UNIVERSITI PUTRA MALAYSIA

THREE-POINT DIAGONALLY IMPLICIT BLOCK METHODS FOR SOLVING ORDINARY AND FUZZY DIFFERENTIAL EQUATIONS

NURZEEHAN BINTI ISMAIL

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By

NURZEEHAN BINTI ISMAIL

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

THREE-POINT DIAGONALLY IMPLICIT BLOCK METHODS FOR SOLVING ORDINARY AND FUZZY DIFFERENTIAL EQUATIONS

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December 2014

Chairman: Zarina Bibi binti Ibrahim, PhD

Faculty: Science

The focus of this thesis is on the derivations of Diagonally Implicit Block Backward Differentiation Formulas (DBBDF) of constant step size. The first part of the thesis discusses on the modification of Fully Implicit Block Backward Differentiation Formulas (FBBDF) to solve first order fuzzy differential equations (FDEs). The subsequent part of the thesis focuses on the derivations of Diagonally Implicit Three-point BBDF of order two and three (DBBDF (3, 2) and DBBDF (3, 3)) for solving first order ordinary differential equations (ODEs) and FDEs.

The convergence properties for DBBDF methods and the adequate stability regions for the proposed methods are presented to show that the methods are capable of solving stiff ODEs. The derived methods are then implemented using Newton iteration which is normally used since the methods derived are implicit in nature.

Numerical results are presented to verify the efficiency of DBBDF methods for ODEs. The derived methods are then compared with Diagonally Implicit Two-point BBDF of order two, three and four (DBBDF (2, 2), DBBDF (2, 3) and DBBDF (2, 4)). The accuracy of the proposed methods outperformed DBBDF (2, 3) and DBBDF (2, 4) as the step size gets smaller while the computational time of the proposed methods are smaller than the existing methods.

Since there are very few block methods used to solve fuzzy differential equations, the derived methods are extended to solve first order fuzzy initial value problems (FIVPs). The fuzzification of DBBDF methods is proposed and the convergence of the corresponding methods when applied to FDEs is also proven. Numerical results of DBBDF methods when solving FDEs are provided and compared with the existing methods. On the whole, this study reveals that the FBBDF and DBBDF methods are capable and efficient for solving first order ordinary and fuzzy differential equations.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

FORMULA BLOK TERSIRAT PEPENJURU TIGA MATA UNTUK MENYELESAIKAN PERSAMAAN PEMBEZAAN BIASA DAN KABUR

Oleh

NURZEEHAN BINTI ISMAIL

Disember 2014

Pengerusi: Zarina Bibi binti Ibrahim, PhD

Fakulti: Sains

Fokus tesis ini adalah penerbitan Formula Blok Beza Ke Belakang Tersirat Pepenjuru (FBBBTPP) menggunakan saiz langkah yang berterusan. Bahagian pertama tesis membincangkan pengubahsuaian Formula Blok Beza Ke Belakang Tersirat Penuh (FBBBTP) untuk menyelesaikan persamaan pembezaan kabur (PPK) peringkat pertama. Bahagian seterusnya member tumpuan kepada penerbitan FFBBTPP tiga mata (3FBBBTPP) untuk menyelesaikan persamaan pembezaan kaku biasa (PPB) peringkat pertama dan PPK.

Sifat-sifat penumpuan untuk 3FBBBTPP dibentangkan di dalam bab yang berikutnya. Rantau kestabilan yang mencukupi untuk 3FBBBTPP digunakan untuk menunjukkan bahawa kaedah ini mampu menyelesaikan PPB. 3FBBBTPP kemudiannya dilaksanakan menggunakan lelaran Newton yang biasanya digunakan memandangkan 3FBBBTPP adalah tersirat secara semulajadi.

Keputusan berangka dikemukakan untuk mengesahkan kecekapan 3FBBBTPP untuk PPB. 3FBBBTPP kemudiannya dibandingkan dengan FBBBTPP dua mata (2FBBBTPP). Ketepatan 3FBBBTPP mengatasi 2FBBBTPP apabila saiz langkah semakin kecil dan masa komputasi bagi 3FBBBTPP lebih kecil daripada kaedah yang sedia ada.

Disebabkan terlalu sedikit penyelidikan dilakukan dalam menyelesaikan persamaan pembezaan kabur menggunakan kaedah blok, 3FBBBTPP diperluaskan untuk menyelesaikan masalah nilai awal persamaan pembezaan kabur (NAPPK). Pengkaburan 3FBBBTPP dicadangkan dan penumpuan 3FBBBTPP apabila menyelesaikan masalah PPK juga terbukti. Keputusan berangka bagi 3FBBBTPP dalam menyelesaikan PPK disediakan dan dibandingkan dengan kaedah yang sedia ada. Pada keseluruhannya, kajian ini menunjukkan bahawa kaedah FBBBTP dan FBBBTPP berkebolehan dan cekap untuk menyelesaikan masalah persamaan pembezaan biasa dan kabur peringkat pertama.

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The Most Compassionate and Most Merciful.

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I certify that a Thesis Examination Committee has met on (9 December 2014) to conduct the final examination of Nurzeehan binti Ismail on her thesis "Three-Point Diagonally Implicit Block Methods for Solving Ordinary and Fuzzy Differential Equations" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U. (A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

Members of the Thesis Examination Committee were as follows:

Mohd Rizam Abu Bakar, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Norazak Senu, PhD
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Daud Mohamad, PhD
Professor
Fakulti Sains Komputer dan Matematik
Universiti Teknologi MARA
Malaysia
(External Examiner)

NORITAH OMAR, PhD
Assoc. Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree on Master of Science. The members of the Supervisory Committee were as follows:

Zarina Bibi Ibrahim, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia

Fudziah Ismail, PhD
Professor
Faculty of Science
Universiti Putra Malaysia

Khairil Iskandar Othman, PhD
Associate Professor
Faculty of Computer & Mathematical Sciences
Universiti Teknologi MARA

BUJANG KIM HUAT, PhD
Professor and Dean
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LIST OF ABBREVIATIONS

ODEs	Ordinary Differential Equations
FDEs	Fuzzy Differential Equations
IVPs	Initial Value Problems
FIVPs	Fuzzy Initial Value Problems
BDF	Backward Differentiation Formulas
BBDF	Block Backward Differentiation Formulas
FBBDF	Fully Implicit Three-point Block Backward Differentiation Formulas
LMM	Linear Multistep Methods
MS	Modified Simpson method
DBBDF (2, 2)	Diagonally Implicit Two-point BBDF of order two
DBBDF (2, 3)	Diagonally Implicit Two-point BBDF of order three
DBBDF (2, 4)	Diagonally Implicit Two-point BBDF of order four
DBBDF (3, 2)	Diagonally Implicit Three-point BBDF of order two
DBBDF (3, 3)	Diagonally Implicit Three-point BBDF of order three

CHAPTER 1

INTRODUCTION

1.1 Introduction

The study of differential equations are commonly applied in many fields, especially in pure and applied mathematics, physics, biology, geology, economics and many branch of engineering. Differential equations play an important role in modelling natural phenomena and engineering systems; from measuring population growth and radioactive decay to measuring height of falling objects, solving problems in electrical circuit and many more. Although these real life problems cannot be directly solvable, numerical methods that are commonly used in applied mathematics are suitable to find approximate solutions for those problems.

Ordinary Differential Equation (ODE) is one of the most commonly used differential equations in real life problems, which describes changes mathematically, i.e. rate of change and gradients of quantities. Numerical methods are used not only to find approximate solutions of ODEs but also to help in understanding the behavior of the solutions.

Another type of differential equation that is usually arise in engineering field and real life applications is Fuzzy Differential Equation (FDE). FDEs are used to model uncertain, vague, imprecise, partially true range of computing problems. A solution in finding the range of quantities of nuclei in a radioactive model is one of the applications of FDEs. Similarly to ODEs, variety numerical methods are used in approximating the solutions of FDEs.

1.2 Objective of the Thesis

The ideas of Block Backward Differentiation Formulas given by Ibrahim et al. (2007) are the center of the studies and extensions have been made based on these ideas. The objective of the thesis are:

i) to derive second and third order Diagonally Implicit Three-point Block Backward Differentiation Formulas suitable for solving first order stiff ODEs.
ii) to extend and modify the Fully Implicit Block Backward Differentiation Formulas for solving first order FDEs.
iii) to construct the stability region for the derived methods and determine the step size restriction.
iv) to develop a code to implement methods as in i) and ii) using fixed step sizes capable for solving FDEs and stiff ODEs.

1.3 Scope of the Thesis

In this study we aim to formulate the Diagonally Implicit Three-point Block Backward Differentiation Formulas for solving first-order ordinary and fuzzy differential equations. The advantages of these methods are they require less computational time and produced more accurate solutions, hence, these methods are able to be alternative solvers for ordinary and fuzzy differential equations.

1.4 Outline of the Thesis

Chapter 1 provides a brief introduction of differential equations and the applications of numerical methods in finding solutions to differential equations.

Chapter 2 consists of literature review that are related to the background of this study. In this chapter, some definitions and theorem on numerical methods are also included.

In Chapter 3, an overview of the derivation of Fully Implicit Three-point Block Backward Differentiation Formulas as well as the modification of the corresponding method in order to solve first-order fuzzy differential equations is given.

Chapter 4 focuses on the derivation of Diagonally Implicit Three-point Block Backward Differentiation Formulas to solve first-order ordinary differential equations. The order of both methods are determined and the implementation of Newton iteration is discussed. The methods are then used to solve first-order stiff ordinary differential equations and their performance are compared with the existing methods.

In Chapter 5, convergence properties for the derived methods are discussed, i.e. consistency and stability of the methods. The restriction of step size of the methods are also studied.

Chapter 6 focuses on the modification of the derived methods, and the convergence properties of both methods are discussed. In this chapter, first-order fuzzy initial value problems are tested using these methods and then the results are compared with Diagonally Implicit Two-point Block Backward Differentiation Formulas of order two, three and four and Fully Implicit Three-point Block Backward Differentiation Formulas.

Finally, the summary of the whole thesis, conclusion and future research are presented in Chapter 7.

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