UNIVERSITI PUTRA MALAYSIA

INTERACTION BETWEEN TWO CRACKS IN CIRCULAR POSITIONS IN PLANE ELASTICITY

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By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Master of Science

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DEDICATIONS

To all of my love;

Mak & Along

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Master of Science

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February 2017

Chairman: Nik Mohd Asri bin Nik Long, PhD
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In this thesis, the interaction between two cracks in circular positions in plane elasticity is formulated into a system of hypersingular integral equations by using the complex variable function method. The center of the cracks are placed at the edge of a circle with radius R. The first crack is fixed on the x-axis while the second crack is located on the boundary of a circle with the varying angle, θ. By using the curved length coordinate method, the cracks are mapped into a straight line which require less collocation points, hence give faster convergence. With the help of particular quadrature rules, the unknown coefficients are solved numerically from the resulting system of hypersingular integral equations. The obtained unknown coefficients are then used for determining the stress intensity factor (SIF).

Three different domains for the problems of two cracks in circular positions will be considered which is the varying angle of positions of the second crack, the length ratio of the second to the first crack, and the length ratio of the first crack to the radius of a circle. The results for the tested problem presented here agree with the previous reported results.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Sarjana Sains

INTERAKSI ANTARA DUA REKAHAN PADA KEDUDUKAN MEMBULAT DALAM SATAH KEKENYALAN

Oleh

RAHIMAH BINTI ABDUL RAFAR

Februari 2017

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Dalam tesis ini, interaksi antara dua rekahan pada kedudukan membulat dalam satah kekenyalan dirumuskan kepada sistem persamaan kamiran hipersingular dengan menggunakan kaedah fungsi pembolehubah kompleks. Pusat rekahan diletakkan di atas sempadan bulatan yang mempunyai jejari R. Rekahan pertama berada tetap pada paksi-x manakala rekahan kedua diletakkan di atas sempadan bulatan dengan sudut θ yang berbeza. Dengan menggunakan kaedah koordinat panjang lengkung, rekahan dipetakan pada satu garis lurus yang hanya memerlukan titik kolokasi yang sedikit, dengan itu memberi penumpuan yang lebih cepat. Pekali yang tidak diketahui diselesaikan dengan menggunakan kaedah kuadratur tertentu bagi menyelesaikan persamaan kamiran hipersingular secara berangka. Pekali yang terhasil digunakan untuk mengira faktor keamatan tekanan.

Tiga domain yang berbeza bagi masalah dua rekahan pada kedudukan membulat dipertimbangkan iaitu sudut kedudukan rekahan, nisbah panjang antara rekahan pertama dan rekahan kedua, dan nisbah panjang antara rekahan pertama dan jejari bulatan. Hasil untuk masalah yang diuji mempunyai persetujuan yang bagus dengan keputusan lepas.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
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LIST OF ABBREVIATIONS

SIF	Stress Intensity Factor
WS	Weakly Singular
S1	Singular type 1
S2	Singular type 2
HS	Hypersingular
COD	Crack Opening Displacement
DISD	Derivative In a Specific Direction
SIE	Singular Integral Equation

CHAPTER 1

INTRODUCTION

1.1 Introduction

The existence of flaws or cracks in a structure usually reduces the structure fatigue and static strength because the stresses and strains are highly magnified at the crack tip. To ensure the stress values are never close to the maximum received stress, large concentrations of stress are avoided and some reasonable securities are taken. However, the imperfections of material that caused at the production process or the material usage are inevitable, and hence must be taken into consideration.

Brittle fracture is the study that concerned the failure of a structure made from a normally ductile material at a load below that required to cause common yielding, and without obvious plastic deformation (Rolfe and Barsom, 1977). This study has led to the development of fracture mechanics. Fracture mechanics is the field that deals with the study of the cracks propagation in materials. The study uses analytical solid mechanics methods in order to calculate the force that apply on a crack and to characterize the resistance of materials to fracture.

Predicting the fatigue life of cracked components is one of the most important tasks in engineering of fracture mechanics. Based on the theories of elasticity, the stress and strain are applied to the materials in order to predict the mechanical failure of bodies. The parameters analyzed from linear elastic fracture mechanics can be used to determine the stress and strain magnification at the crack tip. This parameter known as the stress intensity factor (SIF), combines applied stress levels, geometry and crack size in a systematic manner and may be evaluated from the elastic stress analysis of cracked structures.

Many investigators gave their full attention in evaluating the SIFs in order to solve the crack problems. For instance, the regularization of the singular integral equation in a crack problem is an important study in the theory of integral equations and in the solid mechanics. In the past few years, the hypersingular integral equation was also used to solve the crack problems, involving the multiple crack problems. The multiple crack problems become an important topic in fracture mechanis since there exists the interaction among the cracks. Generally, the difficulty in the solution technique for this problem must be expected because of the interactions between the cracks.

An infinite plate crack problems, including the multiple cracks problem can be formulated into the integral equations. For the problems that involve a dislocation distribution or a dislocation doublet along a crack path, the integral equation may be generally expressed as

$$\int_L K(t,t_0)f(t)dt + ... = p(t_0) \;\; (or \;\; p(t_0) + c), \;\; (t_0 \in L), \qquad (1.1.1)$$

where L represents the crack configuration, $f(t)$ denotes the unknown function, $K(t,t_0)$ denotes the kernel, and $p(t_0)$ denotes the right hand term in the equation.

There are two possibilities for the choices of right hand term, one is the traction along the crack and second is the resultant force along the crack. The property of the kernel $K(t,t_0)$ depends on the choice of the functions $f(t)$ and $p(t_0)$. Table 1.1 lists the possibilities of the functions $f(t)$ and $p(t_0)$, and the property of $K(t,t_0)$.

Table 1.1: The classification of the integral equations.

Type	$f(t)$	$p(t_0)$	Property of $K(t,t_0)$
WS	Dislocations	Resultant forces	Weakly singular
S1	Dislocations	Tractions	Cauchy singular
S2	Displacement jump (COD)	Resultant forces	Cauchy singular
HS	Displacement jump (COD)	Tractions	Hypersingular

In weakly singular (WS) integral equations, the unknown function is the dislocation distribution, and the right hand term is the resultant force (Cheung and Chen, 1987). This integral equation is named weakly singular because the kernel is a logarithmic function, that has the weaker singularity for integration. This type of integral equation has not received a widespread concern for solving the multiple crack problems. Even though, the straightforward solution of this integral equation is unknown, but it can be solved numerically with the boundary element method.

In the first kind of Cauchy singular integral equations (S1), the unknown function is the dislocation distribution, and the right hand term is traction applied on the crack face. This integral equation is named S1 because the integral belongs to Cauchy principal value integral. For second kind Cauchy singular integral equations (S2), the unknown function is the crack opening displacement (COD) function, and the right hand term is the resultant force on the crack face. This type of integration is also a Cauchy principle value integral.

For hypersingular (HS) integral equations, the unknown function is the COD, and the right hand term is the traction applied on the crack face. This integral equation is names HS because the equation has a hypersingular kernel. The advantage for using this type of integral equation is that one can get the COD function directly from the solution. In general, the integration rule for the hypersingular integral along a curve crack is quite complicated.

1.2 Motivation

In fracture mechanics, there are several situations which involve complicated arrangement and configuration of cracks that can actually be solved by using suitable method of analysis. On the other hand, the presence of a crack will affect the stability and safety of a component and reduce the life of a components or structures significantly. The stresses in the vicinity of the crack tips governing the failure of cracked components. The singular stress contribution is defined by the stress intensity factor (SIF) which determined the stability and safety of the component. To this end, efficient and accurate approaches are required for evaluating the SIFs. Thus, the focus of this research is to investigate the interaction between two cracks in circular positions in plane elasticity. The hypersingular integral equation are used to solve this problem.

1.3 Research questions

In this thesis, we investigate two cracks problems lie on the boundary of a virtual circle in plane elasticity. The research questions of these problems are:

1. how the mathematical model for the interaction between two cracks in circular positions can be built?

2. how the hypersingular integral equations can be formulated for the above mentioned problems?

3. how the obtained hypersingular integral equations be solved to find the stress intensity factor (SIF) at the cracks tips?

4. how the two cracks behave as the distance between both cracks are close together or far apart?

1.4 Objectives

Based on the identified problem, the objectives of this investigation are to:

1. Formulate the mathematical model for the interaction between two cracks in circular positions.

2. Obtain the hypersingular integral equations for the above mentioned problems.

3. Solve the obtained hypersingular integral equations for the stress intensity factor (SIF) at the cracks tips.

4. Investigate the behavior of the cracks tips with respect to their positions and length.

1.5 Outline of thesis

The thesis covers seven chapters. Chapter 1 gives a brief introduction on the research subject. Chapter 2 is dedicated for literature review. Chapter 3 focuses on the methodology for solving the multiple crack problems. Chapter 4 studies the interaction between two straight cracks problem in circular positions in plane elasticity. Chapter 5 discusses the curved crack and inclined crack problems in circular positions. Chapter 6 studies the interaction between curved or inclined crack with straight crack in circular position. Chapter 7 contains the summary of the study and the suggestion for the future research.

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