UNIVERSITI PUTRA MALAYSIA

MULTIPLE SOLUTIONS OF CONVECTION BOUNDARY LAYER FLOW FOR DIFFERENT TYPES OF FLUIDS WITH VARIOUS BOUNDARY CONDITIONS

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By

SITI HIDAYAH BINTI MUHAD SALEH

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

April 2017

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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By

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April 2017

Chair: Associate Professor Norihan Md. Arifin, PhD
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In this study, similarity solutions of boundary layer flow and heat transfer in viscous fluid, micropolar fluid and nanofluid are considered for either mixed convection or magnetohydrodynamic (MHD)-forced convection. The objectives of the thesis are to analyse mathematical models of heat and mass transfer problems and to obtain the numerical results of each problem. The scope of this study is limited to two-dimensional or three-dimensional, steady or unsteady, incompressible, laminar boundary layer flows in viscous fluid, micropolar fluid or nanofluid. The first two problems are restricted to mixed convection effects while the rest are explored on forced convection. These problems are modeled to investigate and study their effects on a choice of fluids with various boundary conditions. The studies on stagnation point flow behavior have also been integrated including the non-aligned stagnation point. Besides, the effects of stretching/shrinking surface, permeable surface and also convective boundary condition have also been considered. Moreover, the consequence of moving wall also has been studied. The mathematical models for this problem are formulated, analyzed and simplified, and further transformed to non-dimensional form using non-dimensional variables. Next, the governing nonlinear partial differential equations are transformed to a system of ordinary differential equations using the similarity variables and are solved numerically using the shooting technique. Numerical results presented include the velocity, temperature, nanoparticle fraction (nanofluid) and angular velocity (micropolar fluid) profiles as well as the fluid flow and heat transfer characteristics for a range of the governing parameters. All the numerical solutions are presented in the form of tables and figures. Additionally, the existences of multiple solutions are contributed by the applied numerical method (shooting method) and the involvement of certain parameters in the system. The multiple solutions are reached for shrinking sheet case. Besides, it was also occur when mixed convection, suction and unsteadiness parameter added into the system of equations. The numerical results presented constitute an invaluable reference against which other exact or approximate solutions can be compared in the future.

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Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

PENYELESAIAN BERGANDA BAGI ALIRAN LAPISAN SEMPADAN OLAKAN BAGI JENIS BENDALIR BERBEZA DENGAN PELBAGAI SYARAT SEMPADAN

Oleh

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Dalam kajian ini, penyelesaian persamaan aliran lapisan sempadan dan pemindahan haba dalam bendalir likat, bendalir mikropolar dan nanobendalir telah dipertimbangkan sama ada untuk olakan campuran dan juga olakan paksa magnetohidrodinamik (MHD). Objektif tesis ini adalah untuk membina model matematik bagi masalah pemindahan haba dan jisim serta mendapatkan penyelesaian berangka bagi setiap permasalahan. Skop kajian adalah tertumpu kepada dua dimensi atau tiga dimensi, mantap atau tak mantap, tak termampat, aliran lapisan sempadan dalam bendalir likat, bendalir mikrokutub atau nanobendalir. Dua permasalahan pertama mengkaji tentang olakan campuran manakala empat daripada masalah yang lain bertumpu kepada olakan paksa. Model matematik bagi permasalahan-permasalahan ini adalah bertujuan untuk mengkaji dan mempelajari tentang kesannya terhadap beberapa jenis bendalir pilihan dengan jenis syarat sempadan tertentu. Kajian terhadap aliran titik genangan turut disatukan termasuk titik genangan tak sejajar. Selain itu, kesan permukaan meregang atau mengecut, permukaan separa medium dan juga olakan syarat sempadan olakan juga dipertimbangkan. Tambahan lagi, kesan dinding bergerak turut dikaji. Model-model matematik bagi masalah ini diformulasi, dianalisis serta dipermudahkan dan kemudiannya dijelmakan kepada bentuk tak berdimensi dengan menggunakan pemboleh ubah tak berdimensi. Seterusnya, sistem persamaan menakluk dalam bentuk persamaan pembezaan separa tak linear tersebut kemudiannya dijelmakan ke bentuk sistem persamaan pembezaan biasa dengan menggunakan pemboleh-pemboleh ubah keserupaan. Kemudian, persamaan-persamaan ini kemudiannya diselesaikan secara berangka menggunakan kaedah ‘tembakan’. Penyelesaian berangka yang diperoleh mengandungi profil-profil halaju, suhu, dan halaju sudut (bendalir dwikutub), di samping ciri-ciri aliran bendalir dan pemindahan haba bagi julat parameter tertakluk. Penyelesaian berangka diberikan dalam bentuk jadual dan rajah. Selain itu, kewujudan penyelesaian berganda telah disumbangkan oleh aplikasi kaedah berangka (kaedah tembakan) dan penglibatan parameter-parameter tertentu di dalam sistem. Penyelesaian berganda tersebut diperoleh pada kes permukaan mengecut. Selain itu juga, penyelesaian berganda juga berlaku apabila parameter olakan campuran, sedutan dan

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LIST OF ABBREVIATIONS

a, b, c, d	constant
A	unsteadiness parameter
B_0	magnetic field normal to the wall
Bi	Biot number
C	constant nanoparticle fraction
C_f	skin friction coefficient
C_{fx} / C_{fy}	skin friction along x and y directions, respectively
C_m	couple stress coefficient
C_p	specific heat at constant pressure
C_w	constant wall nanoparticle fraction
C_∞	constant nanoparticle fraction (inviscid flow)
D	mass diffusivity
D_B	Brownian diffusion coefficient
D_T	thermophoresis diffusion coefficient
E	constant for the boundary layer displacement
g	acceleration due to gravity
Gr_x	local Grashof number
$h(t)$	position of the wall
h_f	convective heat transfer coefficient
h_m	convective mass transfer coefficient
j	microinertia density
k	dimensionless curvature parameter
k_t	thermal conductivity
K	micropolar or material parameter
l	characteristic length of the flat plate
L	reference length
Le	Lewis number
Ln	Lewis number for nanofluid
m	parameter associated with plane flow and axissymmetric flow
M	magnetic parameter
n	ratio of the microrotation vector component and the fluid skin friction at the wall
N	microrotation or angular velocity
Nb	Brownion motion parameter
Nt	thermophoresis parameter
Nu_x	local Nusselt number

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O	origin of the coordinate system
p / P	pressure
p_e	constant pressure
Pe	Peclet number
Pr	Prandtl number
q_m	mass flux
q_w	heat flux
R	radius / distance of the sheet from origin
R_0	characteristic radius
Re_x	local Reynolds number
r, s	curvilinear coordinates
S	constant mass flux
Sh_x	local Sherwood number
t	time
T	constant temperature
T_f	convective fluid temperature below the surface of the wall
T_w	temperature of the surface of the wall/sheet
T_∞	constant temperature (inviscid flow)
u, v, w	velocity component in the x, y and z direction, respectively
U	constant velocity
U_e / U_∞	freestream velocity of external flow
v_w / w_w	mass flux velocity
x, y, z	Cartesian coordinates along system

Greek Symbols

α	thermal diffusivity
β	linearity constant
β_t	thermal expansion coefficient
γ	constant velocity ratio parameter (stretching / shrinking)
ξ	spin gradient viscosity
ς	accelerated / decelerated surface coefficient
ζ_T	temperature coefficient of the surface tension
δ	non-alignment distance
δ_h	momentum boundary layer thickness
δ_T	thermal boundary layer thickness
η	pseudo-similarity variable
θ	dimensionless temperature
κ	vortex viscosity (or microrotation viscosity)

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λ	constant mixed convection parameter
μ	dynamic viscosity
ν	kinematic viscosity
ρ	fluid density
ε	electric conductivity
τ	ratio between the effective heat capacity of nanoparticle and heat capacity of the fluid
$\tau_{s/w}$	skin friction or shear stress
σ	electrical conductivity
φ	solid volume fraction of the nanofluid
ψ	stream function
$\bar{\nabla}^2$	Laplacian in Cartesian coordinates (x, y)
$(\rho C_p)_f$	heat capacity of the fluid
$(\rho C_p)_p$	heat capacity of the nanoparticle material

Superscript

'	differentiation with respect to η
$-$	dimensional variables

Subscript

e	condition at the external flow
w	condition at the wall
∞	condition at infinity

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CHAPTER 1

INTRODUCTION

1.1 Boundary Layer

The theory of boundary layer was first presented by German engineer, Ludwig Prandtl in 1904 at the Third International Congress of Mathematicians at Heidelberg. According to Prandtl's concept, when a real fluid flows past a stationary solid boundary, the flow was separated into two main regions (see Figure 1.1). The larger part concerns a free stream of fluid, far from any solid surface, which is considered to be inviscid. The flow behavior at this larger part is similar to the upstream flow where the effect of viscosity can be neglected. The smaller part is a thin layer adjacent to the solid boundary where the effects of skin friction, viscous force and rotation cannot be ignored is called the boundary layer (Acheson, 1990; Schlichting, 1979). All the way through experimental observations, Prandtl discovered that large velocity gradients normal to the streamlines merely happen in area that is close to the surface. Prandtl concluded that it might be adequate to concede viscosity effect within these boundary layers in an examination of a flow field, while the outer surface flow of the boundary layers may be considered inviscid. Most importantly, Prandtl shows that the Navier-Stokes equations can be simplified to obtain an approximate set of boundary layer equations (Bejan, 1984).

There are many reasons why the boundary layer theory is employed very often in solving fluid flow and heat transfer problems (Bejan, 2013b; Cebeci and Bradshaw, 1984). This is because the boundary layer equations are parabolic which is easier to solve compared to the full Navier-Stokes equations, either in elliptic or hyperbolic form and, are more complex and difficult. Further, the boundary layer theory also gives more information about the flow separation from the surface of a body than full Navier-Stokes equations.

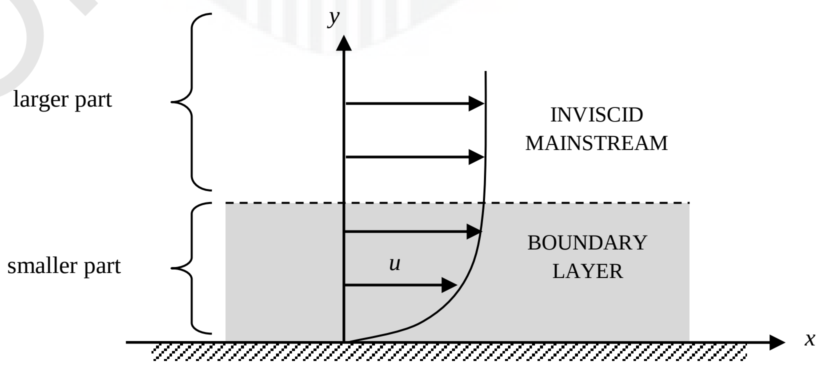

Figure 1.1 : Boundary Layer (Acheson, 1990)

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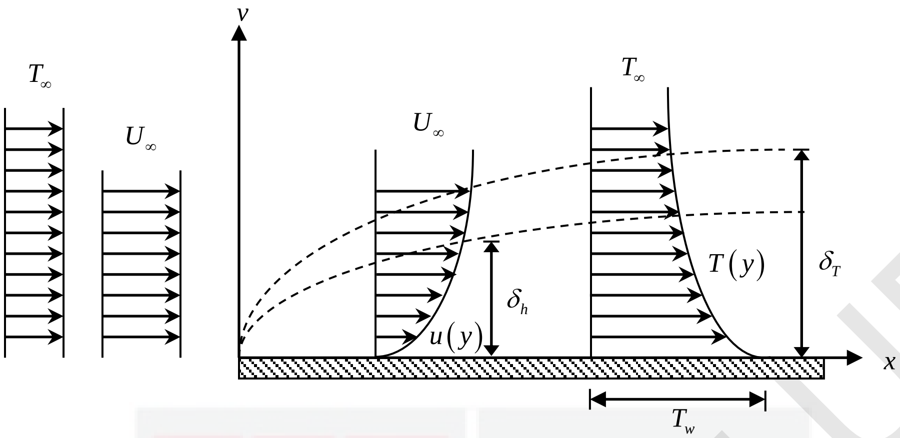

Figure 1.2 : Velocity and Thermal Boundary Layer (Ozisik, 1985)

Boundary layer was divided into two types which is velocity boundary layer and thermal boundary layer (Ozisik, 1985). In Figure 1.2, the experiment shows that all the fluid molecules that close to the wall was not move when the viscous fluids flow through the surface. For that reason, the velocity at the boundary was zero. Distant from the surface, fluid molecules velocity approximately approach to uniform value U_∞ when A is higher than δ_h. The area of δ_h thickness is called velocity boundary layer where the velocity acceleration and shear stress lies in this area.

Basically, thermal boundary layer happens when surface temperature T_w is different from the surrounding temperature T_∞. From Figure 1.2, if $T_w > T_\infty$, where T_w and T_∞ are surface temperature and surrounding temperature respectively, then the heat will move or transfer from solid surface to the fluid molecules on the surface. This energy will increase the inner energy of the fluid molecules and automatically will make the molecule temperature goes to T_∞ asymptotically at δ_T from the surface. Therefore, this area is named thermal boundary layer with δ_T thickness.

1.2 Boundary Layer Stagnation Point Flow

The stagnation point notes the position in the fluid flow where the approaching flow divides and get ahead to both sides along a surface. At a stagnation point, the fluid velocity is zero and all of the kinetic force has been transformed to an internal force and is added to the local static enthalpy. In terms of fluid mechanics, the point in the flow field where the local velocity of the fluid becomes zero is called a stagnation-point. This point located at the surface of the object where the fluid is brought to be at rest because of a force exerted by the object. The Bernoulli equation shows that the total pressure in provisions of static pressure is entitled stagnation where the pressure is at maximum value when the fluid velocity is zero (Jafar et al., 2011).

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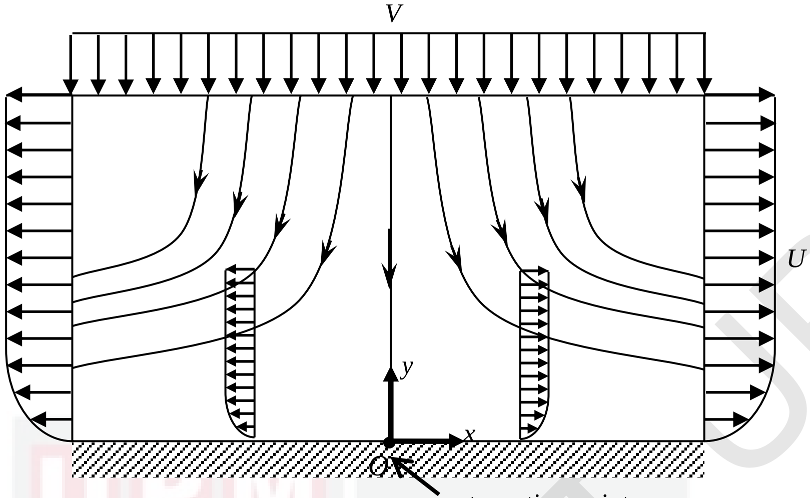

Figure 1.3 : Stagnation point on a plane flow (Schlichting, 1979)

Stagnation-point flows take place when a fluid approaches the impermeable boundary of a body, for example, on an aircraft wing or on an oscillating cylinder immersed in fluid. These flows have a stagnation-point exist in the fluid, about which the streamlines locally look like a saddle point. Another example of particular application is blood flow at a joint within an artery.

Figure 1.3 is the simplest model of this stagnation point flow, is that leading to a stagnation point in plane that is two-dimensional flow. The velocity allocation in the region of the stagnation point at $x = y = 0$ in frictionless potential flow is specified by $U = ax$ and $V = -ay$ where a denotes a constant. The figure clarifies a plane potential flow which comes from the y-axis and collides on a flat wall place at $y = 0$, splits into two streams on the wall and passes in both directions. The viscous flow must stick on to the wall, whereas the potential flow moves along it (Schlichting, 1979). The stagnation regions come across the highest pressure, the highest rate of heat transfer and the highest rate of mass decomposition (Nandy and Mahapatra, 2013).

The two dimensional stagnation point which flows moving towards a stationary plate was first studied by Hiemenz (1911), to transform the Navier-Stokes equations to nonlinear ordinary differential equations by applying similarity transformation.

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1.3 Steady and Unsteady Flow

Steady flow is defined as that in which the conditions such as pressure, velocity and cross-section may differ from point to point but remain unchange with time. The flow is called as unsteady or non-steady if at any point in the fluid, the conditions amend with time (Massey, 1998; Biswas, 2003).

As mentioned above, the steady state form for a flow-field indicates that the velocity field and any ability or character related with the flow field stay unaffected with time. In other words, local derivative of the velocity is zero. Mathematically,

$$\frac{\partial u}{\partial t} = 0. \tag{1.1}$$

The concentration and temperature fields, if related will also be under steady state:

$$\frac{\partial T}{\partial t} = \frac{\partial C}{\partial t} = 0. \tag{1.2}$$

Extensively, the flows in all manufacturing appliances such as the components of a steam power plant (turbines, heat exchangers, compressor and pumps) are randomly assumed to be steady. The flow in the real-life systems, as an example, the helicopter rotor, the ship propeller, the cascades of turbo machinery blade, etc., usually is different with time and thus the flows are unsteady. Indeed, there is no real flow circumstance, natural or imitation, that does not include some unsteadiness. It is renowned that in practice many devices come across an even or rapid change in their aerodynamic situation. In many industrial purposes, unsteadiness is an important element of the problem. The helicopter rotor, the blades of turbomachinery, the ship propeller, etc., usually work in unsteady surroundings. Most of the basic ideas of unsteady viscous flows are expressed by Telionis (1981). The study of unsteady flows is basically more complicated than that of steady flows because unsteady flow situation may differ regarding both space and time, i.e., they are function of both space and time. For that reason, partial differential equations describe unsteady flows as the dependent variables are functions of more than one independent variable.

1.4 Viscous Fluid

The viscous term come from the Latin word, ‘viscum’ which means glue. Viscous fluid has an ability to cling at the solid’s surface. This is one of the most important boundary condition, that is in mechanic of viscous fluid. Fluid friction was invented for the first time by Mariotte, 1620-1684 (Darus, 1989). It had been realized even before Prandtl that the discrepancies between the results of classical hydrodynamics and experiment were, in very many cases, due to the fact that the theory neglected fluid friction (Schlichting, 1979) .

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A Newtonian fluid is considered as a viscous fluid for which the shear stress is equivalent to the velocity slope or gradient (i.e. to the time-rate of strain), $\tau = \mu\left(\partial u/\partial y\right)$, τ is the shear stress, μ is the constant dynamic or absolute viscosity of the fluid and $\partial u/\partial y$ is the velocity slope or gradient that perpendicular to the direction of shear, while a kinematic viscosity is $\nu = \mu/\rho$ (Pavlov, 1974).

For a non-Newtonian fluid, the viscosity alters with the adapted strain rate (velocity gradient). Consequently, non-Newtonian fluids may not have a well-defined viscosity (Peddieson and Mcnitt, 1970). The Reynolds number is a dimensionless parameter defined as $\mathrm{Re} = UL/\nu$ where U indicates a typical flow velocity, L is a characteristic length range of the flow and ν is the kinematic viscosity of the fluid. In addition, the Reynolds number represents a basic indication of the relative amplitudes of two important concept in the equations of motion (John, 1972). For the high Reynolds number flow, $\mathrm{Re} >> 1$, indicates a motion of a fluid with low viscosity. Therefore, the viscous effects can be insignificant. While for the low Reynolds number flow, $\mathrm{Re} << 1$ means a very high viscosity of flow.

In this thesis, we applied the Boussinesq approximation (Tritton, 1977) to formulate the mathematical model. The Boussinesq approximation is a method to solve nonisothermal flow, for example natural convection problems, without having to solve for the full compressible formulation of the Navier-Stokes equations. The Boussinesq approximation was a well-liked method for solving nonisothermal flow, particularly in previous years, as computational costs were lower when solving this method and convergence was more likely to be achieved. The approximation is accurate when density variations are small as this reduces the nonlinearity of the problem. It assumes that variations in density have no effect on the flow field, except that they give rise to buoyancy forces. In more practical terms, this approximation is typically used to model liquids around room temperature, natural ventilation in buildings, or dense gas dispersion in industrial set-ups.

While the Boussinesq approximation has been used to simplify the implementation of some Computational Fluid Dynamic solvers, its use these days is becoming less prevalent. This is because it only slightly reduces the nonlinearity of the system and, with today's solvers and computational hardware, consequently leads to marginal reduction in computational costs. A larger computational cost difference between the full Navier-Stokes equations and the Boussinesq approximation may indicate that the Boussinesq approximation is not valid.

1.5 Micropolar Fluid

Micropolar fluids can be described as fluids with microstructure. They can be allied to a category of fluids with nonsymmetric stress tensor called polar fluids. In physical point of view, micropolar fluids correspond to fluids which consist of rigid, randomly oriented (or spherical) elements suspended in a viscous medium, where the

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deformation of fluid particles is negligible. The model of micropolar fluids established in Eringen (1966) by C. A. Eringen is worth to investigate as a very well balanced one. First, it is a well-studied and considerable generalization of the classical Navier-Stokes model, including both in theory and applications, many more development than the classical one. Furthermore, it is well-designed and not too complex, in other words, convenient to mathematicians who explore its theory and, physicists and engineers who employ it. (Lukaszewicz, 1999).

The idea of the micropolar fluid flow theory established in the expansion of the constitutive equations for Newtonian fluids, in order that more intricated fluid such as lubrication and turbulent shear flows can be represent by this theory. The fundamental of micropolar fluid, first clarified by Eringen (1966). This theory has caught much significance, and many classical flows are being reviewed to verify the reactions of the microstructure fluid. This theory is a special class of the theory of micropolar fuids. The elements are permitted to go through only rigid rotations without stretch. Practically, the theory of micropolar fluid have need of that we must put in a transport equation that described the law of conservation of local angular momentum to the normal transport equations for the conservation of momentum and mass. Therefore, some extra local constitutive parameters are brought in.

The main idea in the growth of Eringen's microcontinum mechanics are the essentials of new kinematic variables, as an example, the microinertia moment tensor and gyration tensor, and the accumulation of the notion of stress moments, body moments, and microstress averages to classical continuum mechanics. These special notes of micropolar fluids were studied in a complete review paper of the theory and application of micropolar fluid mechanics by Ariman et al. (1973). Books by Lukaszewicz (1999) and Eringen (2001) offer a helpful explanation of the theory and wide-ranging studies of the literature of the micropolar fluid theory.

1.6 Nanofluid

Nanofluids is a fluid by dispersing solid nanoparticles in base fluid such as water and oil. Nanofluids are used to increase thermal conductivity, which goes up with increasing volumetric fraction of nanoparticles and it is concept to represent a fluid in which nanometer-sized particles are suspended in conventional heat transfer basic fluids. The nanofluid concept which was firstly introduced by Choi (1995), have remarkable characteristics that cause them have many practical applications in heat transfer, inclusive of microelectronics, pharmaceutical processes, fuel cells, and hybrid-powered engines. There have been published several recent papers in nanofluids.

Convectional heat transfer fluids, including water, oil, and ethylene glycol mixture are low heat transfer fluids, because the thermal conductivity of these fluids plays a significant function in determining the coefficient of heat transfer between the heat transfer surface and the heat transfer medium. Consequently, various methods have been used to develop the thermal conductivity of these fluids by suspending nanometer/micrometer-sized particle materials in liquids (Hamad et al., 2011). There

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are two models used by researchers; Tiwari and Das model and Buongiorno model. Buongiorno model applied for nanofluid incorporated the effets of thermophoresis and Brownian motion and this model depended on seven slip mechanisms: inertia, Brownian diffusion, diffusiophoresis, thermophoresis, Magnus effect, gravity settling, and fluid drainage. Buongiorno (2006) proceeded to write down conservation equations based on these two effects (Kuznetsov and Nield, 2010). His analysis however did not consider the influence of local velocity on the diffusion coefficients. Tiwari and Das (2007) have proposed a theoretical model to analyze the behaviour of nanofluids considering the solid volume fraction. It is found that both the Richardson number and the direction of the moving walls affect the fluid flow and heat transfer in the cavity.

1.7 Magnetohydrodynamic (MHD) Fluid Flow

Magnetohydrodynamics is one of the latest fluid mechanics branches. It is mainly concerned with the study of electrically conducting fluids and how they are influenced by magnetic field. Based on Faraday’s laws of electromagnetism, when a conductor is passed through a magnetic flux, a current gets induced in the conductor. This current is in a direction that is mutually perpendicular to both the direction of the motion of the conductor and magnetic field. On the other hand, when a conductor carrying an electric current is placed in a magnetic flux, a conductor experiences a significant force that is in a direction that is mutually perpendicular to both the direction of the current and the magnetic field. Based on these it is, therefore, true to state that electromagnetic forces result within an electrically conducting fluid when in the influence of a magnetic field. Hydrodynamic forces within the fluid combine with the electromagnetic forces resulting to what is termed as magnetohydrodynamic (MHD) flow.

The model of MHD flow can be described by considering that the equations of motion account for the effects of electromagnetic forces and other forces such as inertial and hydrodynamic forces. The equations of motion are a combination of the Maxwell’s equations of electromagnetism and Navier-Stokes equations of fluid dynamics. Therefore, they need to be solved simultaneously. Electrofluid mechanical energy conversion is linked to the interaction of the magnetic fluids with the electrically conducting fluids. The impacts of this interaction can clearly be observed in plasmas, two-phase mixtures, gases and liquids.

The latter presented applications have diverse technological applications ranging from heating and flow control in metal processing, two-phase mixtures resulting in power generation and the magnetic confinement of high temperature plasmas. Magnetogasdynamics, magnetofluidmechanics, and the widely used magnetohydrodynamics can be used to describe the extensive effects of electromagnetism in the electrically conducting fluids.

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1.8 Types of Boundary Conditions

In this thesis, the effect of suction or injection, stretching or shrinking and convective boundary conditions at the boundary conditions are included in mathematical formulation of the problems.

1.8.1 Suction

Suction is one of the factors that influence the boundary layer control. Reduction of the pull on bodies in an external flow or reduction of the losses of energy in channels is one of the methods in the impediment of boundary layer separation. Suction implementation requires the surface to have holes which can be expounded to refer as perforations, slots and porous sections. The holes are vital for the sucking the portion of the boundary layer that is closest to the wall and which is travelling to the lowest possible velocity.

Practically, to increase the efficiency of diffusers that have a greater compression ratio of the working fluid (with large convergence angles), suction is applied to delay early boundary layer separation. Additionally, the increase in the lift and decrease drag of aerofoil operating at great incidence angles occur when the boundary layer suction through slots is exerted located close the trailing edge. Practically it has been demonstrated that suction through slots is less effective compared to suction in a porous wall. For instance, aerofoil, a similar increase of lift force can be attained by sucking a smaller amount of fluid through pores and slots.

Permeability is a measure of the potential of a porous media to transport fluids. It is an important feature in defining the flow capacity of a rock sample. The permeability of a rock is a degree of the easiness in which the rock will allow the passageway of fluids (see Figure 1.4). In other words, medium are permeable by cause of the presence of interrelated gaps through which water can move from high energy points to low energy points.

1.8.2 Stretching or Shrinking

Stretching sheet flow defined as the flow of fluid is induced when the elastic sheet in the incompressible fluid is being extended by an application of stress. This sheet has an elasticity behavior, means by an ability of a sheet to resist a distorting stress and to return to its original size and shape when the stress is removed. The movement of the stretched or shrinked sheet has velocity that alters or changes with the distance from a fixed point. In spite of that, shrinking sheet has an opposite nature with stretching one; the sheet is compressed and influences the fluid flow and the rate of transferring heat.

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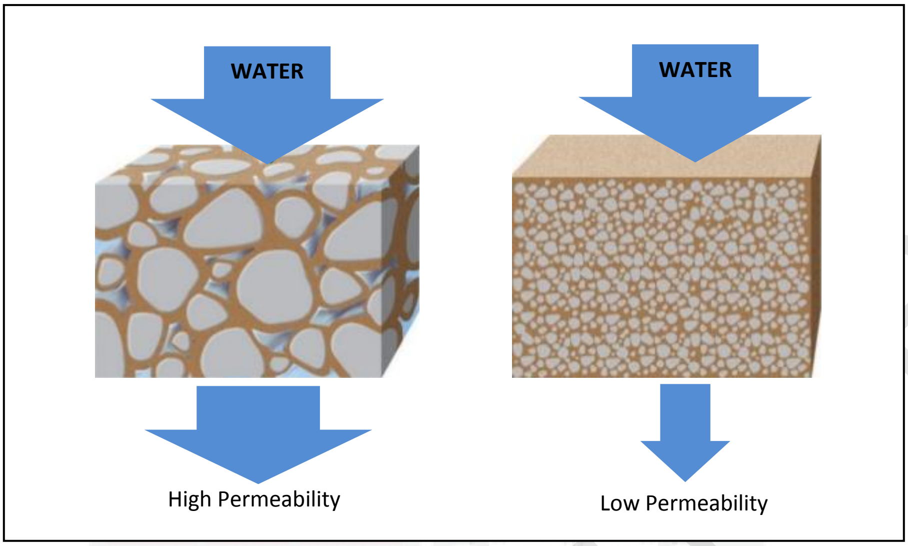

Figure 1.4 : Permeability (Wang, 2000)

The theoretical studies on boundary layer flow and heat transfer, driven by a stretching or shrinking sheet have been in numerous investigations, for the reason that this field has many industrial applications. Some of the aforementioned industrial applications of stretching sheet flow are extrusion of polymer sheets from a die, drawing of plastic films, wire drawing, polyester thin wall heat shrink tubing, and in glass as well as paper production.

It is worth noting that the quality of the final product in industrial applications depends largely on the heat transfer rate at the stretching or shrinking surface. Therefore, in order to achieve the desired properties of the material being manufactured, proper cooling fluid should be chosen and the flow of the cooling fluid cause by the stretching or shrinking sheet must be controlled. As a result, this calls for extra attention to be drawn for both flow and heat transfer characteristics of the cooling fluid medium in the manufacturing processes involving stretching or shrinking sheet.

1.8.3 Convective Boundary Condition

This convective boundary condition (sometimes called the Robin condition) says that conduction is equal to the convection. Consider a fluid over a sheet along the *x*-axis. The lower face of the sheet is in contact with another fluid at temperature T_f . The sheet is stretched and the fluid starts moving, this situation is called convective boundary condition and the boundary condition is,

$$-k_t \frac{\partial T}{\partial y} = h_f \left(T_f - T_w\right)$$

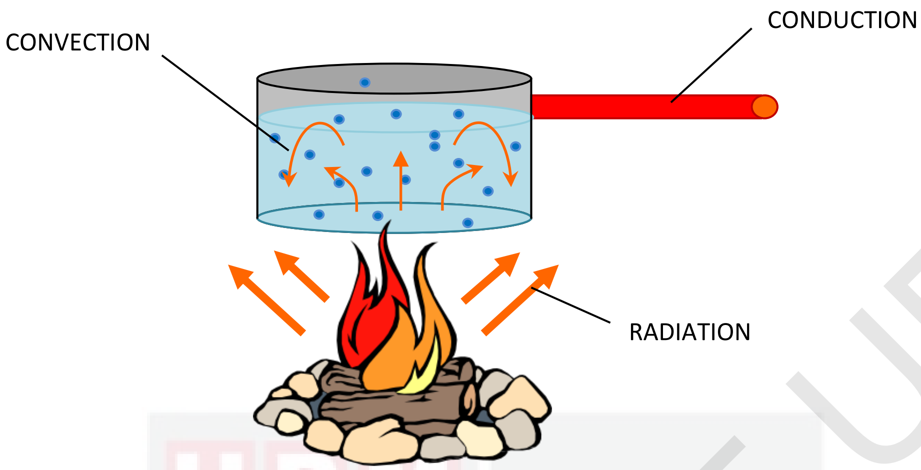

Figure 1.5 : The modes of heat transfer (Welty et al., 1976)

where T_f is the temperature of the hot fluid and h_f is the heat transfer coefficient. Due to the increase in the need for small-size units, the focus has been casted on the effects of the interaction between developments of the thermal boundary layer in both fluid streams, and of axial wall conduction, which usually affects heat exchange performance. In the practical applications of laser processing or laser damage, it often encounters the convective boundary conditions.

1.9 Heat Transfer

Heat transfer is determined as the thermal energy transfer from high temperature point to low temperature point. Heat transfer mechanisms are restricted into three kinds. The first is conduction, which is described as transport of heat going on via interceding substance without bulk movement of the substance. This kind of heat conduction can happen, for instance, through a turbine blade in a jet engine. The outer surface, which is opened to gases from the combustor, is at a higher temperature than the internal surface, which has cooling air close to it. The degree of the wall temperature is crucial for a turbine blade.

The second type is called radiation which is defined as transportation of energy through space or area without any required existence of substance. Radiation is the only process for heat transfer in space. Radiation can be significant even in conditions in which there is an intervening medium; a common example is the heat transfer from a fire (see Figure 1.5). The third heat transfer process is convection, or heat transfer by cause of a flowing fluid. The fluid can be a liquid or a gas. In convection heat transfer, the heat is changed through bulk transport of a non-uniform temperature fluid. Figure 1.5 shows the modes of heat transfer: conduction, radiation and convection.

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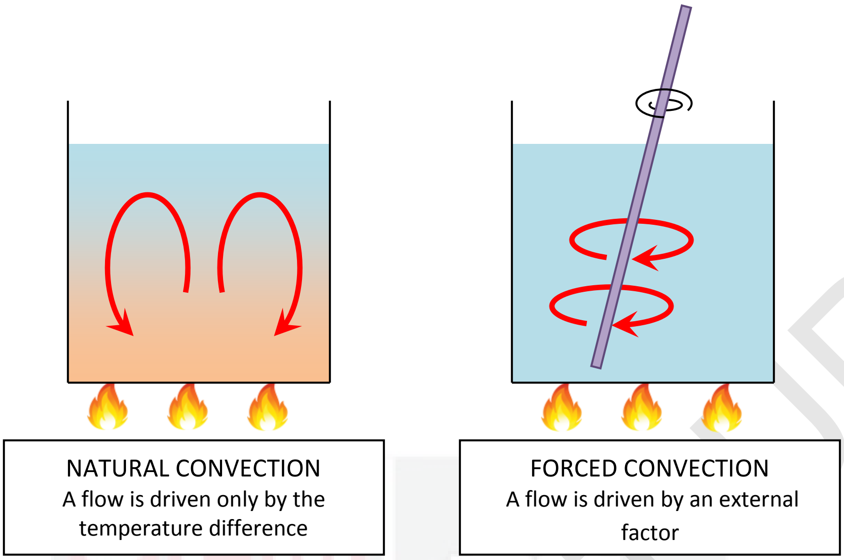

Figure 1.6 : Types of Convection (Baehr and Stephan, 1994)

The convective mode of heat transfer basically occurs into three elementary processes, which are free, forced and mixed convection (Baehr and Stephan, 1994). Forced convection happens when fluid motion is generated mechanically by external forced like a fan, blower, nozzle or jet. Fluid motion related to a surface can be generated by moving an object, such as a missile, through a fluid.

Otherwise, the free convection happens when the fluid motion is generated by gravitational field. Occurrence of free convection requires fluid density change. In free convection, temperature changes are primarily due to variations in density. Whilst, combination of forced convection and natural convection, called mixed convection occurs when both mechanism act together to transfer heat. Figure 1.6 explains the natural and forced convection.

Fluid flow and heat transfer link to each other because of this continuity process from buoyancy to a difference in temperature. An increase in the rate of heat exchange normally uses forced convection. Heat radiator systems and regulatory temperature systems in the body's circulatory system, are examples of forced convection (Merkin and Pop, 2011). Convective heat transfer can also be classified as having either internal or external flow.

Free, forced and mixed convection processes may be divided into having an external flow over immersed body such as flat plates, cylinder, sphere or an internal flow in ducts such as pipes, channels and enclosures. The resultant flow can further be categorized as laminar (stable) or turbulent (unstable) flow. Laminar flow is smooth, with a particle of fluid moving steadily in a smooth line parallel to a surface, while on the other hand, turbulent flow is described as chaotic of fluid moving unsteadily.

Table 1.1 : Selected dimensionless parameter of heat and mass transfer

Dimensionless Parameter	Symbol	Definition	Interpretation
Biot number	Bi	$\frac{h_f L}{k_t}$	Ratio of the internal thermal resistance of a solid to the boundary layer thermal resistance
Coefficient of Friction	C_f	$\frac{\tau_w}{\rho v^2/2}$	Dimensionless shear stress
Grashof number	Gr	$\frac{g\beta(T-T_\infty)L^3}{v^2}$	Ratio of buoyancy to viscous forces
Lewis number	Le	$\frac{\alpha}{D}$	Ratio of thermal and mass diffusivities
Nusselt number	Nu	$\frac{hL}{k_t}$	Dimensionless temperature gradient at the surface
Prandtl number	Pr	$\frac{v}{\alpha}$	Ratio of the momentum and thermal diffusivities
Reynolds number	Re	$\frac{vL\rho}{\mu}$	Ratio of the inertial and viscous forces
Sherwood number	Sh	$\frac{h_m L}{D}$	Dimensionless concentration gradient at the surface

In this thesis, there are dimensionless parameters that exist throughout the formulation and calculations. For references, these parameters are listed in Table 1.1

1.10 Motivation of Study

Studies on boundary layer flows have significantly increased our understanding of effective velocity and temperature within the zone of the boundary layer. Application of these studies include: high speed flows, pollutants emission, conveyor belts for materials handling as shown below and so forth (Zarrini and Pralhad, 2010).

The analysis of boundary layer flow and heat transfer of an incompressible fluid across a stretching sheet has gained attention of many researchers. Nowadays, a large amount of work has been placed to focus on this topic in view of its several applications in engineering and industrial processes. The cooling of electronic devices by the fan and

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nuclear reactor, polymer extrusion, wire drawing, etc are examples of such flows in engineering and industrial processes. The list of importance of flows in fluid mechanics has motivated researchers to continue the study in different types of fluid as well as in different physical aspects.

In a continuation study of flow over a stretching sheet, considerable interest has been placed on fluid flow over a shrinking sheet. For such problem, the movement of the sheet is in the opposite direction to that of the stretching case, and thus the flow moves towards a slot. It is worth noting that the quality of the final product in industrial applications depends largely on the heat transfer rate at the stretching or shrinking surface.

Despite of that, injection or suction of a liquid or fluid throughout the bounding surface, for instance, in the mass transfer cooling has a significant variation in the flow field which consequently affects the heat transfer rate from the plate. Generally suction alleviates the heat transfer coefficients and skin frictions (Al-Sanea, 2004).

The “nanofluid” term was first introduced by Choi and Eastman (1995) to describe the mixture of nanoparticles and base fluid such as water and oil. The addition of nanoparticle into the base fluid is able to change the transport properties, flow and heat transfer capability of the liquids and indirectly increase the low thermal conductivity of the base fluid which is identified as the main obstacle in heat transfer performance. This mixture has attracted the interest of numerous researchers because of its many significant applications such as in the medical applications, transportations, microelectronics, chemical engineering, aerospace and manufacturing.

Motivated by the importance and great influence of each effect and characters above, this study was encouraged to provide experimental data that will help on the improvement and validation of producing good quality of final product and efficient process in either industrial, manufacturing or engineering applications as well as .

Therefore, it would be useful to have a solution (or a better solution) for the problem involving those kind effect and parameter mentioned above, and to explore some new findings and idea on each characteristics found. That is why in the present study, we propose to investigate such a boundary layer flow and heat transfer.

1.11 Problem Statement

Boundary layer flow and heat transfer is an important type of flow occurring in several engineering processes. The boundary layer equations must be used in order to solve boundary layer problem. There are many ways in order to solve boundary layer equation where the result sometimes is not really accurate. The problem may occur if the calculations made have errors and are not accurate. So in order to solve the boundary layer equations, suitable numerical methods will be used.

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Therefore, in this study, we want to investigate and to find any possible information on fluid flow as well as the characteristics of the fluid flow and heat transfer that is immensely useful in producing good quality products and efficient process. In other words, we will be examining the equations governing the mass and momentum balance as well as the appropriate similarity variables as well as its fundamental transformation.

Besides, some effects or additional characters that are injected to the system of boundary layer either on the governing equations or the boundary conditions will be studied to see which parameter shows significant effects and changes. In this thesis, we study and observe for steady or unsteady, stretching or shrinking, permeable or impermeable, moving or stationary effect.

1.12 Objectives and Scope

The objectives of the present study are to develop mathematical model, to provide mathematical formulation and analysis for the computation and to solve numerically the following problems:

(a) The steady, two dimensional mixed convection boundary layer flow and heat transfer over a vertical plate in a viscous fluid. Also, the stagnation point and the stretching/shrinking axis are considered as non-aligned for this objective. The effect of the non-aligned on flow behavior will be discussed. In this problem, we analyze the effects of mixed convection parameter, λ, non-alignment distance parameter, δ, stretching/shrinking parameter, γ, as well as Prandtl number, $\Pr$.

(b) The unsteady, two dimensional mixed convection boundary layer flow and heat transfer of a viscous fluid in the vicinity of the stagnation point over a vertical plate moving along the direction of flow impingement. The main aim is to examine the fluid flow approaching normally onto a body which moves along the oncoming flow direction with a time-dependent velocity. Consequently, the effects of unsteadiness, A, λ, $\Pr$, will be analyzed associated to plane flow or axissymmetric flow parameter, m.

(c) Unsteady two dimensional micropolar fluid flow over a horizontal permeable stretching/shrinking surface with the influence of the nondimensional curvature radius on the physical quantities of interest including velocity and microrotation velocity. For this problem objective, we discussed the effect of A, micropolar parameter, K, curvature parameter, k, according to weak or strong concentration of micropolar fluid parameter, n, with the boundary condition effects of suction parameter, S and γ.

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(d) Two dimensional steady magnetohydrodynamics (MHD) flow and heat transfer of a nanofluid under the application of a constant applied magnetic field at the forward stagnation point of an infinite horizontal permeable stretching/shrinking wall with a convective boundary condition effects. By using Buongiorno model of nanofluid (Buongiorno, 2006), we also apply new boundary condition where the nanoparticle at boundary considered as zero. Accordingly, we explore the effects of magnetic parameter, M, Brownion motion parameter, Nb, thermophorosis parameter, Nt, Lewis number of nanofluid parameter, Ln, Biot number, Bi, Pr, S, and γ, in the system.

(e) Steady flow and heat transfer of nanofluid over a non-linearly horizontal permeable shrinking sheet. Instead of using two-dimensional flow as stated in objectives (a) to (d), this problem looks at the three dimensional case. Also by using Buongiorno model of nanofluid, similar with (d) objectives where no nanoparticle flux boundary conditions applied. In view of that, the effects of linearity parameter, β, Nb, Nt, Ln, Bi, Pr, S, and γ will be examine.

The scope of study is limited to problems involving steady and unsteady, two and three dimensional forced and mixed convection, stagnation point boundary layer flow, immersed in viscous, micropolar fluids and nanofluids with the following boundary condition effects:

(a) stretching and shrinking surface
(b) permeable wall
(c) convective boundary condition.

1.13 Outline of the Thesis

This thesis is divided into nine chapters including this introductory chapter. Chapter 1 is the preliminary chapter consisting of general introduction of boundary layer theory, stagnation point flow, and few important types of flow and fluid that will be considered in this thesis as well as some important parameters that are related to this study. Further, the problem statement, objectives, scope and the thesis outline that described briefly about this thesis also included in this chapter.

Literature review that contributes as references to this related topic was presented in Chapter 2. Next in Chapter 3, the derivation of mathematical model for steady mixed convection flow, unsteady micropolar fluid flow and MHD stagnation point flow are given.

Chapter 4 to Chapter 8 discussed the five main problems mentioned in previous section. Basically, each chapter started with introduction, follows by mathematical formulation or basic equation. Then, the results are presented and discussed in the next section. Finally, the conclusion will be stated at the end of each chapter.

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Chapter 4 investigated the case of steady mixed convection flow close to a non-alignment stagnation point and a vertical shrinking surface. The problem of unsteady mixed convection stagnation point flow over a plate moving along the direction of flow impingement have been solved and discussed in Chapter 5. The next three chapters delivered the effect of permeable case.

In the next Chapter 6 will described on an unsteady micropolar fluid over a permeable curved stretching/shrinking surface. The study of MHD flow of a nanofluid at the forward stagnation point of an infinite permeable stretching/shrinking wall with a convective boundary condition effect have been done in Chapter 7. The three dimensional problem was described in Chapter 8 for the case of nanofluid flow over non-linearly permeable shrinking sheet. The last chapter of this thesis, namely Chapter 9, outlined the conclusions of the whole thesis and recommended future study related to this thesis.

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REFERENCES

Abbas, Z., Naveed, M., & Sajid, M. (2013). Heat transfer analysis for stretching flow over a curved surface with magnetic field. *Journal of Engineering Thermophysics, 22*(4), 337-345.

Abbas, Z., Naveed, M., & Sajid, M. (2016). Hydromagnetic slip flow of nanofluid over a curved stretching surface with heat generation and thermal radiation. *Journal of Molecular Liquids, 215*(2016), 756-762.

Abbas, Z., Wang, Y., Hayat, T., & Oberlack, M. (2010). Mixed convection in the stagnation-point flow of a Maxwell fluid towards a vertical stretching surface. *Nonlinear Analysis: Real World Applications, 11*(4), 3218-3228.

Acheson, D. J. (1990). *Elementary fluid dynamics (Oxford Applied Mathematics and Computing Science Series)*: Oxford : Oxford University Press.

Agarwal, S. (2014). Natural convection in a nanofluid-saturated rotating porous layer: a more realistic approach. *Transport in Porous Media, 104*(3), 581-592.

Akbar, N. S., Nadeem, S., Ul Haq, R., & Khan, Z. H. (2013). Radiation effects on MHD stagnation point flow of nano fluid towards a stretching surface with convective boundary condition. *Chinese Journal of Aeronautics, 26*(6), 1389-1397.

Ali, M. E. (2006). The effect of variable viscosity on mixed convection heat transfer along a vertical moving surface. *International Journal of Thermal Sciences, 45*(1), 60-69.

Alsaedi, A., Awais, M., & Hayat, T. (2012). Effects of heat generation/absorption on stagnation point flow of nanofluid over a surface with convective boundary conditions. *Communications in Nonlinear Science and Numerical Simulation, 17*(11), 4210-4223.

Andersson, V. P. H. (1992). MHD flow of a viscoelastic fluid past a stretching surface. *Acta Mechanica, 95*(1), 227-230.

Al-Sanea, S. A. (2004). Mixed convection heat transfer along a continuously moving heated vertical plate with suction or injection. *International Journal of Heat and Mass Transfer, 47*(6), 1445-1465.

Arifin, N. M., Nazar, R., & Pop, I. (2010). Marangoni driven boundary layer flow past a flat plate in nanofluid with suction/injection. *Proceedings in AIPConference Proceedings, Edited by Cenap Ozel and Adem Kilicman, 1309*(1), 94-99.

Arifin, N. M., Nazar, R., & Pop, I. (2011a). Non-isobaric Marangoni boundary layer flow for Cu, Al2O3 and TiO2 nanoparticles in a water based fluid. *Meccanica, 46*(4), 833-843.

© COPYRIGHT UPM

Arifin, N. M., Nazar, R., & Pop, I. (2011b). Viscous flow due to a permeable stretching/shrinking sheet in a nanofluid. *Sains Malaysiana, 40*(12), 1359-1367.

Arifin, N. M., Nazar, R., & Pop, I. (2013). Similarity solution of marangoni convection boundary layer flow over a flat surface in a nanofluid. *Journal of Applied Mathematics,Vol. 2013*, Article ID 634746, 8 pages, 2013.

Ariman, T., Turk, M., & Sylvester, N. (1973). Microcontinuum fluid mechanics - a review. *International Journal of Engineering Science, 11*(8), 905-930.

Ariman, T., Turk, M. A., & Sylvester, N. D. (1974). Applications of microcontinuum fluid mechanics. *International Journal of Engineering Science, 12*(4), 273-293.

Attia, H. A. (2008). Stagnation point flow and heat transfer of a micropolar fluid with uniform suction or blowing. *Brazil Society of Mechanical Science and Engineering, 5*(1), 51-55.

Aurangzaib, Bhattacharyya, K., & Shafie, S. (2016). Effect of partial slip on an unsteady MHD mixed convection stagnation-point flow of a micropolar fluid towards a permeable shrinking sheet. *Alexandria Engineering Journal, 55*(2), 1285-1293.

Aziz, A. (2009). A similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition. *Communications in Nonlinear Science and Numerical Simulation, 14*(4), 1064-1068.

Aziz, A., & Khan, W. A. (2012). Natural convective boundary layer flow of a nanofluid past a convectively heated vertical plate. *International Journal of Thermal Sciences, 52*(2012), 83-90.

Bachok, N., Ishak, A., & Pop, I. (2010). Melting heat transfer in boundary layer stagnation-point flow towards a stretching/shrinking sheet. *Physics Letters A, 374*(40), 4075-4079.

Bachok, N., Ishak, A., & Pop, I. (2011). Stagnation-point flow over a stretching/shrinking sheet in a nanofluid. *Nanoscale Research Letters, 6*(1), 1-10.

Bachok, N., Ishak, A., & Pop, I. (2012). Unsteady boundary-layer flow and heat transfer of a nanofluid over a permeable stretching/shrinking sheet. *International Journal of Heat and Mass Transfer, 55*(7), 2102-2109.

Baehr, H. D., & Stephan, K. (1994). *Wärme-und Stoffübertragung* (Vol. 6): Berlin, Springer.

Bejan, A. (1984). *Convection heat transfer*: John Wiley & Sons.

© COPYRIGHT UPM

Bejan, A. (2013a). Technology evolution, from the constructal law. In Y. I. C. J. P. A. Ephraim M. Sparrow & M. G. John (Eds.), *Advances in Heat Transfer, 45*(3), 183-207.

Bejan, A. (2013b). *Convection heat transfer*: John wiley & sons.

Bhargava, R., Sharma, S., Takhar, H., Bég, O. A., & Bhargava, P. (2007). Numerical solutions for micropolar transport phenomena over a nonlinear stretching sheet. *Nonlinear Anal. Model. Control, 12*(2007), 45-63.

Bhattacharyya, K. (2013). Heat transfer analysis in unsteady boundary layer stagnation-point flow towards a shrinking/stretching sheet. *Ain Shams Engineering Journal, 4*(2), 259-264.

Biswas, G. (2003). *Introduction to fluid mechanics and fluid machines, 2e*: Tata McGraw-Hill Education.

Buongiorno, J. (2006). Convective transport in nanofluids. *Journal of heat transfer, 128*(3), 240-250.

Cebeci, T., & Bradshaw, P. (1984). *Physical and computational aspects of convective heat transfer*: Springer Science & Business Media.

Chakrabarti, A., & Gupta, A. (1979). Hydromagnetic flow and heat transfer over a stretching sheet, *Quarterly of Applied Mathematics, 37*(1), 73-78.

Chamkha, A. J., Takhar, H. S., & Nath, G. (2001). Effect of buoyancy forces on the flow and heat transfer over a continuous moving vertical or inclined surface. *International Journal of Thermal Sciences, 40*(9), 825-833.

Chand, R., & Rana, G. (2014). Thermal instability in a Brinkman porous medium saturated by nanofluid with no nanoparticle flux on boundaries. *Special Topics & Reviews in Porous Media: An International Journal, 5*(4), 277-286.

Char, M. I. (1994). Heat transfer in a hydromagnetic flow over a stretching sheet. *Wärme-und Stoffübertragung, 29*(8), 495-500.

Chen, H. (2014). Mixed convection unsteady stagnation-point flow towards a stretching sheet with slip effects. *Mathematical problems in Engineering, 2014*(2014), ID435679.

Choi, S., Zhang, Z., Yu, W., Lockwood, F., & Grulke, E. (2001). Anomalous thermal conductivity enhancement in nanotube suspensions. *Applied physics letters, 79*(14), 2252-2254.

Choi, S. U. S., & Eastman, J. A. (1995). Enhancing thermal conductivity of fluids with nanoparticles. *ASME FED, 231*(66), 99-105.

Crane, L. J. (1970). Flow past a stretching plate. *Zeitschrift für angewandte Mathematik und Physik ZAMP, 21* (4), 645-647.

© COPYRIGHT UPM

Darus, A. N. (1989). *Aliran Bendalir Likat, Suatu Pengenalan*. Kuala Lumpur: Dewan Bahasa dan Pustaka.

Damseh, R. A., Al-Odat, M. Q., Chamkha, A. J., & Shannak, B. A. (2009). Combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface. *International Journal of Thermal Sciences, 48*(8), 1658-1663.

Devi, C., Takhar, H., & Nath, G. (1991). Unsteady mixed convection flow in stagnation region adjacent to a vertical surface. *Wärme-und Stoffübertragung, 26*(2), 71-79.

Dey, J., & Nath, G. (1981). Mixed convection row on vertical surface. *Wärme-und Stoffübertragung, 15*(4), 279-283.

Eringen, A. C. (1966). Theory of micropolar fluids. *Journal of Mathematics & Mechanics, 16*, 1-18.

Eringen, A. C. (1972). Theory of thermomicropolar fluids. *Journal Mathematical Analysis & Applications, 38*(2), 480-496.

Eringen, A. C. (2001). *Microcontinuum field theories: II. Fluent media* (Vol. 2): Springer Science & Business Media.

Fan, T., Xu, H., & Pop, I. (2010). Unsteady stagnation flow and heat transfer towards a shrinking sheet. *International Communications in Heat and Mass Transfer, 37*(10), 1440-1446.

Fang, T. (2014). Flow and mass transfer for an unsteady stagnation-point flow over a moving wall considering blowing effect. *ASME Journal of Fluids Engineering, 136*(7), 071103-1 - 071103-7.

Goodarz, A. (1976). Self-similar solution of imcompressible micropolar boundary layer flow over a semi-infinite plate. *International Journal of Engineering Science, 14*(7), 639-646.

Gorla, R. S. R. (1983). Micropolar boundary layer flow at a stagnation point on a moving wall. *International Journal of Engineering Science, 21*(1), 25-33.

Gorla, R. and Sidawi, I. (1994). Free convection on a vertical stretching surface with suction and blowing. *Applied Scientific Research, 52* (3), 247-257.

Guram, G. S., & Smith, A. C. (1980). Stagnation flows of micropolar fluids with strong and weak interactions. *Computers Mathematics with Applications, 6*(2), 213-233.

Hamad, M. A. A., & Ferdows, M. (2012). Similarity solution of boundary layer stagnation-point flow towards a heated porous stretching sheet saturated with a nanofluid with heat absorption/generation and suction/blowing: A Lie group analysis. *Communications in Nonlinear Science and Numerical Simulation, 17*(1), 132-140.

© COPYRIGHT UPM

Hamad, M. A. A., Pop, I., & Md Ismail, A. I. (2011). Magnetic field effects on free convection flow of a nanofluid past a vertical semi-infinite flat plate. *Nonlinear Analysis: Real World Applications, 12*(3), 1338-1346.

Hamad, M. A. A., Uddin, M. J., & Ismail, A. I. M. (2012). Radiation effects on heat and mass transfer in MHD stagnation-point flow over a permeable flat plate with thermal convective surface boundary condition, temperature dependent viscosity and thermal conductivity. *Nuclear Engineering and Design, 242*(2012), 194-200.

Hassani, M., Mohammad Tabar, M., Nemati, H., Domairry, G., & Noori, F. (2011). An analytical solution for boundary layer flow of a nanofluid past a stretching sheet. *International Journal of Thermal Sciences, 50*(11), 2256-2263.

Hayat, T., Abbas, Z., & Pop, I. (2008a). Mixed convection in the stagnation point flow adjacent to a vertical surface in a viscoelastic fluid. *International Journal of Heat and Mass Transfer, 51*(11), 3200-3206.

Hayat, T., Abbas, Z., & Javed, T. (2008b). Mixed convection flow of a micropolar fluid over a non-linearly stretching sheet. *Physics Letters A, 372*(5), 637-647.

Hayat, T., Muhammad, T., Shehzad, S. A., & Alsaedi, A. (2016a). On three-dimensional boundary layer flow of Sisko nanofluid with magnetic field effects. *Advanced Powder Technology, 27*(2), 504-512.

Hayat, T., Aziz, A., Muhammad, T., & Alsaedi, A. (2016b). On magnetohydrodynamic three-dimensional flow of nanofluid over a convectively heated nonlinear stretching surface. *International Journal of Heat and Mass Transfer, 100*(2016), 566-572.

Hayat, T., Shehzad, S., & Qasim, M. (2011). Mixed convection flow of a micropolar fluid with radiation and chemical reaction. *International Journal for Numerical Methods in Fluids, 67*(11), 1418-1436.

Hieber, C. (1973). Mixed convection above a heated horizontal surface. *International Journal of Heat and Mass Transfer, 16*(4), 769-785.

Hiemenz, K. (1911). *Die Grenzschicht an einem in den gleichformigen Flussigkeitsstrom eingetauchten geraden Kreiszylinder*: Berlin : Weber.

Ibrahim, W., & Shankar, B. (2013). MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions. *Computers & Fluids, 75*(2013), 1-10.

Ibrahim, W., Shankar, B., & Nandeppanavar, M. M. (2013). MHD stagnation point flow and heat transfer due to nanofluid towards a stretching sheet. *International Journal of Heat and Mass Transfer, 56*(1), 1-9.

Ingham, D. (1986). Singular and non-unique solutions of the boundary-layer equations for the flow due to free convection near a continuously moving vertical plate. *Zeitschrift für angewandte Mathematik und Physik ZAMP, 37*(4), 559-572.

© COPYRIGHT UPM

Ishak, A. (2010). Similarity solutions for flow and heat transfer over a permeable surface with convective boundary condition. *Applied Mathematics and Computation, 217*(2), 837-842.

Ishak, A., Nazar, R., Bachok, N., & Pop, I. (2010). MHD mixed convection flow near the stagnation-point on a vertical permeable surface. *Physica A: Statistical Mechanics and its Applications, 389*(1), 40-46.

Ishak, A., Nazar, R., & Pop, I. (2006a). Mixed convection boundary layers in the stagnation-point flow toward a stretching vertical sheet. *Meccanica, 41*(5), 509-518.

Ishak, A., Nazar, R., & Pop, I. (2006b). Moving wedge and flat plate in a micropolar fluid. *International Journal of Engineering Science, 44*(18), 1225-1236.

Jackson, J. D., Cotton, M. A., & Axcell, B. P. (1989). Studies of mixed convection in vertical tubes. *International Journal of Heat and Fluid Flow, 10*(1), 2-15.

Jafar, K., Ishak, A., & Nazar, R. (2011). MHD stagnation-point flow over a nonlinearly stretching/shrinking sheet. *Journal of Aerospace Engineering, 26*(4), 829-834.

Jena, S. K., & Mathur, M. N. (1981). Similarity solutions for laminar free convection flow of a thermomicropolar fluid past a non-isothermal vertical flat plate. *International Journal of Engineering Science, 19*(11), 1431-1439.

John, P., Jr. (1972). An application of the micropolar fluid model to the calculation of a turbulent shear flow. *International Journal of Engineering Science, 10*(1), 23-32.

Kandasamy, R., Gunasekaran, K., & Hasan, S. b. H. (2011). Scaling group transformation on fluid flow with variable stream conditions. *International Journal of Non-Linear Mechanics, 46*(7), 976-985.

Kandasamy, R., Muhaimin, I., & Mohamad, R. (2013). Thermophoresis and Brownian motion effects on MHD boundary-layer flow of a nanofluid in the presence of thermal stratification due to solar radiation. *International Journal of Mechanical Sciences, 70*(2013), 146-154.

Karwe, M., & Jaluria, Y. (1988). Fluid flow and mixed convection transport from a moving plate in rolling and extrusion processes. *Journal of heat transfer, 110*(3), 655-661.

Katagiri, M. (1969). Magnetohydrodynamic Flow with Suction or Injection at the Forward Stagnation Point. *Journal of the Physical Society of Japan, 27*(6), 1677-1685.

Khan, J. A., Mustafa, M., Hayat, T., & Alsaedi, A. (2015). Three-dimensional flow of nanofluid over a non-linearly stretching sheet: An application to solar energy. *International Journal of Heat and Mass Transfer, 86*(2015), 158-164.

© COPYRIGHT UPM

Khan, W. A., & Aziz, A. (2011). Natural convection flow of a nanofluid over a vertical plate with uniform surface heat flux. *International Journal of Thermal Sciences, 50*(7), 1207-1214.

Khan, W. A., & Pop, I. (2010). Boundary-layer flow of a nanofluid past a stretching sheet. *International Journal of Heat and Mass Transfer, 53*(11), 2477-2483.

Kimiaeifar, A., Bagheri, G. H., Rahimpour, M., & Mehrabian, M. A. (2009). Analytical solution of a two-dimensional stagnation flow in the vicinity of a shrinking sheet by means of the homotopy analysis method. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering, 223*(3), 133-143.

Kumari, M., Takhar, H. S., & Nath, G. (1990). MHD flow and heat transfer over a stretching surface with prescribed wall temperature or heat flux. *Heat and Mass Transfer, 25*(6), 331-336.

Kuznetsov, A. V., & Nield, D. A. (2010). Natural convective boundary-layer flow of a nanofluid past a vertical plate. *International Journal of Thermal Sciences, 49*(2), 243-247.

Kuznetsov, A. V., & Nield, D. A. (2013). The Cheng–Minkowycz problem for natural convective boundary layer flow in a porous medium saturated by a nanofluid: A revised model. *International Journal of Heat and Mass Transfer, 65*(2013), 682-685.

Kuznetsov, A. V., & Nield, D. A. (2014). Natural convective boundary-layer flow of a nanofluid past a vertical plate: A revised model. *International Journal of Thermal Sciences, 77*(2014), 126-129.

Labropulu, F., Dorrepaal, J. M., & Chandna, O. P. (1996). Oblique flow impinging on a wall with suction or blowing. *Acta Mechanica, 115*(1-4), 15-25.

Lok, Y. Y., Amin, N., & Pop, I. (2003). Unsteady boundary layer flow of a micropolar fluid near the rear stagnation point of a plane surface. *International Journal of Thermal Sciences, 42*(11), 995-1001.

Lok, Y. Y., Amin, N., & Pop, I. (2006). Unsteady mixed convection flow of a micropolar fluid near the stagnation point on a vertical surface. *International Journal of Thermal Sciences, 45*(12), 1149-1157.

Lukaszewicz, G. (1999). *Micropolar fluids: theory and applications*: Springer Science & Business Media.

Magyari, E. (2008). The moving plate thermometer. *International Journal of Thermal Sciences, 47*(11), 1436-1441.

Magyari, E., & Aly, E. H. (2006). Mechanical and thermal characteristics of a mixed convection boundary-layer flow in a saturated porous medium. *International Journal of Heat and Mass Transfer, 49*(21), 3855-3865.

© COPYRIGHT UPM

Mahanthesh, B., Gireesha, B. J., & Gorla, R. S. R. (2016). Nonlinear radiative heat transfer in MHD three-dimensional flow of water based nanofluid over a non-linearly stretching sheet with convective boundary condition. *Journal of the Nigerian Mathematical Society, 35*(1), 178-198.

Mahapatra, T. R., & Gupta, A. (2001). Magnetohydrodynamic stagnation-point flow towards a stretching sheet. *Acta Mechanica*, *152*(1-4), 191-196.

Mahapatra, T. R., & Gupta, A. S. (2002). Heat transfer in stagnation-point flow towards a stretching sheet. *Heat and Mass Transfer, 38*(6), 517-521.

Mahapatra, T. R., Nandy, S. K., & Gupta, A. S. (2009). Analytical solution of magnetohydrodynamic stagnation-point flow of a power-law fluid towards a stretching surface. *Applied Mathematics and Computation, 215*(5), 1696-1710.

Mahdy, A. (2012). Unsteady mixed convection boundary layer flow and heat transfer of nanofluids due to stretching sheet. *Nuclear Engineering and Design, 249*(2012), 248-255.

Makinde, O., Khan, W., & Khan, Z. (2013). Buoyancy effects on MHD stagnation point flow and heat transfer of a nanofluid past a convectively heated stretching/shrinking sheet. *International Journal of Heat and Mass Transfer, 62*(2013), 526-533.

Makinde, O. D., & Aziz, A. (2010). MHD mixed convection from a vertical plate embedded in a porous medium with a convective boundary condition. *International Journal of Thermal Sciences, 49*(9), 1813-1820.

Makinde, O. D., & Aziz, A. (2011). Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. *International Journal of Thermal Sciences, 50*(7), 1326-1332.

Massey, B. S., & Ward-Smith, J. (1998). *Mechanics of fluids* (Vol. 1): CRC Press.

McNitt, J. P. a. R. P. (1970). *Boundary-layer theory for a micropolar fluid. Recent Advances in Engineering Science*: Gordon and Breach.

Meade, D. B., Haran, B. S., & White, R. E. (1996). The shooting technique for the solution of two-point boundary value problems. *Maple Technical Newsletter, 3*(1), 1-8.

Merkin, J. (1986). On dual solutions occurring in mixed convection in a porous medium. *Journal of engineering Mathematics, 20*(2), 171-179.

Merkin, J. H., & Pop, I. (2011). The forced convection flow of a uniform stream over a flat surface with a convective surface boundary condition. *Communications in Nonlinear Science and Numerical Simulation, 16*(9), 3602-3609.

Minkowycz, W., Cheng, P., & Chang, C. H. (1985). Mixed convection about a nonisothermal cylinder and sphere in a porous medium. *Numerical Heat Transfer, 8*(3), 349-359.

© COPYRIGHT UPM

Mureithi, E., & Mason, D. (2010). Local non-similarity solutions for a forced-free boundary-layer flow with viscous dissipation. *Mathematical and Computational Applications, 15*(4), 558.

Mustafa, M., Ahmad Khan, J., Hayat, T., & Alsaedi, A. (2015). Analytical and numerical solutions for axisymmetric flow of nanofluid due to non-linearly stretching sheet. *International Journal of Non-Linear Mechanics, 71*(2015), 22-29.

Mustafa, M., Hayat, T., Pop, I., Asghar, S., & Obaidat, S. (2011). Stagnation-point flow of a nanofluid towards a stretching sheet. *International Journal of Heat and Mass Transfer, 54*(25–26), 5588-5594.

Mustafa, M., Hina, S., Hayat, T., & Ahmad, B. (2014). Influence of induced magnetic field on the peristaltic flow of nanofluid. *Meccanica, 49*(3), 521-534.

Na, T.-Y., & Pop, I. (1996). MHD flow over a moving flat plate with a step change in the magnetic field. *Acta Mechanica, 116*(1), 235-238.

Nadeem, S., & Lee, C. (2012). Boundary layer flow of nanofluid over an exponentially stretching surface. *Nanoscale Research Letters, 7*(1), 1-6.

Nandy, S. K., & Mahapatra, T. R. (2013). Effects of slip and heat generation/absorption on MHD stagnation flow of nanofluid past a stretching/shrinking surface with convective boundary conditions. *International Journal of Heat and Mass Transfer, 64*(2013), 1091-1100.

Naveed, M., Abbas, Z., & Sajid, M. (2016). Hydromagnetic flow over an unsteady curved stretching surface. *Engineering Science and Technology, an International Journal, 19*(2), 841-845.

Nazar, R., Amin, N., & Pop, I. (2004a). Unsteady mixed convection boundary layer flow near the stagnation point on a vertical surface in a porous medium. *International Journal of Heat and Mass Transfer, 47*(12), 2681-2688.

Nazar, R., Amin, N., Filip, D., & Pop, I. (2004b). Stagnation point flow of a micropolar fluid towards a stretching sheet. *International Journal of Non-Linear Mechanics, 39*(7), 1227-1235.

Nield, D., & Kuznetsov, A. (2010). The onset of convection in a horizontal nanofluid layer of finite depth. *European Journal of Mechanics-B/Fluids, 29*(3), 217-223.

Nield, D. A., & Kuznetsov, A. V. (2009). Thermal instability in a porous medium layer saturated by a nanofluid. *International Journal of Heat and Mass Transfer, 52*(25), 5796-5801.

Nield, D. A., & Kuznetsov, A. V. (2014). Thermal instability in a porous medium layer saturated by a nanofluid: A revised model. *International Journal of Heat and Mass Transfer, 68*(2014), 211-214.

© COPYRIGHT UPM

Özışık, M. N. (1985). *Heat transfer: a basic approach* (Vol. 1): McGraw-Hill College.

Pal, D., Mandal, G., & Vajravalu, K. (2015). Mixed convection stagnation-point flow of nanofluids over a stretching/shrinking sheet in a porous medium with internal heat generation/absorption. *Communications in Numerical Analysis, 2015*.1-21.

Pavlov, K. (1974). Magnetohydrodynamic flow of an incompressible viscous fluid caused by deformation of a plane surface. *Magnitnaya Gidrodinamika*, *4*(1), 146-147.

Peddieson J. and Mcnitt R. P. (1970). Boundary layer theory for a micropolar fluid . *Recent Adv. Engng. Sci*. *5*(1), 405-426.

Pop, I., & Ingham, D. B. (2001). Chapter 2 - Mixed convection boundary-layer flow along a vertical flat plate. In I. P. B. Ingham (Ed.), *Convective Heat Transfer* (pp. 45-85). Oxford: Pergamon.

Postelnicu, A., & Pop, I. (2011). Falkner–Skan boundary layer flow of a power-law fluid past a stretching wedge. *Applied Mathematics and Computation, 217*(9), 4359-4368.

Raju, C. S. K., Sandeep, N., Jayachandra Babu, M., & Sugunamma, V. (2016). Dual solutions for three-dimensional MHD flow of a nanofluid over a nonlinearly permeable stretching sheet. *Alexandria Engineering Journal, 55*(1), 151-162.

Rama Subba Reddy, G. (1983). Micropolar boundary layer flow at a stagnation point on a moving wall. *International Journal of Engineering Science, 21*(1), 25-33.

Ramachandran, N., Chen, T. S., & Armaly, B. F. (1988). Mixed convection in stagnation flows adjacent to vertical surfaces. *Journal of Heat Transfer (Transcations of the ASME (American Society of Mechanical Engineers), Series C); (United States); 110*(2), 373-377.

Rana, P., & Bhargava, R. (2012). Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: A numerical study. *Communications in Nonlinear Science and Numerical Simulation, 17*(1), 212-226.

Rana, P., Bhargava, R., & Bég, O. A. (2012). Numerical solution for mixed convection boundary layer flow of a nanofluid along an inclined plate embedded in a porous medium. *Computers & Mathematics with Applications, 64*(9), 2816-2832.

Rashidi, M., Abelman, S., & Mehr, N. F. (2013). Entropy generation in steady MHD flow due to a rotating porous disk in a nanofluid. *International Journal of Heat and Mass Transfer*, *62*(2013), 515-525.

Rashidi, M., Kavyani, N., & Abelman, S. (2014). Investigation of entropy generation in MHD and slip flow over a rotating porous disk with variable properties. *International Journal of Heat and Mass Transfer*, *70*(2014), 892-917.

© COPYRIGHT UPM

Rohni, A. M., Ahmad, S., & Pop, I. (2014). Flow and heat transfer at a stagnation-point over an exponentially shrinking vertical sheet with suction. *International Journal of Thermal Sciences, 75*(2014), 164-170.

Rohni, A. M., Ahmad, S., Pop, I., & Merkin, J. H. (2012). Unsteady mixed convection boundary-layer flow with suction and temperature slip effects near the stagnation point on a vertical permeable surface embedded in a porous medium. *Transport in Porous Media, 92*(1), 1-14.

Rosali, H., Ishak, A., Nazar, R., Merkin, J. H., & Pop, I. (2014). The effect of unsteadiness on mixed convection boundary-layer stagnation-point flow over a vertical flat surface embedded in a porous medium. *International Journal of Heat and Mass Transfer, 77*(2014), 147-156.

Roşca, N. C., & Pop, I. (2014). Boundary layer flow past a permeable shrinking sheet in a micropolar fluid with a second order slip flow model. *European Journal of Mechanics - B/Fluids, 48*(2014), 115-122.

Roşca, N. C., & Pop, I. (2015). Unsteady boundary layer flow over a permeable curved stretching/shrinking surface. *European Journal of Mechanics-B/Fluids, 51*(2015), 61-67.

Sajid, M., Ali, N., Abbas, Z., & Javed, T. (2011). Flow of a micropolar fluid over a curved stretching surface. [journal article]. *Journal of Engineering Physics and Thermophysics, 84*(4), 864-871.

Schlichting, H. (1979). *Boundary-layer theory, 6th edn. Translation by Kestin J. Chaps 14 and 20*: McGraw-Hill, New York.

Schlichting, H., & Gersten, K. (2000). *Boundary-layer theory*: Springer Science & Business Media.

Schneider, W. (1979). A similarity solution for combined forced and free convection flow over a horizontal plate. *International Journal of Heat and Mass Transfer, 22*(10), 1401-1406.

Seshadri, R., Sreeshylan, N., & Nath, G. (2002). Unsteady mixed convection flow in the stagnation region of a heated vertical plate due to impulsive motion. *International Journal of Heat and Mass Transfer, 45*(6), 1345-1352.

Sheikholeslami, M., & Ganji, D. D. (2014). Ferrohydrodynamic and magnetohydrodynamic effects on ferrofluid flow and convective heat transfer. *Energy, 75*(2014), 400-410.

Sui, J., Zheng, L., Zhang, X., & Chen, G. (2015). Mixed convection heat transfer in power law fluids over a moving conveyor along an inclined plate. *International Journal of Heat and Mass Transfer, 85*(2015), 1023-1033.

Takhar, H. S., Chamkha, A. J., & Nath, G. (2005). Unsteady mixed convection on the stagnation-point flow adjacent to a vertical plate with a magnetic field. *Heat and Mass Transfer, 41*(5), 387-398.

© COPYRIGHT UPM

Telionis, D. P. (1981). *Unsteady viscous flows* (Vol. 9): Springer.

Tritton, D. J. (1977). *Physical fluid dynamics.* New York: Van Nostrand Reinhold.

Tiwari, R. K., & Das, M. K. (2007). Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *International Journal of Heat and Mass Transfer, 50*(9), 2002-2018.

Turkyilmazoglu, M. (2012). Exact analytical solutions for heat and mass transfer of MHD slip flow in nanofluids. *Chemical Engineering Science, 84*(2012), 182-187.

Turkyilmazoglu, M. (2016). Flow of a micropolar fluid due to a porous stretching sheet and heat transfer. *International Journal of Non-Linear Mechanics, 83*(2016), 59-64.

Ul Haq, R., Nadeem, S., Hayat Khan, Z., & Sher Akbar, N. (2015). Thermal radiation and slip effects on MHD stagnation point flow of nanofluid over a stretching sheet. *Physica E: Low-dimensional Systems and Nanostructures, 65*(2015), 17-23.

Vajravelu, K. (1986). Hydromagnetic flow and heat transfer over a continuous, moving, porous, flat surface. *Acta Mechanica, 64*(3), 179-185.

Wang, C. Y. (1989). Free Convection on a vertical stretching surface. *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik, 69*(11), 418-420.

Wang, C. Y. (2008). Stagnation flow towards a shrinking sheet. *International Journal of Non-Linear Mechanics, 43*(5), 377-382.

Wang, H. F. (2000). *Theory of linear poroelasticity with applications to geomechanics and hydrogeology*: Princeton University Press.

Watanabe, T., & Pop, I. (1995). Hall effects on magnetohydrodynamic boundary layer flow over a continuous moving flat plate. *Acta Mechanica, 108*(1), 35-47.

Welty, J. R., Wicks, C. E., Rorrer, G., & Wilson, R. E. (1976). *Fundamentals of momentum, heat, and mass transfer*. John Wiley & Sons.

White, F. M., & Corfield, I. (2006). *Viscous fluid flow* (Vol. 3): McGraw-Hill New York.

Yacob, N. A., Ishak, A., & Pop, I. (2010). Non newtonian micropolar fluid flow towards a vertical plate with prescribed wall heat flux. *Australian Journal of Basic and Applied Sciences, 4*(8), 2267-2273.

Yacob, N. A., Ishak, A., Pop, I., & Vajravelu, K. (2011). Boundary layer flow past a stretching/shrinking surface beneath an external uniform shear flow with a convective surface boundary condition in a nanofluid. *Nanoscale Research Letters, 6*(1), 1-7.

© COPYRIGHT UPM

Yadav, D., Agrawal, G., & Bhargava, R. (2011). Thermal instability of rotating nanofluid layer. *International Journal of Engineering Science, 49*(11), 1171-1184.

Zaimi, K., Ishak, A., & Pop, I. (2014). Flow past a permeable stretching/shrinking sheet in a nanofluid using two-phase model, *Plos One*. *9*(11), e111743.

Zarrini, M., Pralhad, R. N (2010), Numerical Investigation of two-dimensional Boundary Layer Flow Over a Moving Surface, *International Journal of Mathematical and Computer Sciences*, *6*(1), 182-184.

Zhong, Y., & Fang, T. (2011). Unsteady stagnation-point flow over a plate moving along the direction of flow impingement. *International Journal of Heat and Mass Transfer*, *54*(15), 3103-3108.

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