-column joints with and without Fiber Reinforced Polymers (FRPs) strengthening were carried out. Three beam specimens with rectangular cross-section were tested. Two beams were strengthened with externally bonded FRP sheets for flexure or shear strengthening and one control beam were considered. One beam was externally bonded with FRP sheet at the bottom of the beam and another one was bonded with FRP sheet in the shear span i.e. the two sides of the beam. The beams were subjected to two point loads and tested to failure. The experimental results were compared to the present model predictions based on conventional fracture models carried out using commercial finite element software (ABAQUS). The results indicated that the use of FRP composites for flexural and shear–strengthened beams decreased crack propagation for approximately 55% and 37%, respectively, in comparison to the control beam. It was observed that the length of FPZ increased by using of FRP for shear–strengthening. The present model showed that the main diagonal crack formed at the support in the control beam whereas it appeared through the shear span in the shear–strengthened beam.

The developed fracture mechanics modeling was also applicable for identifying crack propagation in FRP-strengthened beam column joints. For this purpose, two beam column joints were made and tested to validate the present model. The results of the FRP-strengthened beam column joints by using present study showed good agreement with the experimental results (7 to 11%), whereas the results from numerical analysis using finite element software were considerably greater than experimental results (16 to 20 %). The results revealed that cracks formed in the joint area in the control specimen, while extensive cracks appeared in the beam in the specimen strengthened by FRP.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

SIMULASI PEMANJANGAN REKAHAN DALAM SENDI RASUK-TIANG KONKRIT BERTETULANG YANG DIPERKUKUHKAN DENGAN FRP DI BAHAGIAN LENTURAN DAN RICIH

By

SHAHRIAR SHAHBAZPANAHI

Disember 2015

Pengerusi : Profesor Abang Abdullah Abang Ali, PhD
Fakulti : Kejuteraan

Pada masa kini, pemulihan bahagian-bahagian struktur merupakan isu yang mencabar untuk jurutera awam (struktural) dan usaha telah dipergiatkan untuk meramal pemanjangan rekahan dalam bahagian-bahagian struktur . Dalam kajian ini, sebuah matriks ketegaran telah dihasilkan untuk zon proses rekahan (FPZ). Berdasarkan formulasi yang didapati, satu elemen baru telah dibentuk untuk pemodelan pemanjangan rekahan menggunakan analisa elemen terhingga. Kesan-kesan saiz seperti kedalaman, ketebalan alang dan kepanjangan rekahan telah dipertimbangkan dalam pengiraan kepanjangan FPZ dan pelanjutan rekahan. Berdasarkan elemen baru itu, kaedah pembezaan tenaga Griffith telah digunakan untuk meramal ciri-ciri pemanjangan rekahan dengan ketepatan yang tinggi.

yang diperkuatkan dengan pelbagai kaedah seperti polimer yang diperkukuhkan dengan gentian (FRPs).

Oleh itu, dalam kajian ini, sebuah model yang berasaskan angka telah dihasilkan untuk pemodelan pemanjangan rekahan dalam alang konkrit yang diperkukuhkan dari segi kelenturan atau ricihan dengan FRP. Bagi mengesahkan ketepatan model ini, ujikaji eksperimen keatas rasuk dan sambungan rasuk-tiang konkrit bertetulang yang tidak diperkukuh dan yang diperkukuh dengan FRP dijalankan. Tiga spesimen rasuk dengan keratan rentas segi empat tepat telah diuji. Dua rasuk diperkukuhkan dengan kepingan FRP yang dilekat di luar untuk pengukuhan kelenturan atau ricihan dan satu rasuk sebagai sebagai kawalan. Pada salah satu rasuk, kepingan FRP diikat bersama secara luaran di bawah rasuk dan pada rasuk yang lain, kepingan FRP diikat pada rentang ricihan iaitu kepada 2 sisi rasuk. Rasuk tersebut dikenakan dua beban terpumpun sehingga gagal. Keputusan eksperimen telah diperbandingan dengan ramalan model sedia ada berdasarkankeputusan ujian dengan model rekahan konvensional yang dijalankan menggunakan perisian elemen terhingga komersil (ABAQUS). Keputusan menunjukkan bahawa penggunaan komposit FRP untuk alang yang diperkukuhkan dari segi kelenturan dan ricihan telah mengurangkan pemanjangan rekahan sebanyak kira-kira 55% dan 37% masing-masing berbanding

dengan alang kawalan. Kepanjangan FPZ didapati bertambah dengan penggunaan FRP untuk pengukuhan kelenturan. Model ini menunjukkan bahawa rekahan pepenjuru utama terhasil di penyokong alang kawalan manakala terbentuk di kawasan ricihan dalam alang yang diperkukuhkan secara ricihan.

Pemodelan mekanik rekahan yang dihasilkan ini boleh digunakan untuk mengenalpasti pemanjangan rekahan dalam sendi rasuk-tiang yang diperkukuhkan dengan FRP. Untuk penggunaan sedemikian tujuan tersebut, dua sendi rasuk-tiang dihasilkan dan diuji untuk mengesahkan ketepatan model ini. Keputusan kajian ini adalah selaras dengan keputusan eksperimental (7 hingga 11%), manakala keputusan dari analisa berasaskan angka menggunakan perisian elemen terhingga didapati lebih besar dari keputusan eksperimental (16 hingga 20%). Keputusan menunjukkan bahawa rekahan terbentuk di bahagian sendi untuk spesiman kawalan manakala rekahan yang luas terbentuk di alang spesimen yang diperkukuhkan dengan FRP.

ACKNOWLEDGEMENTS

I would like to express my gratitude to my supervisor, Professor Abang Abdullah Abang Ali for his support and encouragement throughout my doctoral study. This thesis would have been impossible to be completed without his editorial and academic advice. I would also like to thank him for editing my publications and spending the time.

I would also like to thank my co-supervisor, Dr Farah Nora Aznieta Abdul Aziz, for her guidance, support and expertise in editing my thesis. I also would like to thank Dr Raizal Saifulnaz Bin Muhammad Rashid, and Dr Anwar for their remarkable advices and for serving in my PhD committee.

I would like to thank Dr. Frazad Hejazi for his support through my Ph.D. study and his guidance and time for designing the experimental set-up despite his busy schedule.

It is also an honor and privilege to have my brother Professor Shahram Shahbazpanahi to serve on my Ph.D. I am very grateful for the continuous help, encouragement, and cooperation to edit of my publications and spending the time.
I would like to express my appreciation to my wife Alaleh Kamgar for her great love, patience, understanding and support throughout this work. She is truly my best friend and partner.

I would also like to special thanks to my precious friend, Dr Nima Farzadnia, for editing of my publications, guidance and help.

I am very grateful for the continuous help and encouragement of my parents, my brothers, and my daughter, Rozhan.

I would like to express my gratitude to Dr Masoud Pakanhad, who encouraged, help and guidance me to pursue my Ph.D. at University Putra Malaysia.

I wish to express my thanks to my friends and colleagues Dr Xaksar, Dr Shahedeh Taiamon, Dr Zareii, Dr Hakim, Dr Samira Jilani, Dr Asra Amidi and staff of Housing Research Center in UPM for all the discussions, cooperation and for the wonderful time we have shared.
You will always be in my heart.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

Abang Abdullah Abang Ali, PhD
Professor, Ir
Housing Research Center
Universiti Putra Malaysia
(Chairman)

Farah Nora Aznieta, PhD
Senior Lecturer
Faculty of Engineering
Universiti Putra Malaysia
(Member)

S. Raizal, PhD
Professor, Ir
Faculty of Engineering
Universiti Putra Malaysia
(Member)

BUJANG BIN KIM HUAT, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

Declaration by Members of Supervisory Committee

This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) were adhered to.

Signature: ______________________
Name of Chairman of Supervisory Committee: Dr. Abang Abdullah Abang

Signature: ______________________
Name of Member of Supervisory Committee: Dr. Farah Nora Aznieta

Signature: ______________________
Name of Member of Supervisory Committee: Dr. S. Raizal

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF NOMENCLATURES

P	Applied load
h	Beam depth
L	Beam length
B	Beam width
f_c'	Compressive strength of concrete
$_n$,	Constant values
	Crack extension
u	Crack opening displacement
w_c	Critical crack opening displacement
$_c$	Critical strain energy release rate for concrete
j	Degrees of freedom
u_1, u_2	Displacement node 1 to the local coordinates and
u_3, u_4	Displacement node 2 to the local coordinates and
u_5, u_7	Displacement node in x direction for nodes 3 and 4
u_9, u_{11}	Displacement node in x direction for nodes 5 and 6
U	Displacement vector
d^i**,**	Displacement, the corresponding increment after iteration
q( )	Distribution of stress in FPZ
a	Effective crack length
$_c$	Effective strain for FRP sheet
d_e, d_s	Elastic opening and softening opening
	Energy dissipation rate by the flexural– FRP
$_P$	Energy dissipation rate by the shear– FRP (side bonding)
	Energy release rate of concrete for Mode I
	Fracture resistance of the material
,$K_{s_{II}}$	Fracture toughness due to force and slip of FRP sheet for Mode II
$_C$,	Fracture toughness of plain concrete and FPZ for Mode II
A_F	FRP area section
f (a/h)	Geometric shape function
(a/h)	Geometric shape function due to the shear stress of the FPZ

i	Iteration
l_{com}	Length of the FPZ in composite material
l_p	Length of FRP
a_0	Length of initial notch
$_{\sigma}$	Length stress-free region
u	Maximum crack opening displacement
s_m	Maximum of the slip between concrete and FRP
$_{max}$	Maximum shear stress between concrete and FRP
	Mesh size
	Modulus of composite
	Nodal force due to external load in x direction
	Nodal force due to strain energy in x direction
	Normal force due to the FPZ
σ	Normal stress in the FPZ
	Number of elements that have failed behind FPZ
n'	Number of FRP elements with no existing record of rupture
n	Number of side face FRP sheets
δ_n	Opening displacement
	Orientation angle
$_c$	Poisson’s r tio of concrete
	Poisson’s r tio of P
$_s$	Shear force due to FRP sheet slip
	Shear modulus
$_n$	Shear strength of the four-point loading RC beam
$_c$, $_P$	Shear strength contributions of the concrete and FPZ
	Shear stress between concrete and FRP
τ_{FPZ}	Shear stress distributions in the damage zone
δ_t	Slide displacement
s	Slip between concrete and FRP
S_e, S_s	Slopes in the elastic and softening zones
S	Softening parameter
Δd^i	Specified tolerance
K	Stiffness matrix for local element

$\mathbf{K_s}$	Stiffness matrix for the element
$\frac{\partial U}{\partial A}$	Strain energy release rate
$_p$	Stress intensity factor due to the external load
$_f$	Tensile force due to the extension of FRP
f_t	Tensile strength of concrete
ole r nce	Tolerance
	Transpose
d	Total crack opening
b_f, l_f	Width and length of FRP
	Work done by externally applied loading
$_c$	Young's modulus of concrete
	Young's modulus of FRP

CHAPTER 1

INTRODUCTION

1.1 General

Sudden failure occurs in concrete structural members, such as beams and beam column joints, due to the quasi-brittle behavior of concrete. In general, sudden failures in concrete initiate with crack propagation in the tension zone because of an increase in stress or occurring initial crack. Therefore, these failures should be accurately predicted.

Material strength and fracture mechanics theories are two major groups of theories in crack growth analysis. In material strength theory, cracks are detected based on strain, stress, or a combination of stress and strain. The disadvantage of this theory is that eliminating the damaged elements can produce a stress singularity. Material strength theory, which existed prior to fracture mechanics, uses a crack propagation criterion with no strain energy effect. It explains the propagation of a crack as an unavoidable method of energy transfer between the strain energy of an elastic body and the fracture energy required to produce a new crack. The fracture mechanics theory is considered to be a more accurate method for predicting crack growth, because this method is similar to the physical reality of crack propagation (Shi, 2009). Accurate prediction of crack propagation in concrete is essential for improving its reliability, durability, and serviceability.

Two methods are now available for fracture analysis in concrete. These can be broadly categorized into Linear Elastic Fracture Mechanics (LEFM) and nonlinear elastic fracture mechanics.

LEFM was first used to study crack propagation in warships deployed during World War II (Esfahani, 2007). In this method, a coefficient is applied to the stress in the vicinity of the crack tip. This coefficient is called the stress intensity factor. Because the stress intensity factor depends on the material properties, size of the crack, load, and geometry of the structure, it presents a relationship between the material and the reaction of the structure. Stress singularity at crack-tip is the characteristic of Linear LEFM. Later, some studies used LEFM in crack propagation analysis, but Kaplan (1961) found that deploying LEFM was not acceptable when it came to solving crack problems with normal concrete. To solve the aforementioned problem, the first model based on nonlinear fracture mechanics for concrete was proposed by Hillerborg et al. (1976). Their study introduced a region, often termed fracture process zone (FPZ), which has the ability to transfer normal and shear stress to close the crack. The FPZ plays a remarkable role in the behavior of cracks and their propagation under load. Hence a study on the role of the FPZ to predict and prevent crack propagation under static monotonic load (Esfahani, 2007) is indispensable. Although more techniques of crack propagation have been developed in fracture mechanics, crack modeling to predict the behavior of concrete structures is still far from satisfactory.

On the other hand, the crack patterns of concrete changes by using different types of reinforcement. Fiber reinforced polymer (FRP) composites have been increasingly used in concrete structures in recent years because of their corrosion resistance, low weight, high tensile strength and large strain. Nowadays, strengthening using FRP is of interest from an economic point of view. The use of FRP composites is now identified as a successful, suitable, and efficient technique to strengthen structures.

Because FRP modifies cracks propagation in concrete structural members, it is essential to study the crack behavior in structural members with FRP. Much effort has been made to model and test members flexural strengthened with FRP for different types of cracks, debonding, anchorage, and beam behavior (Arduini, et al., 1997; Yang et al., 2003; Bruno et al., 2007 ; Pavan et al., 2005; Achintha and Burgoyne, 2011). In addition, there have been many experimental studies on beams shear–strengthened using FRP (Adhikary and Mutsuyoshi, 2004; Coronado and Lopez, 2008; Teng and Hollaway, 2008; Siddiqui, 2009). Conventional theoretical and numerical analyses in fracture mechanics have been applied to study concrete flexural beams strengthened using FRP composites (Jae et al., 2010; Chen et al., 2012; Kesavan et al., 2013; Lin et al., 2014). However, there is still little in the way of theoretical analysis to calculate the fracture resistance of a shear–cracked beam strengthened with external FRP.

One of the most vulnerable structural elements to sudden failure is the beam column joint (Figure 1.1) whose crack propagation behavior needs to be predicted (Deaton, 2013). Cracks in the joint start where the beam and column intersect because of stress increase (Said and Nehdi, 2004). Thus, the fracture mechanics theory is the best method for predicting this type of crack. Beam column joints, which provide for the continuity of a structure, are a critical and significant part of concrete structures. Therefore, the study of their behavior, crack patterns, failure mode, and strengthening is essential. Presenting an accurate fracture mechanics model for beam column joints is of great importance owing to the complex behavior of the joints.

Figure 1.1. Damage to Beam Column Joint (Said and Nehdi, 2004)

Taylor (1974) was among the first to study joint behavior, and the American Concrete Institute (ACI) (1976) was the first to recommend codes for joint design. The behavior of a structural joint is very complex because of the interaction between different mechanisms. These different mechanisms are flexure, shear, and confinement of the joint. A crack may happen in the joint (shear failure) (Figure 1.2), in the beam, or in the column. Shear cracks in the joint are generally more significant than other types of cracks because shear failure in joints can break down the whole structure. These types of cracks have been observed in many studies (Pantelides et al., 2008). When a crack happens in a joint, the column and beam rotate and lose their load capacity. This mechanism is undesirable and it should be prevented. To prevent shear failure in the joint, it is better that the failure be shifted to an adjacent beam.

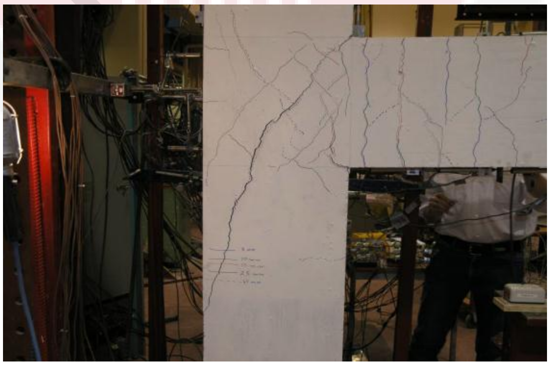

Figure 1.2. Cracks in the Joint Tested (Corazao and Durrani, 1989)

The strengthening of joints can be carried out using jacketing steel, bolted steel, or plate steel or by stitching and drilling with grout. Corrosion, confinement in the joint, and cost are reasons why these techniques are rarely used. Nowadays, to retrofit a beam column joint, external FRP is used. However, use of fracture mechanics modeling to analyze crack propagation in the joints and in joints strengthened by FRP has not been reported.

The prediction of crack propagation in concrete structures fails to conform to experimental tests.

Given these issues, the elastic stiffness of a fracture process zone is not applicable to the prediction of crack propagation in concrete. Thus, new fracture process zone stiffness and crack propagation criterion for concrete have been valuable to the study of concrete and have provided theoretical support to proposed empirical models.

The other contributions of this study have been summarized as follow:

a) Development of a fracture mechanics model to calculate the dissipation of the release rate on beams with flexural and shear strengthened with FRP, which is essential for predicting crack propagation in concrete
b) Proposed of a theoretical method to obtain the fracture resistance of shear–cracked concrete beams and beams shear strengthened using FRP
c) Development of numerical model to predict crack propagation in beam column joints and FRP-strengthened joints

It is also believed that this investigation will impact professional practice. Also, the findings have informed the effect of FRP on crack propagation in concrete structures such as beams and joints and will contribute to the development of design guidelines.

1.2 Problem Statements

Modeling of crack propagation is critical for simulation of failure in concrete structures. The common challenge for the modeling of crack propagation is the lack of a precise numerical solution for modeling material softening because the prediction of crack propagation in concrete structures fails to conform to experimental tests. In previous models, the stiffness of the FPZ was estimated from the Young’s modulus (Gerstle and Xie, 1992; Elices et al., 2009; Sagaresan, 2012). However, the FPZ has softening behavior as well, which is not considered. Therefore, previous studies did not provide an accurate prediction of crack propagation. Hypothetically, in order to obtain accurate results, the softening effect of the FPZ needs to be considered when using the finite element method to model a material. Then, based on an accurate stiffness matrix, an improved Griffith energy approach (1921) can be used to predict the propagation criterion.

Recently, the application of FRP to strengthen concrete structures has significantly increased, and the stiffness and strength of concrete structural elements have been improved by applying FRP in the member. However, there is concern about crack propagation and crack patterns in concrete sections strengthened by FRP. There are many numerical models to predict cracks between FRP and concrete for various strengthened structures. However, most of them are based on stiffness and strength criteria. These numerical models cannot properly predict crack propagation in structural members retrofitted with FRP based on an energy approach.

It is essential to consider effect of FRPs on cracks propagation in concrete to prevent crack propagation. In previous models, the authors focused primarily on the occurrence of slip in FRP composites (Dai and Ng, 2014; Pan and Wu, 2014). Therefore, previous investigations did not take the influence of FRP on the FPZ and on crack propagation into account. Also, the effect of FRP on crack propagation based on an energy approach has not been investigated yet. It is a challenge that needs further study. This is what previous authors have done, and here is our contribution.

There are many theoretical models for calculating the loading capacity for various flexural–strengthened beams (Ohno et al., 2014). However, shear capacity of a beam shear strengthened with FRP composites has not been reported yet.

Many studies have been conducted on beam column joints with bonded FRP. However, most of these studies have been done experimentally; there is no research available on numerical modeling owing to the lack of proper finite element models of the fracture process in joints.

Cracks begin where columns and beams intersect owing to tension stress. Thus, fracture mechanics that takes into account tension softening is needed to analyze cracks in joints. Therefore, an accurate FPZ model of joints with and without FRP strengthening is required.

The aim of this study is to develop more accurate prediction of crack propagation in concrete structures particular reinforced concrete (RC) beams and RC beam column joints strengthened with FRP.

1.3 Objectives

In order to achieve the above mention aims, the following objectives are outline:

1. To develop new formulation for FPZ stiffness and crack propagation criterion in RC beams and RC beams strengthened with FRP under static load.
2. To develop a new element and numerical model for the prediction of crack propagation in RC beams with FRP shear strengthened under static load.
3. To identify shear capacity theoretically for cracked RC beams with FRP shear– strengthened.
4. To implement the new developed FPZ stiffness and numerical model on beam column joints strengthened by FRP.
5. To validate the new proposed model by experimental tests on RC beam and RC beam column joints with and without flexural and shear FRP strengthening.

1.4 Scope and limitations

The scope of the current research is as follows:

a) This research focused on the FPZ stiffness and crack propagation criterion in concrete beams and the flexural and shears strengthening of concrete beams using FRP. It is also focused on the development of a numerical model for beam column joints strengthened by FRP under static load.
b) Three beams were built and tested to validate the results obtained by the present model. All beams were tested under four-point loading with simple support. The beam B-0 was used as the control beam. Specimen B-1 was flexure-strengthened with CFRP sheets. The third specimen B-2 was strengthened in the shear span. Two full scale reinforced concrete beam column joints were tested to validate the results obtained by the present model. One beam column joint was tested in an un-strengthened condition to act as the control beam column joint. One beam column joint was strengthened with CFRP sheets.
c) Analysis was carried out using fracture mechanics theory. In fracture mechanics, a crack is assumed to start in the tension face in concrete with strain-softening behavior. The stress–strain behavior of concrete in the compression zone and crushing are neglected in the fracture mechanics theory.
d) Four-node isoparametric elements were used to model bulk concrete as a linear elastic material. The nonlinear behavior of bulk concrete and plastic deformation is ignored owing to the small deformation.
e) Program code was developed for analysis of two-dimensional (2D) plane stress for fast convergence. Based on the literature review, when displacements are small, 2D modeling is sufficient for analyzing cracks (Gerstle and Xie, 1992). A three-dimensional (3D) model increases complexity, resulting in inaccuracy.
f) An investigation of the effect of various material and geometric parameters (parameter study) was carried out and is discussed based on the numerical, experimental and theoretical results.
g) Validation of the numerical model was carried out through numerical modeling by use of commercial finite element software and experimental tests for three types of reinforcement concrete beams with and without implementing FRP for flexural strengthening of beam under four-point loading. No internal stirrup was provided in the desired shear failure region. Also, two full-scale beam column joint specimens, one of which was strengthened using a CFRP sheet, were cast and tested under static load.

1.5 Layout of the Thesis

The thesis is prepared in five chapters as follows:

Chapter 1 describes the importance of the current study, contribution of thesis, provides the problem statements, objectives, scopes and limitations.

Chapter 2 reviews earlier studies modeling crack propagation in concrete and previous efforts on the use of fracture mechanics to analyze FRP-strengthened concrete. This chapter also includes a literature review of previous efforts to analyze beam column joints strengthened using FRP composites.

Chapter 3 describes the proposed model of cracks in concrete in order to obtain accurate results under static load and the program code developed to model beams flexural strengthened and shear strengthened with FRP under static load. It also describes the experimental tests conducted on an RC beam and on concrete beams flexural strengthened and shear strengthened with FRP.

Chapter 3 also describes the development of the proposed theoretical method to estimate the fracture resistance of shear cracks in RC beams shear strengthened with FRP.

In addition, in Chapter 3 the developed numerical model is applied to predict crack propagation in beam column joints and in FRP-strengthened joints. The experimental tests on the beam column joints and the FRP-strengthened joints used to validate the present numerical model are also described.

Chapter 4 includes the program code used to estimate and validate the analytical and experimental results obtained from the current study, the results reported in previous studies, and the results obtained from the finite element analysis (FEA) software ABAQUS. Also, a number of beams were considered as benchmarks and their respective behaviors were observed. The effect of various material and geometric parameters (parameter study) was investigated and the results are discussed in this chapter. This chapter also validates and discusses a proposed theoretical model of a shear–cracked beam and provides theoretical analysis of the beam shear strengthened with an external FRP.

Chapter 4 also discusses the ability of the present nonlinear finite element code to estimate fracture mechanics and to model control beam column joints and joints strengthened with FRP. In this study, to validate the present model, the FEA software ABAQUS was used to model crack propagation by conventional cohesive elements (COH2D4P).

Chapter 5 provides conclusions from the present study with suggestions for future investigations.

REFERENCES

Abbass, M., Matsagar, V., and Nag, A. (2014). Bond-slip response of FRP sheets or plates bonded to reinforced concrete beam (pp. 1959–1969). *Advances in Structural Engineering, Springer India.*

Achintha, M., and Burgoyne, C. (2011). Fracture mechanics of plate debonding: Validation against experiment. *Construction and Building Materials*, 25(6): 2961–2971.

ACI Committee, 318-08. *Building Code Requirements for Structural Concrete.* American Concrete Institute, Farmington Hills Michigan, USA, 2007.

ACI Committee, 440.2R-02. *Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures*. American Concrete Institute Farmington Hills Michigan, USA, 2003.

ACI Committee, 352R-76. *Recommendations for design of beam-column joints in monolithic reinforced concrete structures.* American Concrete Institute Farmington Hills Michigan, USA, 2007.

Adhikary, B. B., and Mutsuyoshi, H. (2004). Behavior of concrete beams strengthened in shear with carbon-fiber sheets. *Journal of Composites for Construction, ASCE,* 8(4), 258–264.

Akguzel, U., and Pampanin, S. (2011). Effects of variation of axial load and bidirectional loading on seismic performance of GFRP retrofitted reinforced concrete exterior beam-column joints. *Journal of Composites for Construction*, 14(1), 94–104.

Akguzel, U., and Pampanin, S. (2012). Assessment and design procedure for the seismic retrofit of reinforced concrete beam-column joints using FRP composite materials. *Journal of Composites for Construction*, 16(1), 21–34.

Alhaddad, M., Siddiqui, N., Abadel, A., Alsayed, S., and Al-Salloum, Y. (2012). Numerical investigations on the seismic behavior of FRP and TRM upgraded RC exterior beam-column joints. *ASCE, Journal of Composites for Construction*, 16(3), 308–321.

Alva, G. M., and Ferreira, M. A. (2009). Partially restrained beam-column connections in reinforced concrete structures. *IBRACON Structures and Material Journal*, 2(4), 356–379.

Anderson, T. L., and Glinka, G. (2006). A closed-form method for integrating weight functions for part-through cracks subject to Mode I loading. *Engineering Fracture Mechanics*, 73(15), 2153–2165.

Ansari, F. (1988). Mechanism of microcrack formation in concrete. *ACI Materials Journal*, 86(5), 459–464.

Antonopoulos, C. P., and Triantafillou, T. C. (2003). Experimental investigation of FRP-strengthened RC beam-column joints. *Journal of Composites for Construction, ASCE*, 7(1), 39–49.

Aravind, N., Amiya, K., Samanta, K., Singha, K., and Thanikal, J. (2015). Flexural strengthening of reinforced concrete (RC) beams retrofitted with corrugated glass fiber reinforced polymer (GFRP) laminates. *Curved and Layered Structures*, 2(1): 244–253.

Arduini, M., Di Tommaso, A., and Nanni, A. (1997). Brittle failure in FRP plate and sheet bonded beams. *ACI Struct Juornal*, 94(4): 363–370.

Arrea, M., and Ingraffea, A. R. *Mixed-mode crack propagation in mortar and concrete*. Department of Structural Engineering. Cornell University: Report No. 81–13, 1982.

Baggio, D., Soudki, K., and Noel, M. (2014). Strengthening of shear critical RC beams with various FRP systems. *Construction and Building Materials*, 66(15): 634–644.

Baji, H., Eslami, A., and Ronagh, H. (2015). Development of a nonlinear FE modelling approach for FRP-strengthened RC beam-column connections. *Structures*, In Press.

Baluch, M. H., Azad, A., and Ashmawi, W. (1992). Fracture mechanics application to reinforced concrete concrete members in flexure. In *Applications of Fracture Mechanics to Reinforced Concrete*. London: Elsevier Applied Science.

Bazant, Z. P. (1984). Size effect in blunt fracture: Concrete, rock, metal. *Journal of Engineering Mechanics, ASCE*, 110(4), 518–535.

Bazant, Z., and Gambarova, P. (1980). Rough cracks in reinforced concrete. *Journal of the Structural Division*, 106(4), 819–842.

Bazant, Z., and Oh, B. (1983). Crack band theory for fracture of concrete. *Materials and Structures*, 16(3), 155–177.

Bencardino, F., and Condello, A. (2015). SRG/SRP–concrete bond–slip laws for externally strengthened RC beams. *Composite Structures*, 132, 804–815.

Beres, A., El-Borgi, S., and White, R. N. *Experimental results of repaired and retrofitted beam-column joint tests in lightly reinforced concrete frame* *buildings*. Technical report NCEER-92-0025, State University of Buffalo, Buffalo, NY,1992.

Bhargava, J., and Rehnström, A. (1975). High speed photography for fracture studies of concrete. *Cement Concrete Research, 5,* 239–248.

Bindhu, K. R., and Jaya, K. P. (2010). Strength and behaviour of exterior beam column joints with diagonal cross bracing bars. *Asian Journal of Civil Engineering,* 11(3), 397–410.

Biscaia, H. C., Chastre, C., and Silva, M. A. (2014). Linear and nonlinear analysis of bond-slip models for interfaces between FRP composites and concrete. *Composites: Part B,* 45, 1554–1568.

Bocca, P., Carpinteri, A., and Valente, S. (1991). Mixed-mode fracture of concrete. *International Journal of Solid and Structures,* 27(9), 1139–1153.

Bresler, B., and Scordelis, A. (1963). Shear strength of reinforced concrete beams, *ACI journal,* 60(40), 51–72.

Bruno, D., and Greco, F. (2001). Delamination in composite plates: influence of sheardeformability on interfacial debonding. *Cement and Concrete Composites,* 23, 33–45.

Bruno, D., and Greco, F. (2001). Mixed mode delamination in plates: a refined approach. *International Journal of Solids and Structures,* 38, 9149–77.

Bruno, D., Carpino, R., and Greco, F. (2007). Modelling of mixed mode debonding in externally FRP reinforced beams. *Composites Science and Technology,* 67, 1459–1474.

Bsisu, K., and Hiari, B. (2015). Finite element analysis of retrofitting techniques for reinforced concrete beam-column joint. *Journal of American Science,* 11(8), 48–56.

Carloni, C., and Subramaniam, K. V. (2013). Investigation of sub-critical fatigue crack growth in FRP/concrete cohesive interface using digital image analysis. *Composites: Part B,* 51, 35–43.

Carpinteri, A. (1990). A catastrophe theory approach to fracture mechanics. *International Journal of Fracture,* 44, 57–69.

Carpinteri, A. (1992). Reinforced concrete beam behaviour. In *Applications of fracture mechanics to reinforced concrete*. Amsterdam: Elsevier.

Carpinteri, A., Spagnoli, A., and Vantadori, S. (2010). A multifractal analysis of fatigue crack growth and its application to concrete. *Engineering Fracture Mechanics*, 87, 36-47.

Cendón, D. A., Gálvez, J. C., Elices, M., and Planas, J. (2000). Modelling the fracture of concrete under mixed loading. *International Journal of Fracture,* 103, 293–310.

Chajes, M., Finch, J., and Januszka, W. (2003). Bond and force transfer of composite material plates bonded to concrete. *ACI Structural Journal*, 93(2), 208–217.

Chen, G., Chen, J., and Teng, J. (2012). On the finite element modelling of RC beams shear-strengthened with FRP. *Construction and Building Materials*, 13–26.

Chen, J. F., and Teng, J. G. (2001). Anchorage strength models for frp and steel plates bonded to concrete. *Journal of Structural Engineering*, 127(7), 784-791.

Chen, J. F., and Teng, J. G. (2003). Shear capacity of fiber-reinforced polymer-strengthened reinforced concrete beams: Fiber reinforced polymer rupture. *Journal of Structural Engineering*, 129(5), 615-625.

Clyde, C., and Pantelides, C. P. Seismic evaluation and rehabilitation of R/C exterior building joints. *Proceedings of the Seventh U.S. National Conference on Earthquake Engineering.* Boston, 2002.

Colotti V, V., Spadea, G., and Swamy, R. N. (2004). Analytical model to evaluate failure behavior of plated reinforced concrete beams strengthened for shear. *ACI Structur Journal*, 101(6), 755–64.

Corazao, M., and Durrani, A. J. *Repair and strengthening of beam–to column connections.* Technical report NCEER-89–0013, State University of New York, Buffalo, 1989.

Coronado, C. A., and Lopez, M. A. (2008). Experimental characterization of concrete-epoxy interfaces. *Journal of Materials in Civil Engineering,* 20(4), 303–312.

Costa, R., and Providência, P. (2015). Considering the size and strength of beam-column joints in the design of RC frames. *Structural Concrete,* 16(2): 233–248.

Dai, J., Ueda, T., and Sato, Y. (2005). Development of the nonlinear bond stress–slip model of fiber reinforced plastics sheet–concrete interfaces with a simple method. *Journal of Composites for Construction,* 9(1): 52–62.

Dai, Q., and Ng, K. (2014). 2D cohesive zone modeling of crack development in cementitious digital samples with microstructure characterization. *Construction and Building Materials,* 54: 584–595.

Davidson, B. D., Hu, H., and Schapery, R. A. (1995). An analytical crack–tip element for layered elastic structures. *Journal of Applied Mechani,* 62: 294–305.

De Moura, M., Campilho, R., and Goncalves, J. (2009). Pure mode II fracture characterization of composite bonded joints. *International Journal of Solids and Structures,* 46, 1589–1595.

De O., I. External beam–column joints–the importance of stirrups. *Proceedings of IABSE Symposium*. Rio de Janeiro, 1999.

Deaton, J. (2013). *Nonlinear finite element analysis of reinforced concrete exterior beam–column joints with nonseismic detailing.* School of Civil and Environmental Engineering, PhD dissertation, Georgia Institute of Technology, USA.

Del V., C., Di Ludovico, M., Balsamo, A., Prota, A., Manfredi, G., and Dolce, M. (2014). Experimental investigation of exterior RC beam–column joints retrofitted with FRP systems. *Journal of Composites for Construction,* 18(4), In Press.

Derucher, K. N. (1978). Application of the scanning electron microscope to fracture studies of concrete. *Building and Environment,* 13, 135–141.

Dhir, R. K., and Sangha, R. M. (1974). Development and propagation of microcracks in plain concrete. *Materials and Structures,* 7(1): 17–23.

Diab, H. (2013). Pe Materials and Structures performance of different types of FRP sheets bonded to concrete using flexible adhesive. *Journal of Science and Technology*, 3(2): 124–135.

Diab, H. M., and Farghal, O. A. (2014). Bond strength and effective bond length of FRP sheets/plates bonded to concrete considering the type of adhesive layer. *Composites: Part B,* 58: 618–624.

Dong, W., He, H. N., and Wu, Z. M. (2011). Experimental and numerical study on mode I–II crack propagation for small size specimens of concrete. *Advanced Materials Research,* 163: 908–912.

Elices, M., Rocco, C., and Roselló, C. (2009). Cohesive crack modeling of a simple concrete: Experimental and numerical results. *Engineering Fracture Mechanics, 76*: 1398–1410.

Esfahani, M. (2007). *Fracture Mechanics Of Concrete*. Tehran, Iran: Tehran Polytechnic press.

Fan, G., Song, Y., and Wang, L. (2014). Experimental study on the behavior of reinforced concrete beam–column joints under various strain rates. *Journal of Reinforced Plastics and Composites*, 601–618.

Fischer, G., and Bohse, J. Observation and analysis of fracture rocesses in concrete with acoustic emission (AE) and digital image correlation (DIC). *31th Conference of the European Working Group on Acoustic Emission*, Berlin, 2014.

Gálvez, J. C., C ervenka, J., Cendón, D. A., and Saouma, V. (2002). A discrete crack approach to normal/shearcracking of concrete. *Cement and Concrete Research*, 32: 1567–1585.

Gerstle, W., and Xie, M. (1992). FEM modelling of fictitious crack propagation in concrete. *Journal of Engineering mechanics, ASCE,* 118(2):416–434.

Ghobarah, A., and El–Amoury, T. (2005). Seismic rehabilitation of deficient exterior concrete frame joints. *Journal of Composite construction*, 9(5): 408–416.

Ghobarah, A., and Said, A. (2002). Shear strengthening of beam–column joints. *Engineering Structures: The Journal of Earthquake, Wind and Ocean Engineering,* 2*4*(7): 881–888.

Goto, Y., and Joh, O. An experimental study on shear failure mechanism of RC interior beam–column joint. *Eleventh world conference on earthquake engineering.* Acapulco, México, 1996.

Granata, P. J., and Parvin, A. (2001). An experimental study on kevlar strengthening of beam–column connections. *Composite Structure,* 53: 63–71.

Grassl, P., Grégoire, P., Rojas–Solano, D., and Pija, L. (2012). Meso–scale modelling of the size effect on the fracture process zone of concrete. *International Journal of Solids and Structures*, 49: 1818–1827.

Grassl, P., Xenos, D., Nyström, U., and Rempling, R. (2013). A damage–plasticity approach to modelling the failure of concrete. *International Journal of Solids and Structures*, 50: 3805–3816.

Greco, F., and Lonetti, P. (2009). Mixed mode delamination in fiber reinforced composites. *Composites Part B: Engineering*, 40(5): 379–392.

Grégoire, D., Verdon, L., Lefort, V., and Grassl, P. (2015). Mesoscale analysis of failure in quasi–brittle materials:comparison between lattice model and acoustic emission data. *International Journal for Numerical and Analytical Methods in Geomechanics*, In Press.

Griffith, A. A. (1921). *The phenomena of rupture and flow in solids.* Philosophical Transactions of the Royal Society of London.

Guo, X., Su, R. K., and Young, B. (2012). Numerical investigation of the bilinear softening law in the cohesive crack model for normal–strength and high–strength concrete. *Advances in Structural Engineering*, 15(3): 373–388.

Guryao, Wang. *Study of Mode II Fracture of Rock and its Engineering Application.*Centeral South Univercity of Technology, Changsha, China, 1996.

Haach, V., Debs, E., and Debs, M. (2008). Evaluation of the influence of the column axial load on the behavior of monotonically loaded R/C exterior beam–columnjoints through numerical simulations. *Engineering Structures*, 30: 965–975.

Hadjab, H., and Chabaa, M. (2004). Fracture process zone in notched concrete beams treated by using acoustic emission. *Nondestructive Testing*,1(12):1–12.

Hadjazi, K., Sereir, Z., and Amziane, S. (2012). Cohesive zone model for the prediction of interfacial shear stresses in a composite–plate RC beam with an intermediate flexural crack. *Composite Structures*, 94: 3574–3582.

Harries, K., Eveslage, T., Aidoo, J., and Bro, W. Towards a standard test method for assessing FRP to concrete bond characteristics. *The 5th international conference on FRP composites in civil engineering*, Beijing, 2010.

Harries, K., Hamilton, H., Kasan, J., and Tatar, J. Development of standard bond capacity test for FRP bonded to concrete. *6th international conference on Fiber Reinforced Polymer (FRP) Composites in Civil Engineering (CICE).* Rome, 2012.

He W, W., Wo, Y. F., and Liew, K. M. (2008). A fracture energy based constitutive model for the analysis of reinforced concrete structures understatic loading. *Computer Methods Appl Mechanichs Engineering*, 197:4745–4762.

Hegger, J., Sherif, A., and Roeser, W. (2003). Nonsiesmic design of beam–column joints. *ACI Structural Journal*, 100(5): 654–664.

Hillerborg, A., Modeer, M., and Petersson, P. E. (1976). Analysis of crack formation and crack growth in concrete by means of mechanics and finite element. *Cement and Concrete Research*, 6: 773–782.

Hu, X. Z., and Wittmann, F. H. (1990). Experimental method to determine extension of fracture–process zone. *Journal of Materials in Civil Engineering*, 2(1): 459–464.

Huang, D., Guangda, L., and Liu, Y. (2015). Nonlocal peridynamic modeling and simulation on crack propagation in concrete structures. *Mathematical Problems in Engineering*, In Press.

Hwang, S., and Lee, H. (1999). Analytical model for predicting shear strengths of exterior reinforced concrete beam–column joints for seismic resistance. *ACI Structural Journal*, 96(5): 846–857.

Ingraffea, A., Gerstle, W., Gergely, P., and Saouma, V. (1984). Fracture mechanics of bond in reinforced concrete. *Journal Structal Engeering, ASCE*, 110(4): 871–890.

Jae, H., Lee, H., Chacko, R. M., and Lopez, M. M. (2010). Use of mixed–mode fracture interfaces for the modeling of large–scale FRP–strengthened beams. *Journal of Composites for Construction*, 14(6): 845–855.

Jeang, F., and Hawkins, N. *Non–linear analysis of concrete fracture*. Report No. SM 85–2, University of Washington, USA, 1985.

Jefferson, D. (2002). Tripartite cohesive crack model. *Journal of engineering mechanics*, 128(6): 644–653.

Jenq, Y., and Shah, S. (1985). A two parameter fracture model for concrete. *Journal of Engineering Mechanics*, 111(4): 1227–1241.

Jeremic, B., and Bao, Y. *Modeling of reinforced concrete beam–column joint*. ECI 284 Term Project Report, University of California, 2005.

Kalfat, R., and Al–Mahaidi, R. (2014). A prediction model for bidirectional fiber patch anchors used to enhance the performance of FRP materials bonded to cocrete. *Composite Structures*, 117: 51–58.

Kaliluthin, A., and Kothandaraman, S. (2014). Experimental investigation on behavior of reinforced concrete beam column. *Intrnational Journal of Civil and Structural Engineering*, 4(3): 248–261.

Kaliluthin, A., Kothandaraman, S., and Aha, T. (2014). A review on behavior of reinforced concrete beam column joint. *International Journal of Innovative Research in Science, Engineering and Technology*, 3(4): 11299–11312.

Kang, T., T., and Hung, Y. (2011). Nonlinear finite element analyses of unbonded post–tensioned slab–column connections. *PTI Journal*, 18: 138–148.

Kaplan, M. (1961). Crack propagation and the fracture concrete. *ACI Journal,* 58(5): 591–610.

Karayannis, C. G., and Sirkelis, G. M. Effectiveness of RC beam–column connections strengthening using carbon–FRP jackets. *Proceedings of the Twelfth European Conference on Earthquake Engineering*, London, 2002.

Kesavan, K., Ravisankar, K., and Senthil, R. (2013). Experimental studies on performance of reinforced concrete beam strengthened with CFRP under static loading using FBG array. *Measurement, 46*: 3855–3862.

Khalifa, A., and Nanni, A. (2002). Rehabilitation of rectangular simply supported RC with shear deficiencies using CFRP composites. *Construction and Building Materials,* 16: 135–146.

Khomwan, J. and Foster H. (2006) *Finite element modelling of FRP strengthened beams and walls.* New South Wales, Sydney.

Kim, H., Kim, M., Ko, M., and Lee, Y. (2015). Shear behavior of concrete beams reinforced with GFRP shear reinforcement. *International Journal of Polymer Science*, 1–8.

Kiran, R., and Genesio, G. (2014). A case study on pre 1970s constructed concrete exterior beam–column joints",, *Case Studies in Structural Engineering, Elsevie*,20–25.

Kiranea, K., and Bazant, Z. (2015). Size effect in Paris law for quasibrittle materials analyzed by themicroplane constitutive model. *Mechanics Research Communications*, 60–64.

Kulkarni, S., and Li, B. (2010). behavior of reinforced concrete interior widebeam column joints. *Journal of Earthquake Engineering,* 31: 80–99.

Kulkarni, S., and Patil, Y. (2014). A novel reinforcement pattern for exterior reinforced concrete beam column joint. *Procedia Engineering,* 51, 184–193.

Kumar and Barai (2011). *Concrete fracture models and applications.* Berlin Heidelberg: Springer.

Kwon, S. H., Zhao, Z., and Shah, S. P. (2008). Effect of specimen size on fracture energy and softening curve of concrete: Part II. Inverse analysis and softening curve. *Cement and Concrete Research,* 38: 1061–1069.

Lee, J. H., Chacko, R. M., and Lopez, M. M. (2010). Use of mixed–mode fracture interfaces for the modeling of large–scale FRP–strengthened beams. *Journal of Composites for Construction,* 14(6): 845–855.

Lee, N. K., Mayfield, B., and Snell, C. (1981). Detecting the progress of internal cracks in concrete by using embedded graphite rods. *Magazine of Concrete Research*, 33(106): 180–193.

Li, H. N., and Li, M. (2013). Experimental and numerical study on properties of RC beam. *Magazine of Concrete Research,* 65(12):744 –756.

Li, V. C., and Matsumoto, T. (1998). Fatigue crack growth analysis of fiber reinforced concrete with effect of interfactial bond degradation. *Cement Concrete Composits*, *20,* 339–351.

Lin, C. T., Wu, Y. H., and Chin, W. (2014). Performance of CFRP–strengthened RC beams subjected to repeated loads. *Journal of the Chinese Institute of Engineers*, In Press.

Loreto, G., Babaeidarabad, S., Leardini, L., and Nanni, A. (2015). RC beams shear–strengthened with fabric–reinforced–cementitious–matrix (FRCM) composite. *International Journal of Advanced Structural Engineering,* 7: 341–352.

Lowes, N., and Altoontash, A. (2003). Modeling of reinforced concrete beam–column joint subjected static load. *Juornal of Structural Engineerin, ASCE,* 129(12): 1686–1697.

Lu, W., and Hu, S. (2015). Effect of large crack–depth ratio on three–point bending concrete beam with single edge notch. *Material Research Innovations,* 19(8), 312–317.

Ludovico, M., Lignola, G., Prota, A., and Cosenza, E. (2010). Nonlinear analysis of cross sections under axial load and biaxial bending. *ACI Structural Journal*, 107(4): 390–399.

Mahjuob, H., and Barr, B. I. Imapct shear strength of FRC material. *Fibre reinforced cement and concrete, proceedings of the fourth RILEM international symposium*, 1992.

Maji, A., Ouyang, C., and Shah, S. P. (1990). Fracture mechanisms of concrete based on acoustic emission. *Journal of Materials Research*, 5(1): 206–217.

Masi, A., Santarsiero, G., and Mossucca, A. (2014). Influence of axial load on the seismic behavior of RC beam–column joints with wide beam. *Applied Mechanics and Materials, 508*, In Press.

Matthys, S., and Triantafillou, T. C.Shear and torsion strengthening with externally bonded FRP reinforcement. *Proceedings of the international workshop on composite in construction*, Capri, Italy, 2001.

Meinheit, D. F., and Jirsa, J. O. *The shear strength of reinforced concrete beam–column joints*. CESRL Report No. 77–1, University of Texas, USA, 1977.

Mindess, S., and Diamond, S. (1980). A preliminary SEM study of crack propagation in mortar. *Cement and Concrete Research,* 10: 509–519.

Mitra, N., and Lowes, L. (2007). Evaluation, calibration, and verification of a reinforced concrete beam–column joint model. *Journal of Structural Engineering*, 131(1): 105–120.

Morice, E., and Pommier, S. A novel experimental method to characterize the static response of the crack tip process zone in a quasi–brittle material. *Proceedings of the 8th International Conference on Fracture Mechanics of Concrete and Concrete Structures*, Toledo; Spain, 2013.

Mosallam, A. S., and Banerjee, S. (2007). Shear enhancement of reinforced concrete beams strengthened with FRP composite laminates. *Composites: Part B,* 38(6): 781–793.

Mostofinejad, N., and Talaeitaba, M. (2006). Finite element modeling of RC connections. *Iranian Journal of Science and Technology*, 30(1): 246–256.

Mukherjee, A., and Joshi, M. (2004). FRPC reinforced concrete beam–column joints. *Journal of Composite Structures,* 70: 185–199.

Nakaba, K., Kanakubo, K., Furuta, T., and Yoshizawa, K. (2001). Bond behavior between fiber reinforced polymer laminates and concrete. *ACI Structural Journal*, 98: 359–367.

Nallathambi, P., and Karihaloo, B. L. (1986). Determination of specimen–size independent fracture toughness of plain concrete. *Magazine of Concrete Research*, 38(135): 67–76.

Ohno, K., Uji, K., Ueno, A., and Ohtsu, M. (2014). Fracture process zone in notched concrete beam under three–point bending by acoustic emission. *Construction and Building Materials*, 67: 139–145.

Ooi, E. T., and Yang, Z. J. (2011). Modelling crack propagation in reinforced concrete using a hybrid finite element–scaled boundary finite element method. *Engineering Fracture Mechanics*, 78(2): 252–273.

Owen, J. and Hinton K. (1980) *Finite elements in plasticity, Theory and practice.* Swansea: Pineridage Press Limited.

Ozbolt, J., Sharma, A., and Reinhardt, H. w. (2011). Fracture of concrete – compact tension specimen. *International Journal of Solids and Structures,* 48: 1534–1543.

Palmieri, V., and Lorenzis, L. (2014). Multiscale modeling of concrete and of the FRP–concrete interface. *Engineering Fracture Mechanics*, In Press.

Pan, J., and Wu, Y. (2014). Analytical modeling of bond behavior between FRP plate and concrete. *Composites: Part B,* 61: 17–25.

Pantazopoulou, S., and Bonacci, J. (1994). On earthquake resistant reinforced concrete frame connections. *Canadian Journal of Civil Engineering,* 21: 307–328.

Pantelides, C., Okahashi, Y., and Reaveley, D. (2008). Rehabilitation of reinforced concrete frame interior beam–column joints with FRP composites. *Journal of Composites for Construction*, 12(4): 435–445.

Park, S., and Mosalam, K. M. (2012). Parameters for shear strength prediction of exterior beam–column joints without transverse reinforcement. *Engineering Structures,* 36: 198–209.

Park, S., and Mosalam, M. (2013). Simulation of reinforced concrete frames with nonductile beam–column joints. *The Professional Journal of the Earthquake Engineering Research Institute,* 29(1): 233–257.

Parker, D. E., and Bullman, P. J. (1997). Shear strength within reinforced concrete beam–column joints. *the Structural Engineer, 75*(4): 53–57.

Parvin, A., and Granata, P. (2000). Investigation on the effects of fiber composites at concrete joints. *Composites: Part B,* 31: 499–509.

Parvin, A., and Wu, S. (2008). Ply angle effect on fiber composite wrapped reinforced concrete beam–column connections under combined–axial. *Composites. Structure.,* 82(4): 532–538.

Parvin, A., Altay, S., Yalcin, C., and Kaya, O. (2010). CFRP rehabilitation of concrete frame joints with inadequate shear and anchorage details. *Journal of Composites for Construction,* 14(1): 72–82.

Patel, R., Nambiyanna, B., and Prabhakara, R. (2015). An experimental study on effect of diameter of rebar on exterior beam column joint. *International Journal of Innovative Research in Science,* 4(7): 5984–5991.

Patil, S., and Manekari, S. (2013). Analysis of reinforced beam–column joint subjected to monotonic loading. *International Journal of Engineering and Innovative Technology, 2*(10), 149–157.

Paulay, T., and Priestley, M. (1992) *Design of reinforced concrete and masonry buildings*. John Wiley and Sons.

Pavan, R. C., Oliveira, B. F., and Creus, G. J. (2005). FE analysis of reinforced concrete beams strengthened by composite plates. *Latin American Journal of Solids and Structures*, 2(3): 253–267.

Pessiki, S., Conley, C., Gergely, P., and White, R. (1990). Non-seismic behavior of lightly–reinforced concrete column and beam–column joint details. *National Center for Earthquiche,*4(1): 44–52

Petersson, P. *Crack growth and development of fracture zone in plain concrete and similar materials. Report TVBM–1006,* Division of Building Materials, Lund Institute of Technology, Lund, Sweden, 1981.

Pietruszczak, S., and Haghighat, E. (2015). Modeling of fracture propagation in concrete structures using a constitutive relation with embedded discontinuity. *Studia Geotechnica et Mechanica,* 34(5): 27–33.

Prabavathy, S., and Rajagopal, S. (2013). Study of exterior beam–column joint with different joint core and anchorage details under reversal loading. *Structural Engineering and Mechanics,* 46(6): 809–825.

Prasada, M., and Krishnamoorthy, C. (2002). Computational model for discrete crack growth in plain and reinforced concrete. *Computer Methods in Applied Mechanics and Engineering*, 191(25): 2699–2725.

Rabinovitch, O., and Frostig, Y. (2001). Nonlinear high–order analysis of cracked RC beam strengthened with FRP strips. *Structural Engineering*, 127(4): 381–389.

Raghu, P. B., and Renuka, D. M. (2007). Extension of FCM to plain concrete beams with vertical tortuous cracks. *Engineering Fracture Mechanics, 74*: 2758–2769.

Rajagopal, S., and Prabhavaty, S. (2014). Behaviour of exterior beam column joint using mechanical anchorage under reversal loading: An experimental study. *Transactions of Civil Engineering,* 38(2): 345–358.

Raju, I. (1987). Calculation of strain–energy release rates with higher orde rand singular finite elements. *Engineering Fracture Mechanics,* 28(3): 251–274.

Ramarishna, R., and Ravindra, V. (2012). Experimental investigation on rehabilitation of reinforced cement concrete interior beam–column joints using CFRP and GFRP sheets. *International Journal of Engineering Science and Technology,* 4(3): 0975–5462.

Ray, S., and Chandra, J. M. (2012). Fatigue crack propagation model and size effect in concrete using dimensional analysis. *Mechanics of Materials,* 43: 75–86.

Realfonzo, R., Napoli, A., and Pinilla, J. (2014). behavior of RC beam–column joints strengthened with FRP systems. *Construction and Building Materials,* 54: 282–297.

Rots, J., and Brost, R. (1987). Analysis of mixed–mode fracture in concrete. *Journal of Engineering Mechanics,* 113(11): 1739–1758.

Rushad, S., and Duggal, S. (2015). Experimental investigations of RC beams strengthened with 4–layerd symmetric cross GFRP laminates. *International Journal of Research in Engineering and Technology,* 13: 430–437.

Rybick, E., and Kanninen, M. (1977). A finite element calculation of stress intensity factors by a modified crack closure integral. *Engineering Fracture Mechanics,* 9(4): 931–938.

Sagaresan, N. (2012). Modeling fracture of concrete with a simplified meshless discrete crack method. *KSCE Journal of Civil Engineering,* 16(3): 417–425.

Said, A. M., and Nehdi, M. L. (2004). Use of FRP for RC frames in seismic zones: Part I. Evaluation of FRP beam–column joint rehabilitation techniques. *Applied Composite Materials,* 11: 205–226.

Sakata, Y., and Ohtsu, M. (1995). Crack evaluation in concrete members based on ultrasonic spectroscopy. *ACI Materal Journal*, 92(6): 686–698.

Sarsam, K. F., and Phipps, M. E. (1985). The shear design of in situ reinforced concrete beam–column joints subjected to monotonic loading. *Magazine of Concrete Research*, 37(13): 16–28.

Selman, E., and Alver, N. (2015). A modified fiber–reinforced plastics concrete interface bond–slip law for shear–strengthened RC elements. *Polymer Composites*, In Press.

Shafei, E. (2015). Deformation characteristics of shear–deficient RC beams wrapped with CFRP. *Materials and Structures,* In Press.

Sharma, A., Eligehausen, R., and Hofmann, J. Influence of joint modeling on nonseismic evaluation of evaluation of non–seismically designed RC frame structures. *Second European conference on earthquake engineering and seismology*. Istanbul, 2014.

Sharma, A., Eligehausen, R., and Reddy, G. (2010). A new model tosimulate joint shear behavior of poorly detailed beam–column connections in RC structures. *Engineering Structures*, 33(3): 1034–1051.

Shen, J., Feng, Z., and Wang, X. (2014). Crack growth rate of concrete with pre–cracks in tension–compression load. *Jianzhu Cailiao Xuebao/Journal of Building Materials*, 17(3): 526–531.

Shi, Z. (2009). *Crack analysis in stuctural concrete, theory and aplication.* Burlington, USA: Butterworth–Heinemann.

Shi, Z., Ohtsu, M., Suzuki, M., and Hibino, Y. (2001). Numerical analysis of multiple cracks in concrete using the discrete approach. *Journal of Structural Engineering*, 127(9): 1085–1091.

Siddiqui, N. A. (2009). Experimental investigation of RC beams strengthened with externally bonded FRP composites. *Latin American Journal of Solids and Structures*, 6(4): 343 – 362.

Singh, V., Bansal, P. P., Kumar, M., and Kaushik, S. (2013). Experimental studies on strength and ductility of CFRP jacketed reinforced concrete beam–column joints. *Construction and Building Materials*, 55: 194–201.

Sofi, A., and Phanikumar, B. (2015). An experimental investigation on flexural behaviour of fibre–reinforced pond ash–modified concrete. *Ain Shams Engineering Journal*, 6(4):1133–1142.

Soudki, K., El–Salakawy, E., and Craig, B. (2007). Behavior of CFRP strengthened reinforced concrete beams in corrosive environment. *Journal of Composites for Construction, ASCE*, 11(3), 291–298.

Spada, A., Giambanco, G., and Rizzo, P. (2011). Elastoplastic damaging model for adhesive anchor systems. I: Theoretical formulation and numerical implementation. *American Society of Civil Engineers*, 137(12): 854.

Sumarc, D., Sekulovic, M., and Kra, D. (2003). Fracture of reinforced concrete beams subjected to three point bending. *International Journal of Damage Mechanics*, 12(1): 30–44.

Sundararaman, V., and Davidson, B. D. (1998). An unsymmetric end–notched flexure test for interfacial fracture toughness determination. *Engineering Fracture Mechanics*, 60: 361–377.

Supaviriyakit, T., Pimanmas, A., and Warnitchai, P. (2008). Nonlinear finite element analysis of non–seismically detailed interior reinforced concrete beam–column connection under staitc load. *Science Asia*, 34: 049–058.

Szekrenyes, A., and Uj, J. (2004). Beam and finite element analysis of quasi–unidirectional composite SLB and ELS specimens. *Composites Science and Technology*, 64: 2393–2406.

Tada, H., Paris, P., and Irwin, G. (1985)*The stress analysis of cracks handbook.* Del Research Corporation.

Taylor, H. P. *The behavior of in situ concrete beam–column joints,Technical Report 42.492.* London: Cement and Concrete Association, 1974.

Taylor, L. *FEAPpv source, A finite element analysis program, Personal version.* University of California, Berkeley, 2009.

Teng, J., and Hollaway, L. (2008). *Strengthening and rehabilitation of civil infrastructures using fibre–reinforced polymer (FRP) composites.* Cambridge, UK: Woodhead Publishing Limited.

Thai, C., Stitmanna, B., and Ueda, T. (2014). Investigation of the bond behaviour between PBO–FRCM strengthening material and concrete. *Journal of Advanced Concrete Technology*, 12: 545–557.

Tjhin, T., and Kuchma, D. (2002). Computer–based tools for design by strut–and–tie method: advances and challenges. *ACI Structural Journal*, 99(5): 586–594.

Toudeshky, H. H., Hosseini, S., and Mohammadi, B. (2010). Progressive delamination growth analysis using discontinuous layered element. *Composite Structures*, 92(4): 883–890.

Toutanji, H., Han, M., and Ghorbel, E. (2012). Interfacial bond strength characteristics of FRP and RC substrate. *Journal of Composites for Construction*, 16(1):35–46.

Toutanji, H., Ueno, S., and Vuddandam, R. (2013). Prediction of the interfacial shear stress of externally bonded FRP to concrete substrate using critical stress state. *Composite Structures*, 95: 375–380.

Triantafillou, T. (1998). Shear strengthening of reinforced concrete beams using epoxy–bonded FRP composites. *ACI Structure Journal*, 95(20): 107–15.

Triantafillou, T., Matthys, S., and Taerwe, L. (2001). Design of concrete members strengthened with externally bonded FRP reinforcement. *proceeding*, Cambridge.

Walraven, J. *The infuence of depth on the shear strength of lightweight concrete beams without shear reinforcement.* TU–Delft Report No. 578–4, Stevin Laboratory, Delft University of Technology,1978.

Wang, J. (2006). Cohesive zone model of intermediate crack–induced debonding of FRP–plated concrete beam. *International Journal of Solids and Structures,* 43: 6630–6648.

Wecharatana, M., and Shah., P. S. (1983). Predictions of nonlinear fracture process zone in concrete. *Journal of Engineering Mechanics, ASCE,* 109(5): 1231–1246.

Wong, R., and Vecchio, F. (2003). Towards modelling of reinforced concrete members with externally bonded fiber–reinforced polymer composites. *ACI Structural Journal*, 100: 47–55.

Wu, J., and Davies, Z. (2003). Mechanical analysis of a cracked beam reinforced with an external FRP plate. *Composite Structures,* 62(2)*:* 139–143.

Wu, Y., Zhoub, Z., Yangb, Q., and Ch, W. (2010). On shear bond strength of FRP–concrete structures. *Engineering Structures,* 32(3): 897–905.

Wu, Z., and Bailey, C. G. (2005). Fracture resistance of a cracked concrete beam post–strengthened with FRP sheets. *International Journal of Fracture*, 135: 35–49.

Wu, Z., and Ye, J. (2003). Strength and fracture resistance of FRP reinforced concrete flexural members. *Cement and Concrete Composites,* 25(2): 253–261.

Wu, Z., Islam, S. M., and Said, H. (2009). A three parameter bond strength model for FRP–concrete Interface. *Journal of Reinforced Plastics and Composites, 28*(19), 441–426.

Wu, Z., Rong, H., Zheng, J., and Xu, F. (2011). An experimental investigation on the FPZ properties in concrete using digital image correlation technique. *Engineering Fracture Mechanics*, 78(17): 2978–2990.

Wu, Z., Yang, S., and Wu, Y. An analytical method for three–point bending notched beam strengthened with CFRP sheet. *Fourth International Conference on FRP Composites in Civil Engineering.* Zurich, Switzerland, 2008.

Wu, Z., Yang, S., Hu, X., Zheng, J., Fan, X., and Shan, J. (2010). Analytical solution for fracture analysis of CFRP sheet–strengthened cracked concrete beams. *Journal of Engineering,ASCE,* 136(10): 1202–1219.

Xenosa, D., Grégoireb, D., Morelc, S., and Grassl, P. (2015). Calibration of nonlocal models for tensile fracture in quasi–brittle heterogeneous materials heterogeneous materials. *Journal of the Mechanics and Physics of Solids*, 82: 48–60.

Xiaobing, S., Xianglin, G., Yupeng, L., Chang, T., and Weiping, Z. (2013). Mechanical behavior of FRP–strengthened concrete columns subjected to concentric and eccentric compression loading. *Journal of Composite Construction*, 17(3): 36–346.

Xie, D., and Biggers, S. (2006). Progressive crack growth analysis using interface element based on the virtual crack closure technique. *Finite Elements in Analysis and Design,* 42(11): 977 – 984.

Xie, D., and Waas, A. (2006). Discrete cohesive zone model for mixed–mode fracture using finite element analysis. *Engineering Fracture Mechanics,* 73(13): 1783–1796.

Xie, D., Salvi, A., Sun, C., Waas, A., and Caliskan, A. (2006). Discrete cohesive zone model to simulate static fracture in 2D triaxially braided carbon fiber composites. *Journal of Composite Materials*, 40(22): 2025–2046.

Xie, M., and Gerstle, W. (1995). Energy–based cohesive crack propagation modeling. *Journal of Engineering Mechanics, ASCE,* 121(12): 1349–1458.

Xu, F., Wu, Z., Zheng, J., Zhao, Y., and Liu, K. (2011). Crack extension resistance curve of concrete considering variation of FPZ length. *Journal Materials In Civil Engineering., ASCE,* 23(5): 703–710.

Xu, S. L., and Reinhardt, H. (1998). Crack extension resistance and fracture properties of quasi–brittle softening materials like concrete based on the complete process of fracture. *International Journal of Fracture,* 92(1): 71–99.

Xu, S., and Reinhardt, H. W. (1999). Determination of double–K criterion for crack propagation in quasi–brittle materials, part I: Experimental investigation of crack propagation. *International Journal of Fracture,* 98(2): 111–149.

Xu, S., and Zhang, X. (2008). Determination of fracture parameters for crack propagation in concrete using an energy approach. *Engineering Fracture Mechanics,* 75(15): 4292–4308.

Xu, T., He, Z., Tang, C., Zhu, W., and Ranjith, P. (2014). Finite element analysis of width effect in interface debonding of FRP plate bonded to concrete. *Finite Elements in Analysis and Design*, 93: 30–41.

Yang, W. G., and Xiang, H. J. (2012). A new approach for the simulation of cementitious materials. *Journal of Mechanical Science and Technology :* 26(1), 45–51.

Yang, W. G., Guo, Z. Q., and Xiang, H. J. (2012). Numerical simulation of cementitious granular materials. *Science and Engineering of Composite Materials,* 19(1), 9–17.

Yang, Z., and Chen, j. (2005). Finite element modelling of multiple cohesive discrete crack propagation in reinforced concrete beams. *Engineering Fracture Mechanics,* 72(14): 2280–2297.

Yang, Z., and Liu, G. (2008). Towards fully automatic modelling of the fracture process in quasi–brittle and ductile materials: a unified crack growth criterion. *Journal of Zhejiang university science*, 9(7): 1862–1775.

Yang, Z., Chen, J. F., and Proverbs, D. (2003). Finite element modelling of concrete cover separation failure in FRP plated RC beams. *Construction and Building Materials,* 17(1): 3–13.

Youssef, M., and Ghobarah, A. (2001). Modelling of RC beam–column joints and structural walls. *Journal of Earthquake Engineering*, 5(1): 99–111.

Zhang, D., and Wu, K. (1999). Fracture process zone of notched three–point–bending concrete beams. *Cement and Concrete Research,* 29(12): 1887–1892.

Zhang, H. M., and Zhang, L. X. (2014). Numerical simulation of fluid–structure interaction with water hammer in a vertical penstock subjected to high water head. *Advanced Materials Research*, 86: 1530–1534.

Zhang, S., and Teng, J. (2014). Finite element analysis of end cover separation in RC beams strengthened in flexure with FRP. *Engineering Structures*, Engineering Structures, 75: 550–560.

Zheng., J., Dai, J., and Fan, X. (2015). Fracture analysis of FRP–plated notched concrete beams subjected to three–point bending. *Journal of Engineering Mechanics,* 142(3): 1–10.

Zhou, H., and Zhang, Z. (2012). Interaction of internal forces of exterior beam–column joints of reinforced concrete frames under seismic action. *Structural Engineering and Mechanics,* 44(2): 197–217.

Zou, X. (2003). Flexural behavior and deformability of fiber reinforced polymer prestressed concrete beams. *Journal of composites for Construction,* 7(4): 275–284.