UNIVERSITI PUTRA MALAYSIA

SKEWED DISTRIBUTIONS GENERATED BY TRUNCATED CAUCHY KERNEL

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SKEWED DISTRIBUTIONS GENERATED BY TRUNCATED CAUCHY KERNEL

By

ZAHRA NAZEMI ASHANI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

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DEDICATION

This Thesis is dedicated to my beloved parents for being great pillars of support

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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July 2017

Chairman : **Associate Professor Mohd Rizam Abu Bakar,PhD**
Faculty : **Science**

The aim of the present study is to explore skewed distributions extended from the skew symmetric distributions generated by Cauchy kernel. In the last two decades, there has been a growing interest in the construction of skew symmetric distributions. Different forms of the skewed distributions have been appeared in literature for data analysis and modelling. In particular, different forms of skew Cauchy symmetric distributions have been introduced and applied in different areas from physics to economics where researchers mostly have to deal with asymmetric data with heavy tails. However, the main weakness of skew Cauchy symmetric distributions is that they do not have finite moments and suffer from limited applicability. In the present study, we will introduce and explore the skew truncated Cauchy symmetric distributions to solve the problems related to infinite moments. A random variable X has skew symmetric distribution with probability density function of the form $2f(x)G(\lambda x)$ where f is a density function which is symmetric around 0 and G is distribution function of symmetric density function around 0 and λ is the skewness parameter. In this study we will introduce skew truncated Cauchy normal, skew trunctaed Cauchy uniform, skew truncated Cauchy logistic, skew truncated Cauchy Laplace and skew truncated Cauchy student's t model. For all of these new models, we will provide finite moments of all orders and solve the problems related to infinite moments. We will investigate some other mathematical properties such as distribution functions and characteristic functions. We will apply them to real applications. In particular, we will consider exchage rate data of Japanese Yen to the American Dollar from 1862 to 2003.

On the other hand, the main feature of skew symmetric distribution is the new parameter which controls skewness and kurtosis and provides more flexible models. In this study, we will provide the ranges of possible values of skewness and kurtosis for all these models and compare them with skewness and kurtosis of truncated Cauchy distribution. According to the results, skew models with truncated Cauchy kernel will be more flexible than truncated Cauchy distribution. The simulation studies for these new models and graphical illustrations also will be provided.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

BEBERAPA TABURAN PENCONG YANG DIJANA OLEH KERNEL CAUCHY TERPANGKAS

Oleh

ZAHRA NAZEMI ASHANI

Julai 2017

Pengerusi : **Profesor Madya Mohd Rizam Abu Bakar, PhD**
Fakulti : **Sains**

Tujuan kajian ini adalah untuk meneroka perihal taburan pencong lanjutan daripada taburan simetri yang dijana oleh kernel Cauchy. Pada dua dekad kebelakangan ini, semakin ramai yang meminati perkembangan taburan simetri pencong. Pelbagai bentuk berbeza taburan pencong telah muncul dalam literatur untuk analisis data dan pemodelan. Khasnya, bentuk-bentuk berbeza taburan simetri pencong Cauchy telah diperkenalkan dan diaplikasi dalam aspek-aspek berbeza dari fizik ke bidang ekonomi, di mana pengkaji perlu menjurus kepada data asimetri dengan hujung yang berat. Walaubagaimanapun, kelemahan utama taburan simetri Cauchy pencong Cauchy ini ialah ia tidak mempunyai momen terhingga dan mengalami kebolehaplikasian yang terbatas. Dalam kajian ini, kita akan memperkenalkan dan mengkaji taburan simetri pencong ringkas Cauchy untuk menyelesaikan masalah-masalah berkenaan dengan momen tak terhingga. Satu pembolehubah rawak X mempunyai taburan simetri pencong dengan fungsi kebarangkalian berbentuk $2f(x)G(\lambda x)$ di mana f adalah fungi ketumpatan yang simetri di sekitar 0 dan G adalah fungi taburan bagi fungi ketumpatan yang simetri di sekitar 0 dan λ adalah parameter pencong. Dalam kajian ini, kita akan memperkenalkan Cauchy ringkas pencong yang normal, Cauchy ringkas pencong seragam, Cauchy ringkas pencong logistik, Cauchy ringkas pencong Laplace and Cauchy ringkas pencong model t pelajar. Untuk kesemua model baru ini, kita akan memberikan momen terhingga untuk kesemua susunan dan menyelesaikan masalah-masalah yang berkaitan dengan momen tak terhingga. Kita akan mengkaji beberapa ciri bermatematik seperti fungi taburan dan fungi cirian. Kita akan mengaplikasikannya kepada aplikasi sebenar. Khususnya, kita akan mempertimbangkan data kadar pertukaran Yen Jepun ke Dolar Amerika dari tahun 1862 sehingga 2003. Sebaliknya, ciri utama taburan simetri pencong adalah parameter baharu yang mengawal kepencongan dan kurtosis dan ini menghasilkan model-model yang lebih fleksibel. Dalam kajian ini, kami menyediakan beberapa julat nilai yang mungkin bagi nilai kepencongan dan kurtosis untuk kesemua model dan membandingkannya dengan kepencongan dan kurtosis bagi taburan Cauchy terpangkas. Keputusan menunjukkan, model-model pencong dengan Kernel Cauchy terpangkas akan menjadi lebih fleksibel dari taburan Cauchy terpangkas. Kajian simulasi untuk model-model baharu ini berserta dengan ilustrasi grafik juga disediakan.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy.

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Declaration by Members of Supervisory Committee

This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) are adhered to.

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LIST OF ABBREVIATIONS

pdf	Probability Density Function
cdf	Cumulative Distribution Function
TC	Truncated Cauchy distribution
STC	Skew Truncated Cauchy distribution
STCS	Skew Truncated Cauchy Symmetric distribution
STCN	Skew Truncated Cauchy Normal distribution
STCU	Skew Truncated Cauchy Uniform distribution
STCL	Skew Truncated Cauchy Logistic distribution
STCLS	Skew Truncated Cauchy Laplace distribution
STCT	Skew Truncated Cauchy Student's t distribution
GSTCN	Generalized Skew Truncated Cauchy Normal distribution
GSTCCN	Generalized Skew Truncated Curved Cauchy Normal distribution
SN	Skew Normal distribution
StN	Skew Student's t Normal distribution
SNt	Skew Normal Student's t distribution
GSTN	Gereralized Skew Student's t distribution
G_1StN	Gereralized Skew Student's t distribution (type one)
G_2StN	Gereralized Skew Student's t distribution (type two)
G_3StN	Gereralized Skew Student's t distribution (type three)
G_4StN	Gereralized Skew Student's t distribution (type four)
MSL	Multivariate Skew Laplace
t_n	student's t distribution

CHAPTER 1

INTRODUCTION

1.1 Introduction

In the real world, we usually need to study many phenomena statistically. The quality of the methods which are used in a statistical analysis is heavily related to the considered distributions or probability models. Therefore, remarkable efforts, during the years, have been made to extend classes of standard distributions and to develop proper statistical methods to make suitable models for analyzing a wide range of phenomena in real world. However, there remain many problems where none of classical or standard models can be proper for analyzing the data. Normality of distribution is the most important assumption for analyzing symmetric data. When data are normally distributed, it means that they represent the respective phenomena in the natura state. However, very few phenomena distributed symmetrically, and normal distribution is a poor description of asymmetric data. Using normal distribution for asymmetric data can caused the loss of information. So, it is necessary to introduce and evaluate models which can present different degrees of asymmetry.

1.2 Motivation of the study

Many statistical researchers have sought to introduce flexible statistical models. They always have been trying to eliminate unnecessary assumptions in the process of data analysis. In the case of continuous observations, sometimes, data are not symmetric and have a slight skewness. Genton (2004) stated that many real data sets were asymmetric and that symmetric distributions were not always good models. In such situations, it is better to use mixed distributions instead of symmetric distributions in a way that symmetric distributions are a special case of mixed distributions. The model which includes the skew and symmetric distributions together is called skew symmetric model. It is common for a model to departure from normality to non-normality thereby introducing the new parameter which controls skewness and kurtosis provides a more flexible model to exhibit the data as accurately as possible. Therefore, skew symmetric models have been an attractive subject for many statistical researchers. Many univariate and multivariate skew symmetric models have been introduced during the past two decades. In fact, skew symmetric distributions play an important role in many research fields such as biology, survival analysis, finance, etc. where good skew symmetric distributions are needed. There is a large number of papers which explain about the applications of skew symmetric models. First of all, Hill and Dixon (1982) provided some examples about the applications of skew models. Genton (2004) and Azzalini

(2005) also provided an extensive list of applications of skew symmetric distributions. Reward, assumption of normality is included in many econometrics models. However, in practice, data on insurance losses and financial returns are commonly skewed. Although, the normal distribution is common and easy to use but it is not proper to model data which are naturally skewed.

1.3 Scope of the thesis

The Cauchy distribution has been applied in statistical works for more than three hundred years. Despite the common use of the term, Cauchy distribution, Simeon D. Poisson (1781-1840) was the first person to notice the probability density function of Cauchy distribution and could generate some examples for some accepted results in statistics. It is interesting that the Cauchy distribution was discovered twice with different motivations. In spite of the simplicity of Cauchy distribution, it did not appear as a possible distribution before 1824. The density function of Cauchy distribution with the parameter μ and σ is specified by:

$$f(x) = \frac{1}{\pi\sigma\left(1+\left(\frac{x-\mu}{\sigma}\right)^2\right)}$$

where $-\infty < x < \infty, -\infty < \mu < \infty$ and $\sigma > 0$. The Cauchy distribution is symmetric around μ and the spread of distribution is related to σ. It means when the spread of distribution increases, the value of σ increases as well. The Cauchy distribution is more peaked in the middle and has fatter tails than the normal distribution. As a result, this function can be utilized in a wide range of areas including extreme risk analysis and financial applications. This is because of its tails which are more realistic in the real world of applications. For example,according Nadarajah (2011) used the Cauchy distribution in physics to calculate the distribution of the energy of an unstable state in quantum mechanics with the name of Lorentzian distribution. It is also a solution for differential equation to describe resonance behavior. In statistics, it is heavily related to the Poisson kernel which offers a solution for the Laplace equation. The main weakness of Cauchy distribution is that it does not have any moments. For example, for finding the expectation value of standard Cauchy distribution we have

$$E(X) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{1+x^2}dx.$$

It is obvious that this integral is not convergence. On the other hand, in statistics, moments measure somethings related to the center of values. Moments help in deciding about characteristic of population.Therefore, researchers have tried to handle the problems related to the infinite moments for Cauchy distribution. In the first step, Johnson and Kotz (1970) introduced truncated Cauchy distribution. They discussed estimation issues of the standard truncated Cauchy distribution. Rohatgi (1976) provided the first two moments for standard truncated Cauchy distribution.

Finally, Nadarajah and Kotz (2006) obtained the finite moments of all orders for truncated Cauchy distribution. The truncated Cauchy distribution is applied in various fields such as industrial setting and engineering. For example, Nadarajah (2011), used it in the process of product's inspection before it is sent to customers or inspection process of a product's at every stage of a multistage production process. It is a common prior for Bayesian models particularly for economics data. (Bauwens, 1999). Additionally, it is a model for packet size in many traffic models (Ni, 2001) and in the fields of crystallographic statistics (Mitra and Das (1989)).

Nadarajah and Kotz (2005) introduced skew symmetric distributions with Cauchy kernel. The skew symmetric models with Cauchy kernel are fat-tailed. Therefore, they can be applied as a model for data which are skewed and fat-tailed. For example, in economics, it is considered that the returns on risky financial assets have a normal distribution however, the original data showed that they were not distributed normality and were distributed in a skewed and fat-tailed manner (Balkema and Embrechts, 2007). So, the skew models with Cauchy kernel can be used to model this kind of data. However, this family of distributions has the same problem as Cauchy distribution ,i.e., it does not have finite moments. This family of distributions suffers from limited applicability because of the lack of finite moments. Hence, one of the main purposes of this thesis is to modify the skew symmetric models with Cauchy kernel in a way that they have finite moments of all orders. For this purpose, the family of skew truncated Cauchy symmetric distributions will be introduced and provide the cumulative distribution functions, the finite moments of all orders and the characteristic functions. The flexibility (range of skewness and kurtosis) of skew truncated Cauchy symmetric distributions and truncated Cauchy distribution as the main feature of skew symmetric distributions will be compared. Also, the application of skew truncated Cauchy symmetric models in financial studies will be presented.

1.4 Objective of the thesis

The present thesis adresses two major aims: first, providing new continuous skew symmetric distributions with high flexibility and desirable distributive and inferential properties. Second, providing application of these skew models in statistical data analysis. Other important aims of this thesis can be stated as follows:

1. To develop skew truncated Cauchy normal, skew truncated Cauchy uniform, skew truncated Cauchy logistic, skew truncated Cauchy student's t and skew truncated Cauchy Laplace models.
2. To develop generalized skew truncated Cauchy normal distribution and examining its properties especially the limit behavior of this distribution in terms of model parameters.
3. To obtain some mathematical properties such as finite moments of all orders,

characteristic function and cumulative distribution function of skew truncated Cauchy models.

4. To conduct a simulation study for the family of skew truncated Cauchy symmetric distributions and compare them with truncated Cauchy distribution using p-p plots.

1.5 Outline of the thesis

The outline of this study is as follows:

Chapter 2: Background

This chapter reviews the skew symmetric distributions. Some univariate and multivariate skew symmetric models will be presented with the focus on Cauchy distribution, truncated Cauchy distribution and skew symmetric distributions with Cauchy kernel. Finally some mathematical concepts that will be used in this thesis will be discussed.

Chapter 3: On the skew truncated Cauchy normal distribution

This chapter will focus on the statistical models called skew truncated Cauchy distribution, skew truncated Cauchy normal distribution and generalized skew truncated Cauchy normal distribution. Some mathematical properties of these models such as finite moments of all orders, cumulative distribution function and characteristic function will be explored. An application of skew truncated Cauchy normal distribution in finance using the exchange rate data of Japanese Yen to the US Dollar from 1862 to 2003 will be presented,using the data obtained from the official website of Global Financial DAta Organization accessible at *http://www.globalfinancialdata.com/*.

Chapter 4: On the skew truncated Cauchy uniform distribution

In chapter four, skew truncated Cauchy uniform and some of its mathematical properties will be discussed.

Chapter 5: On the skew truncated Cauchy logistic distribution

Chapter five will present the skew truncated Cauchy logistic distribution and some of its mathematical properties.

Chapter 6: On the skew truncated Cauchy student' s t distribution

The chapter, we will study skew truncated Cauchy student's t distribution and some of its mathematical properties.

Chapter 7: On the skew truncated Cauchy Laplace distribution

This chapter will cover skew truncated Cauchy Laplace distribution and some of its mathematical properties.

Chapter 8: Conclusion

In this chapter, a summary of the findings and some ideas for future research will be presented.

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