

UNIVERSITI PUTRA MALAYSIA

MULTIPLE SOLUTIONS OF CONVECTION BOUNDARY LAYER FLOW FOR DIFFERENT TYPES OF FLUIDS WITH VARIOUS BOUNDARY CONDITIONS

SITI HIDAYAH BINTI MUHAD SALEH

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By

SITI HIDAYAH BINTI MUHAD SALEH

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

April 2017

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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April 2017

Chair: Associate Professor Norihan Md. Arifin, PhD Faculty: Science

In this study, similarity solutions of boundary layer flow and heat transfer in viscous fluid, micropolar fluid and nanofluid are considered for either mixed convection or magnetohydrodynamic (MHD)-forced convection. The objectives of the thesis are to analyse mathematical models of heat and mass transfer problems and to obtain the numerical results of each problem. The scope of this study is limited to two-dimensional or three-dimensional, steady or unsteady, incompressible, laminar boundary layer flows in viscous fluid, micropolar fluid or nanofluid. The first two problems are restricted to mixed convection effects while the rest are explored on forced convection. These problems are modeled to investigate and study their effects on a choice of fluids with various boundary conditions. The studies on stagnation point flow behavior have also been integrated including the non-aligned stagnation point. Besides, the effects of stretching/shrinking surface, permeable surface and also convective boundary condition have also been considered. Moreover, the consequence of moving wall also has been studied. The mathematical models for this problem are formulated, analyzed and simplified, and further transformed to non-dimensional form using non-dimensional variables. Next, the governing nonlinear partial differential equations are transformed to a system of ordinary differential equations using the similarity variables and are solved numerically using the shooting technique. Numerical results presented include the velocity, temperature, nanoparticle fraction (nanofluid) and angular velocity (micropolar fluid) profiles as well as the fluid flow and heat transfer characteristics for a range of the governing parameters. All the numerical solutions are presented in the form of tables and figures. Additionally, the existences of multiple solutions are contributed by the applied numerical method (shooting method) and the involvement of certain parameters in the system. The multiple solutions are reached for shrinking sheet case. Besides, it was also occur when mixed convection, suction and unsteadiness parameter added into the system of equations. The numerical results presented constitute an invaluable reference against which other exact or approximate solutions can be compared in the future.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

PENYELESAIAN BERGANDA BAGI ALIRAN LAPISAN SEMPADAN OLAKAN BAGI JENIS BENDALIR BERBEZA DENGAN PELBAGAI SYARAT SEMPADAN

Oleh

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April 2017

Pengerusi: Profesor Madya Norihan Md. Arifin, PhD Fakulti: Sains

Dalam kajian ini, penyelesaian persamaan aliran lapisan sempadan dan pemindahan haba dalam bendalir likat, bendalir mikropolar dan nanobendalir telah dipertimbangkan sama ada untuk olakan campuran dan juga olakan paksa magnetohidrodinamik (MHD). Objektif tesis ini adalah untuk membina model matematik bagi masalah pemindahan haba dan jisim serta mendapatkan penyelesaian berangka bagi setiap permasalahan. Skop kajian adalah tertumpu kepada dua dimensi atau tiga dimensi, mantap atau tak mantap, tak termampat, aliran lapisan sempadan dalam bendalir likat, bendalir mikrokutub atau nanobendalir. Dua permasalahan pertama mengkaji tentang olakan campuran manakala empat daripada masalah yang lain bertumpu kepada olakan paksa. Model matematik bagi permasalahan-permasalahan ini adalah bertujuan untuk mengkaji dan mempelajari tentang kesannya terhadap beberapa jenis bendalir pilihan dengan jenis syarat sempadan tertentu. Kajian terhadap aliran titik genangan turut disatukan termasuk titik genangan tak sejajar. Selain itu, kesan permukaan meregang atau mengecut, permukaan separa medium dan juga olakan syarat sempadan olakan juga dipertimbangkan. Tambahan lagi, kesan dinding bergerak turut dikaji. Modelmodel matematik bagi masalah ini diformulasi, dianalisis serta dipermudahkan dan kemudiannya dijelmakan kepada bentuk tak berdimensi dengan menggunakan pemboleh ubah tak berdimensi. Seterusnya, sistem persamaan menakluk dalam bentuk persamaan pembezaan separa tak linear tersebut kemudiannya dijelmakan ke bentuk sistem persamaan pembezaan biasa dengan menggunakan pemboleh-pemboleh ubah keserupaan. Kemudian, persamaan-persamaan ini kemudiannya diselesaikan secara berangka menggunakan kaedah 'tembakan'. Penyelesaian berangka yang diperoleh mengandungi profil-profil halaju, suhu, dan halaju sudut (bendalir dwikutub), di samping ciri-ciri aliran bendalir dan pemindahan haba bagi julat parameter tertakluk. Penyelesaian berangka diberikan dalam bentuk jadual dan rajah. Selain itu, kewujudan penyelesaian berganda telah disumbangkan oleh aplikasi kaedah berangka (kaedah tembakan) dan penglibatan parameter-parameter tertentu di dalam sistem. Penyelesaian berganda tersebut diperoleh pada kes permukaan mengecut. Selain itu juga, penyelesaian berganda juga berlaku apabila parameter olakan campuran, sedutan dan

ketakstabilan dimasukkan ke dalam sistem persamaan. Penyelesaian berangka yang dihasilkan boleh dijadikan sumber rujukan berharga untuk tujuan penyemakan keputusan penyelesaian tepat atau penyelesaian hampir pada masa hadapan.



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I certify that a Thesis Examination Committee has met on 21 April 2017 to conduct the final examination of Siti Hidayah binti Muhad Saleh on her thesis entitled "Multiple Solutions of Convection Boundary Layer Flow for Different Types of Fluids with Various Boundary Conditions" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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C

LIST OF ABBREVIATIONS

a,b,c,d	constant
Α	unsteadiness parameter
B_0	magnetic field normal to the wall
Bi	Biot number
С	constant nanoparticle fraction
C_{f}	skin friction coefficient
C_{fx} / C_{fy}	skin friction along x and y directions, respectively
C_m	couple stress coefficient
C_p	specific heat at constant pressure
$C_{_W}$	constant wall nanoparticle fraction
C_{∞}	constant nanoparticle fraction (inviscid flow)
D	mass diffusivity
$D_{\scriptscriptstyle B}$	Brownian diffusion coefficient
D_T	thermophoresis diffusion coefficient
Ε	constant for the boundary layer displacement
g	acceleration due to gravity
Gr_x	local Grashof number
h(t)	position of the wall
h_{f}	convective heat transfer coefficient
h_m	convective mass transfer coefficient
j	microinertia density
k	dimensionless curvature parameter
k _t	thermal conductivity
K	micropolar or material parameter
l	characteristic length of the flat plate
L	reference length
Le	Lewis number
Ln	Lewis number for nanofluid
m	parameter associated with plane flow and axissymmetric flow
Μ	magnetic parameter
n	ratio of the microrotation vector component and the fluid skin friction
N	at the wall
IN N/L	Droumion motion personator
IND NG	Brownion mouon parameter
INT	thermophoresis parameter
Nu _x	local Nusselt number

0	origin of the coordinate system
p / P	pressure
p_{e}	constant pressure
Pe	Peclet number
Pr	Prandtl number
$q_{\scriptscriptstyle m}$	mass flux
$q_{_{W}}$	heat flux
R	radius / distance of the sheet from origin
R_0	characteristic radius
Re _x	local Reynolds number
<i>r</i> , <i>s</i>	curvilinear coordinates
S	constant mass flux
Sh_x	local Sherwood number
t	time
Т	constant temperature
T_{f}	convective fluid temperature below the surface of the wall
T_w	temperature of the surface of the wall/sheet
T_{∞}	constant temperature (inviscid flow)
<i>u</i> , <i>v</i> , <i>w</i>	velocity component in the x , y and z direction, respectively
U	constant velocity
$U_{_e}$ / $U_{_\infty}$	freestream velocity of external flow
v_w / w_w	mass flux velocity
<i>x</i> , <i>y</i> , <i>z</i>	Cartesian coordinates along system
Greek Symbols	
α	thermal diffusivity
β	linearity constant
β_t	thermal expansion coefficient
γ	constant velocity ratio parameter (stretching / shrinking)
ξ	spin gradient viscosity

accelerated / decelerated surface coefficient

temperature coefficient of the surface tension

 δ non-alignment distance

ς

 ζ_T

- δ_h momentum boundary layer thickness
- δ_{T} thermal boundary layer thickness
- η pseudo-similarity variable
- θ dimensionless temperature
- κ vortex viscosity (or microrotation viscosity)

λ	constant mixed convection parameter
μ	dynamic viscosity
V	kinematic viscosity
ρ	fluid density
ε	electric conductivity
τ	ratio between the effective heat capacity of nanoparticle and heat
	capacity of the fluid
$\mathcal{T}_{s/w}$	skin friction or shear stress
σ	electrical conductivity
φ	solid volume fraction of the nanofluid
Ψ	stream function
$\overline{\nabla}^2$	Laplacian in Cartesian coordinates (x, y)
$\left(ho C_{p} ight)_{f}$	heat capacity of the fluid
$\left(ho C_p ight)_p$	heat capacity of the nanoparticle material

Superscript

'	differentiation with respect to a	7
-	dimensional variables	

Subscript

G

е	condition at the external flow
W	condition at the wall
∞	condition at infinity

CHAPTER 1

INTRODUCTION

1.1 Boundary Layer

The theory of boundary layer was first presented by German engineer, Ludwig Prandtl in 1904 at the Third International Congress of Mathematicians at Heidelberg. According to Prandtl's concept, when a real fluid flows past a stationary solid boundary, the flow was separated into two main regions (see Figure 1.1). The larger part concerns a free stream of fluid, far from any solid surface, which is considered to be inviscid. The flow behavior at this larger part is similar to the upstream flow where the effect of viscosity can be neglected. The smaller part is a thin layer adjacent to the solid boundary where the effects of skin friction, viscous force and rotation cannot be ignored is called the boundary layer (Acheson, 1990; Schlichting, 1979). All the way through experimental observations, Prandtl discovered that large velocity gradients normal to the streamlines merely happen in area that is close to the surface. Prandtl concluded that it might be adequate to concede viscosity effect within these boundary layers in an examination of a flow field, while the outer surface flow of the boundary layers may be considered inviscid. Most importantly, Prandtl shows that the Navier-Stokes equations can be simplified to obtain an approximate set of boundary layer equations (Bejan, 1984).

There are many reasons why the boundary layer theory is employed very often in solving fluid flow and heat transfer problems (Bejan, 2013b; Cebeci and Bradshaw, 1984). This is because the boundary layer equations are parabolic which is easier to solve compared to the full Navier-Stokes equations, either in elliptic or hyperbolic form and, are more complex and difficult. Further, the boundary layer theory also gives more information about the flow separation from the surface of a body than full Navier-Stokes equations.



Figure 1.1 : Boundary Layer (Acheson, 1990)



Figure 1.2 : Velocity and Thermal Boundary Layer (Ozisik, 1985)

Boundary layer was divided into two types which is velocity boundary layer and thermal boundary layer (Ozisik, 1985). In Figure 1.2, the experiment shows that all the fluid molecules that close to the wall was not move when the viscous fluids flow through the surface. For that reason, the velocity at the boundary was zero. Distant from the surface, fluid molecules velocity approximately approach to uniform value U_{∞} when A is higher than δ_h . The area of δ_h thickness is called velocity boundary layer where the velocity acceleration and shear stress lies in this area.

Basically, thermal boundary layer happens when surface temperature T_w is different from the surrounding temperature T_∞ . From Figure 1.2, if $T_w > T_\infty$, where T_w and T_∞ are surface temperature and surrounding temperature respectively, then the heat will move or transfer from solid surface to the fluid molecules on the surface. This energy will increase the inner energy of the fluid molecules and automatically will make the molecule temperature goes to T_∞ asymptotically at δ_T from the surface. Therefore, this area is named thermal boundary layer with δ_T thickness.

1.2 Boundary Layer Stagnation Point Flow

The stagnation point notes the position in the fluid flow where the approaching flow divides and get ahead to both sides along a surface. At a stagnation point, the fluid velocity is zero and all of the kinetic force has been transformed to an internal force and is added to the local static enthalpy. In terms of fluid mechanics, the point in the flow field where the local velocity of the fluid becomes zero is called a stagnation-point. This point located at the surface of the object where the fluid is brought to be at rest because of a force exerted by the object. The Bernoulli equation shows that the total pressure in provisions of static pressure is entitled stagnation where the pressure is at maximum value when the fluid velocity is zero (Jafar et al., 2011).



Figure 1.3 : Stagnation point on a plane flow (Schlichting, 1979)

Stagnation-point flows take place when a fluid approaches the impermeable boundary of a body, for example, on an aircraft wing or on an oscillating cylinder immersed in fluid. These flows have a stagnation-point exist in the fluid, about which the streamlines locally look like a saddle point. Another example of particular application is blood flow at a joint within an artery.

Figure 1.3 is the simplest model of this stagnation point flow, is that leading to a stagnation point in plane that is two-dimensional flow. The velocity allocation in the region of the stagnation point at x = y = 0 in frictionless potential flow is specified by U = ax and V = -ay where *a* denotes a constant. The figure clarifies a plane potential flow which comes from the *y*-axis and collides on a flat wall place at y = 0, splits into two streams on the wall and passes in both directions. The viscous flow must stick on to the wall, whereas the potential flow moves along it (Schlichting, 1979). The stagnation regions come across the highest pressure, the highest rate of heat transfer and the highest rate of mass decomposition (Nandy and Mahapatra, 2013).

The two dimensional stagnation point which flows moving towards a stationary plate was first studied by Hiemenz (1911), to transform the Navier-Stokes equations to nonlinear ordinary differential equations by applying similarity transformation.

1.3 Steady and Unsteady Flow

Steady flow is defined as that in which the conditions such as pressure, velocity and cross-section may differ from point to point but remain unchange with time. The flow is called as unsteady or non-steady if at any point in the fluid, the conditions amend with time (Massey, 1998; Biswas, 2003).

As mentioned above, the steady state form for a flow-field indicates that the velocity field and any ability or character related with the flow field stay unaffected with time. In other words, local derivative of the velocity is zero. Mathematically,

$$\frac{\partial u}{\partial t} = 0. \tag{1.1}$$

The concentration and temperature fields, if related will also be under steady state:

$$\frac{\partial T}{\partial t} = \frac{\partial C}{\partial t} = 0. \tag{1.2}$$

Extensively, the flows in all manufacturing appliances such as the components of a steam power plant (turbines, heat exchangers, compressor and pumps) are randomly assumed to be steady. The flow in the real-life systems, as an example, the helicopter rotor, the ship propeller, the cascades of turbo machinery blade, etc., usually is different with time and thus the flows are unsteady. Indeed, there is no real flow circumstance, natural or imitation, that does not include some unsteadiness. It is renowned that in practice many devices come across an even or rapid change in their aerodynamic situation. In many industrial purposes, unsteadiness is an important element of the problem. The helicopter rotor, the blades of turbomachinery, the ship propeller, etc., usually work in unsteady surroundings. Most of the basic ideas of unsteady viscous flows are expressed by Telionis (1981). The study of unsteady flows is basically more complicated than that of steady flows because unsteady flow situation may differ regarding both space and time, i.e., they are function of both space and time. For that reason, partial differential equations describe unsteady flows as the dependent variables are functions of more than one independent variable.

1.4 Viscous Fluid

The viscous term come from the Latin word, 'viscum' which means glue. Viscous fluid has an ability to cling at the solid's surface. This is one of the most important boundary condition, that is in mechanic of viscous fluid. Fluid friction was invented for the first time by Mariotte, 1620-1684 (Darus, 1989). It had been realized even before Prandtl that the discrepancies between the results of classical hydrodynamics and experiment were, in very many cases, due to the fact that the theory neglected fluid friction (Schlichting, 1979).

A Newtonian fluid is considered as a viscous fluid for which the shear stress is equivalent to the velocity slope or gradient (i.e. to the time-rate of strain), $\tau = \mu (\partial u / \partial y)$, τ is the shear stress, μ is the constant dynamic or absolute viscosity of the fluid and $\partial u / \partial y$ is the velocity slope or gradient that perpendicular to the direction of shear, while a kinematic viscosity is $v = \mu / \rho$ (Pavlov, 1974).

For a non-Newtonian fluid, the viscosity alters with the adapted strain rate (velocity gradient). Consequently, non-Newtonian fluids may not have a well-defined viscosity (Peddieson and Mcnitt, 1970). The Reynolds number is a dimensionless parameter defined as Re = UL/v where U indicates a typical flow velocity, L is a characteristic length range of the flow and v is the kinematic viscosity of the fluid. In addition, the Reynolds number represents a basic indication of the relative amplitudes of two important concept in the equations of motion (John, 1972). For the high Reynolds number flow, Re >> 1, indicates a motion of a fluid with low viscosity. Therefore, the viscous effects can be insignificant. While for the low Reynolds number flow, Re << 1 means a very high viscosity of flow.

In this thesis, we applied the Boussinesq approximation (Tritton, 1977) to formulate the mathematical model. The Boussinesq approximation is a method to solve nonisothermal flow, for example natural convection problems, without having to solve for the full compressible formulation of the Navier-Stokes equations. The Boussinesq approximation was a well-liked method for solving nonisothermal flow, particularly in previous years, as computational costs were lower when solving this method and convergence was more likely to be achieved. The approximation is accurate when density variations are small as this reduces the nonlinearity of the problem. It assumes that variations in density have no effect on the flow field, except that they give rise to buoyancy forces. In more practical terms, this approximation is typically used to model liquids around room temperature, natural ventilation in buildings, or dense gas dispersion in industrial set-ups.

While the Boussinesq approximation has been used to simplify the implementation of some Computational Fluid Dynamic solvers, its use these days is becoming less prevalent. This is because it only slightly reduces the nonlinearity of the system and, with today's solvers and computational hardware, consequently leads to marginal reduction in computational costs. A larger computational cost difference between the full Navier-Stokes equations and the Boussinesq approximation may indicate that the Boussinesq approximation is not valid.

1.5 Micropolar Fluid

Micropolar fluids can be described as fluids with microstructure. They can be allied to a category of fluids with nonsymmetric stress tensor called polar fluids. In physical point of view, micropolar fluids correspond to fluids which consist of rigid, randomly oriented (or spherical) elements suspended in a viscous medium, where the deformation of fluid particles is negligible. The model of micropolar fluids established in Eringen (1966) by C. A. Eringen is worth to investigate as a very well balanced one. First, it is a well-studied and considerable generalization of the classical Navier-Stokes model, including both in theory and applications, many more development than the classical one. Furthermore, it is well-designed and not too complex, in other words, convenient to mathematicians who explore its theory and, physicists and engineers who employ it. (Lukaszewicz, 1999).

The idea of the micropolar fluid flow theory established in the expansion of the constitutive equations for Newtonian fluids, in order that more intricated fluid such as lubrication and turbulent shear flows can be represent by this theory. The fundamental of micropolar fluid, first clarified by Eringen (1966). This theory has caught much significance, and many classical flows are being reviewed to verify the reactions of the microstructure fluid. This theory is a special class of the theory of micropolar fluids. The elements are permitted to go through only rigid rotations without stretch. Practically, the theory of micropolar fluid have need of that we must put in a transport equation that described the law of conservation of local angular momentum to the normal transport equations for the conservation of momentum and mass. Therefore, some extra local constitutive parameters are brought in.

The main idea in the growth of Eringen's microcontinum mechanics are the essentials of new kinematic variables, as an example, the microinertia moment tensor and gyration tensor, and the accumulation of the notion of stress moments, body moments, and microstress averages to classical continuum mechanics. These special notes of micropolar fluids were studied in a complete review paper of the theory and application of micropolar fluid mechanics by Ariman et al. (1973). Books by Lukaszewicz (1999) and Eringen (2001) offer a helpful explanation of the theory and wide-ranging studies of the literature of the micropolar fluid theory.

1.6 Nanofluid

Nanofluids is a fluid by dispersing solid nanoparticles in base fluid such as water and oil. Nanofluids are used to increase thermal conductivity, which goes up with increasing volumetric fraction of nanoparticles and it is concept to represent a fluid in which nanometer-sized particles are suspended in conventional heat transfer basic fluids. The nanofluid concept which was firstly introduced by Choi (1995), have remarkable characteristics that cause them have many practical applications in heat transfer, inclusive of microelectronics, pharmaceutical processes, fuel cells, and hybrid-powered engines. There have been published several recent papers in nanofluids.

Convectional heat transfer fluids, including water, oil, and ethylene glycol mixture are low heat transfer fluids, because the thermal conductivity of these fluids plays a significant function in determining the coefficient of heat transfer between the heat transfer surface and the heat transfer medium. Consequently, various methods have been used to develop the thermal conductivity of these fluids by suspending nanometer/micrometer-sized particle materials in liquids (Hamad et al., 2011). There are two models used by researchers; Tiwari and Das model and Buongiorno model. Buongiorno model applied for nanofluid incorporated the effets of thermophoresis and Brownian motion and this model depended on seven slip mechanisms: inertia, Brownian diffusion, diffusiophoresis, thermophoresis, Magnus effect, gravity settling, and fluid drainage. Buongiorno (2006) proceeded to write down conservation equations based on these two effects (Kuznetsov and Nield, 2010). His analysis however did not consider the influence of local velocity on the diffusion coefficients. Tiwari and Das (2007) have proposed a theoretical model to analyze the behaviour of nanofluids considering the solid volume fraction. It is found that both the Richardson number and the direction of the moving walls affect the fluid flow and heat transfer in the cavity.

1.7 Magnetohydrodynamic (MHD) Fluid Flow

Magnetohydrodynamics is one of the latest fluid mechanics branches. It is mainly concerned with the study of electrically conducting fluids and how they are influenced by magnetic field. Based on Faraday's laws of electromagnetism, when a conductor is passed through a magnetic flux, a current gets induced in the conductor. This current is in a direction that is mutually perpendicular to both the direction of the motion of the conductor and magnetic field. On the other hand, when a conductor carrying an electric current is placed in a magnetic flux, a conductor experiences a significant force that is in a direction that is mutually perpendicular to both the direction of the current and the magnetic field. Based on these it is, therefore, true to state that electromagnetic forces result within an electrically conducting fluid when in the influence of a magnetic field. Hydrodynamic forces within the fluid combine with the electromagnetic forces resulting to what is termed as magnetohydrodynamic (MHD) flow.

The model of MHD flow can be described by considering that the equations of motion account for the effects of electromagnetic forces and other forces such as inertial and hydrodynamic forces. The equations of motion are a combination of the Maxwell's equations of electromagnetism and Navier-Stokes equations of fluid dynamics. Therefore, they need to be solved simultaneously. Electrofluid mechanical energy conversion is linked to the interaction of the magnetic fluids with the electrically conducting fluids. The impacts of this interaction can clearly be observed in plasmas, two-phase mixtures, gases and liquids.

The latter presented applications have diverse technological applications ranging from heating and flow control in metal processing, two-phase mixtures resulting in power generation and the magnetic confinement of high temperature plasmas. Magnetogasdynamics, magnetofluidmechanics, and the widely used magnetohydrodynamics can be used to describe the extensive effects of electromagnetism in the electrically conducting fluids.

1.8 Types of Boundary Conditions

In this thesis, the effect of suction or injection, stretching or shrinking and convective boundary conditions at the boundary conditions are included in mathematical formulation of the problems.

1.8.1 Suction

Suction is one of the factors that influence the boundary layer control. Reduction of the pull on bodies in an external flow or reduction of the losses of energy in channels is one of the methods in the impediment of boundary layer separation. Suction implementation requires the surface to have holes which can be expounded to refer as perforations, slots and porous sections. The holes are vital for the sucking the portion of the boundary layer that is closest to the wall and which is travelling to the lowest possible velocity.

Practically, to increase the efficiency of diffusers that have a greater compression ratio of the working fluid (with large convergence angles), suction is applied to delay early boundary layer separation. Additionally, the increase in the lift and decrease drag of aerofoil operating at great incidence angles occur when the boundary layer suction through slots is exerted located close the trailing edge. Practically it has been demonstrated that suction through slots is less effective compared to suction in a porous wall. For instance, aerofoil, a similar increase of lift force can be attained by sucking a smaller amount of fluid through pores and slots.

Permeability is a measure of the potential of a porous media to transport fluids. It is an important feature in defining the flow capacity of a rock sample. The permeability of a rock is a degree of the easiness in which the rock will allow the passageway of fluids (see Figure 1.4). In other words, medium are permeable by cause of the presence of interrelated gaps through which water can move from high energy points to low energy points.

1.8.2 Stretching or Shrinking

Stretching sheet flow defined as the flow of fluid is induced when the elastic sheet in the incompressible fluid is being extended by an application of stress. This sheet has an elasticity behavior, means by an ability of a sheet to resist a distorting stress and to return to its original size and shape when the stress is removed. The movement of the stretched or shrinked sheet has velocity that alters or changes with the distance from a fixed point. In spite of that, shrinking sheet has an opposite nature with stretching one; the sheet is compressed and influences the fluid flow and the rate of transferring heat.



Figure 1.4 : Permeability (Wang, 2000)

The theoretical studies on boundary layer flow and heat transfer, driven by a stretching or shrinking sheet have been in numerous investigations, for the reason that this field has many industrial applications. Some of the aforementioned industrial applications of stretching sheet flow are extrusion of polymer sheets from a die, drawing of plastic films, wire drawing, polyester thin wall heat shrink tubing, and in glass as well as paper production.

It is worth noting that the quality of the final product in industrial applications depends largely on the heat transfer rate at the stretching or shrinking surface. Therefore, in order to achieve the desired properties of the material being manufactured, proper cooling fluid should be chosen and the flow of the cooling fluid cause by the stretching or shrinking sheet must be controlled. As a result, this calls for extra attention to be drawn for both flow and heat transfer characteristics of the cooling fluid medium in the manufacturing processes involving stretching or shrinking sheet.

1.8.3 Convective Boundary Condition

This convective boundary condition (sometimes called the Robin condition) says that conduction is equal to the convection. Consider a fluid over a sheet along the *x*-axis. The lower face of the sheet is in contact with another fluid at temperature T_f . The sheet is stretched and the fluid starts moving, this situation is called convective boundary condition and the boundary condition is,

$$-k_t \frac{\partial T}{\partial y} = h_f \left(T_f - T_w \right)$$



Figure 1.5 : The modes of heat transfer (Welty et al., 1976)

where T_f is the temperature of the hot fluid and h_f is the heat transfer coefficient. Due to the increase in the need for small-size units, the focus has been casted on the effects of the interaction between developments of the thermal boundary layer in both fluid streams, and of axial wall conduction, which usually affects heat exchange performance. In the practical applications of laser processing or laser damage, it often encounters the convective boundary conditions.

1.9 Heat Transfer

Heat transfer is determined as the thermal energy transfer from high temperature point to low temperature point. Heat transfer mechanisms are restricted into three kinds. The first is conduction, which is described as transport of heat going on via interceding substance without bulk movement of the substance. This kind of heat conduction can happen, for instance, through a turbine blade in a jet engine. The outer surface, which is opened to gases from the combustor, is at a higher temperature than the internal surface, which has cooling air close to it. The degree of the wall temperature is crucial for a turbine blade.

The second type is called radiation which is defined as transportation of energy through space or area without any required existence of substance. Radiation is the only process for heat transfer in space. Radiation can be significant even in conditions in which there is an intervening medium; a common example is the heat transfer from a fire (see Figure 1.5). The third heat transfer process is convection, or heat transfer by cause of a flowing fluid. The fluid can be a liquid or a gas. In convection heat transfer, the heat is changed through bulk transport of a non-uniform temperature fluid. Figure 1.5 shows the modes of heat transfer: conduction, radiation and convection.



Figure 1.6 : Types of Convection (Baehr and Stephan, 1994)

The convective mode of heat transfer basically occurs into three elementary processes, which are free, forced and mixed convection (Baehr and Stephan, 1994). Forced convection happens when fluid motion is generated mechanically by external forced like a fan, blower, nozzle or jet. Fluid motion related to a surface can be generated by moving an object, such as a missile, through a fluid.

Otherwise, the free convection happens when the fluid motion is generated by gravitational field. Occurrence of free convection requires fluid density change. In free convection, temperature changes are primarily due to variations in density. Whilst, combination of forced convection and natural convection, called mixed convection occurs when both mechanism act together to transfer heat. Figure 1.6 explains the natural and forced convection.

Fluid flow and heat transfer link to each other because of this continuity process from buoyancy to a difference in temperature. An increase in the rate of heat exchange normally uses forced convection. Heat radiator systems and regulatory temperature systems in the body's circulatory system, are examples of forced convection (Merkin and Pop, 2011). Convective heat transfer can also be classified as having either internal or external flow.

Free, forced and mixed convection processes may be divided into having an external flow over immersed body such as flat plates, cylinder, sphere or an internal flow in ducts such as pipes, channels and enclosures. The resultant flow can further be categorized as laminar (stable) or turbulent (unstable) flow. Laminar flow is smooth, with a particle of fluid moving steadily in a smooth line parallel to a surface, while on the other hand, turbulent flow is described as chaotic of fluid moving unsteadily.

Dimensionless Parameter	Symbol	Definition	Interpretation
Biot number	Bi	$\frac{h_f L}{k_i}$	Ratio of the internal thermal resistance of a solid to the boundary layer thermal resistance
Coefficient of Friction	C_{f}	$\frac{\tau_w}{\rho v^2/2}$	Dimensionless shear stress
Grashof number	Gr	$\frac{g\beta(T-T_{\infty})L^3}{v^2}$	Ratio of buoyancy to viscous forces
Lewis number	Le	$\frac{\alpha}{D}$	Ratio of thermal and mass diffusivities
Nusselt number	Nu	$\frac{hL}{k_t}$	Dimensionless temperature gradient at the surface
Prandtl number	Pr	$\frac{\nu}{\alpha}$	Ratio of the momentum and thermal diffusivities
Reynolds number	Re	$\frac{\nu L \rho}{\mu}$	Ratio of the inertial and viscous forces
Sherwood number	Sh	$\frac{h_m L}{D}$	Dimensionless concentration gradient at the surface

Table 1.1 : Selected dimensionless parameter of heat and mass transfer

In this thesis, there are dimensionless parameters that exist throughout the formulation and calculations. For references, these parameters are listed in Table 1.1

1.10 Motivation of Study

Studies on boundary layer flows have significantly increased our understanding of effective velocity and temperature within the zone of the boundary layer. Application of these studies include: high speed flows, pollutants emission, conveyor belts for materials handling as shown below and so forth (Zarrini and Pralhad, 2010).

The analysis of boundary layer flow and heat transfer of an incompressible fluid across a stretching sheet has gained attention of many researchers. Nowadays, a large amount of work has been placed to focus on this topic in view of its several applications in engineering and industrial processes. The cooling of electronic devices by the fan and nuclear reactor, polymer extrusion, wire drawing, etc are examples of such flows in engineering and industrial processes. The list of importance of flows in fluid mechanics has motivated researchers to continue the study in different types of fluid as well as in different physical aspects.

In a continuation study of flow over a stretching sheet, considerable interest has been placed on fluid flow over a shrinking sheet. For such problem, the movement of the sheet is in the opposite direction to that of the stretching case, and thus the flow moves towards a slot. It is worth noting that the quality of the final product in industrial applications depends largely on the heat transfer rate at the stretching or shrinking surface.

Despite of that, injection or suction of a liquid or fluid throughout the bounding surface, for instance, in the mass transfer cooling has a significant variation in the flow field which consequently affects the heat transfer rate from the plate. Generally suction alleviates the heat transfer coefficients and skin frictions (Al-Sanea, 2004).

The "nanofluid" term was first introduced by Choi and Eastman (1995) to describe the mixture of nanoparticles and base fluid such as water and oil. The addition of nanoparticle into the base fluid is able to change the transport properties, flow and heat transfer capability of the liquids and indirectly increase the low thermal conductivity of the base fluid which is identified as the main obstacle in heat transfer performance. This mixture has attracted the interest of numerous researchers because of its many significant applications such as in the medical applications, transportations, microelectronics, chemical engineering, aerospace and manufacturing.

Motivated by the importance and great influence of each effect and characters above, this study was encouraged to provide experimental data that will help on the improvement and validation of producing good quality of final product and efficient process in either industrial, manufacturing or engineering applications as well as .

Therefore, it would be useful to have a solution (or a better solution) for the problem involving those kind effect and parameter mentioned above, and to explore some new findings and idea on each characteristics found. That is why in the present study, we propose to investigate such a boundary layer flow and heat transfer.

1.11 Problem Statement

Boundary layer flow and heat transfer is an important type of flow occurring in several engineering processes. The boundary layer equations must be used in order to solve boundary layer problem. There are many ways in order to solve boundary layer equation where the result sometimes is not really accurate. The problem may occur if the calculations made have errors and are not accurate. So in order to solve the boundary layer equations, suitable numerical methods will be used.

Therefore, in this study, we want to investigate and to find any possible information on fluid flow as well as the characteristics of the fluid flow and heat transfer that is immensely useful in producing good quality products and efficient process. In other words, we will be examining the equations governing the mass and momentum balance as well as the appropriate similarity variables as well as its fundamental transformation.

Besides, some effects or additional characters that are injected to the system of boundary layer either on the governing equations or the boundary conditions will be studied to see which parameter shows significant effects and changes. In this thesis, we study and observe for steady or unsteady, stretching or shrinking, permeable or impermeable, moving or stationary effect.

1.12 Objectives and Scope

The objectives of the present study are to develop mathematical model, to provide mathematical formulation and analysis for the computation and to solve numerically the following problems:

- (a) The steady, two dimensional mixed convection boundary layer flow and heat transfer over a vertical plate in a viscous fluid. Also, the stagnation point and the stretching/shrinking axis are considered as non-aligned for this objective. The effect of the non-aligned on flow behavior will be discussed. In this problem, we analyze the effects of mixed convection parameter, λ, non-alignment distance parameter, δ, stretching/shrinking parameter, γ, as well as Prandtl number, Pr.
- (b) The unsteady, two dimensional mixed convection boundary layer flow and heat transfer of a viscous fluid in the vicinity of the stagnation point over a vertical plate moving along the direction of flow impingement. The main aim is to examine the fluid flow approaching normally onto a body which moves along the oncoming flow direction with a time-dependent velocity. Consequently, the effects of unsteadiness, A, λ, Pr, will be analyzed associated to plane flow or axissymmetric flow parameter, m.
- (c) Unsteady two dimensional micropolar fluid flow over a horizontal permeable stretching/shrinking surface with the influence of the nondimensional curvature radius on the physical quantities of interest including velocity and microrotation velocity. For this problem objective, we discussed the effect of A, micropolar parameter, K, curvature parameter, k, according to weak or strong concentration of micropolar fluid parameter, n, with the boundary condition effects of suction parameter, S and γ .

- (d) Two dimensional steady magnetohydrodynamics (MHD) flow and heat transfer of a nanofluid under the application of a constant applied magnetic field at the forward stagnation point of an infinite horizontal permeable stretching/shrinking wall with a convective boundary condition effects. By using Buongiorno model of nanofluid (Buongiorno, 2006), we also apply new boundary condition where the nanoparticle at boundary considered as zero. Accordingly, we explore the effects of magnetic parameter, M, Brownion motion parameter, Nb, thermophorosis parameter, Nt, Lewis number of nanofluid parameter, Ln, Biot number, Bi, Pr, S, and γ , in the system.
- (e) Steady flow and heat transfer of nanofluid over a non-linearly horizontal permeable shrinking sheet. Instead of using two-dimensional flow as stated in objectives (a) to (d), this problem looks at the three dimensional case. Also by using Buongiorno model of nanofluid, similar with (d) objectives where no nanoparticle flux boundary conditions applied. In view of that, the effects of linearity parameter, β , *Nb*, *Nt*, *Ln*, *Bi*, Pr, *S*, and γ will be examine.

The scope of study is limited to problems involving steady and unsteady, two and three dimensional forced and mixed convection, stagnation point boundary layer flow, immersed in viscous, micropolar fluids and nanofluids with the following boundary condition effects:

- (a) stretching and shrinking surface
- (b) permeable wall
- (c) convective boundary condition.

1.13 Outline of the Thesis

This thesis is divided into nine chapters including this introductory chapter. Chapter 1 is the preliminary chapter consisting of general introduction of boundary layer theory, stagnation point flow, and few important types of flow and fluid that will be considered in this thesis as well as some important parameters that are related to this study. Further, the problem statement, objectives, scope and the thesis outline that described briefly about this thesis also included in this chapter.

Literature review that contributes as references to this related topic was presented in Chapter 2. Next in Chapter 3, the derivation of mathematical model for steady mixed convection flow, unsteady micropolar fluid flow and MHD stagnation point flow are given.

Chapter 4 to Chapter 8 discussed the five main problems mentioned in previous section. Basically, each chapter started with introduction, follows by mathematical formulation or basic equation. Then, the results are presented and discussed in the next section. Finally, the conclusion will be stated at the end of each chapter.

Chapter 4 investigated the case of steady mixed convection flow close to a nonalignment stagnation point and a vertical shrinking surface. The problem of unsteady mixed convection stagnation point flow over a plate moving along the direction of flow impingement have been solved and discussed in Chapter 5. The next three chapters delivered the effect of permeable case.

In the next Chapter 6 will described on an unsteady micropolar fluid over a permeable curved stretching/shrinking surface. The study of MHD flow of a nanofluid at the forward stagnation point of an infinite permeable stretching/shrinking wall with a convective boundary condition effect have been done in Chapter 7. The three dimensional problem was described in Chapter 8 for the case of nanofluid flow over non-linearly permeable shrinking sheet. The last chapter of this thesis, namely Chapter 9, outlined the conclusions of the whole thesis and recommended future study related to this thesis.

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