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SOLVING PARTIAL AND FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS USING CORRECTED FOURIER SERIES METHOD

NOR HAFIZAH ZAINAL

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SOLVING PARTIAL AND FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS USING CORRECTED FOURIER SERIES METHOD

By

NOR HAFIZAH BINTI ZAINAL

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SOLVING PARTIAL AND FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS USING CORRECTED FOURIER SERIES METHOD

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NOR HAFIZAH BINTI ZAINAL

November 2014

Chair: Professor Adem Kilicman, PhD
Faculty: Science

Partial differential equations (PDE) are often used to construct models of the most basic theories in physics and engineering. Our goal here is to solve the PDEs problem by using Fourier series method that always been used. However, the truncated Fourier series will cause the Gibbs phenomenon. To eliminate this phenomenon, the corrected Fourier series which consists of a uniformly convergent Fourier series and a correction function will be used. The correction function here is referred to the algebraic polynomials and Heaviside step functions. The Fourier series remains uniformly convergent until its m-th derivative without Gibbs oscillation if the order of polynomial in correction functions not exceed $(m+1)$-th order which the Gibbs oscillation of the Fourier series will be terminated until its m-th derivative.

In this study, we use the corrected Fourier series to solve partial differential equations and fractional partial differential equations. The theory of derivatives and integrals of fractional (non-integer) order was started over 300 years ago. In recent years, fractional calculus have been attract in various research due to its extensive application in engineering and science. We solve this problem by using corrected Fourier series method with modified Riemann-Liouville derivatives. The fractional derivatives are described in Riemann-Liouville sense.

For the case PDEs, we compared the result with classical Fourier series method and exact solution. There is some case that classical Fourier series method cannot solve at a certain point. Meanwhile, corrected Fourier series method gives the solution at that point. For the case that cannot solve by using classical Fourier series method, we can

solve by using corrected Fourier series method. For the fractional PDEs, there is no exact solution for order α as a non-integer number. Thus, we compared the result with others method which is variational iteration method (VIM) and homotopy method. The Maple software is used for all calculation in this study.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

MENYELESAIKAN PERSAMAAN PEMBEZAAN SEPARA DAN PECAHAN DENGAN MENGGUNAKAN KAEDAH SIRI TEPAT FOURIER

Oleh

NOR HAFIZAH BINTI ZAINAL

November 2014

Pengerusi: **Professor Adem Kilicman, PhD**
Fakulti: **Sains**

Persamaan pembezaan separa (PPS) sering digunakan untuk membina model teori-teori yang paling asas dalam bidang fizik dan kejuruteraan. Matlamat kami di sini adalah untuk menyelesaikan masalah PPS dengan menggunakan kaedah siri Fourier yang biasa digunakan. Walau bagaimanapun, siri Fourier yang dipenggal akan menyebabkan fenomena Gibbs. Untuk menghapuskan fenomena ini, siri tepat Fourier yang terdiri daripada satu siri Fourier yang menumpu secara seragam dan fungsi pembetulan akan digunakan. Fungsi pembetulan di sini merujuk kepada polinomial algebra dan langkah fungsi Heaviside. Siri Fourier kekal menumpu secara seragam sehingga terbitan ke-m tanpa Gibbs ayunan jika peringkat polinomial dalam fungsi pembetulan tidak melebihi peringkat $(m+1)$ yang ayunan Gibbs daripada siri Fourier akan ditamatkan sehingga terbitan ke-m .

Dalam kajian ini, kami menggunakan siri tepat Fourier bagi menyelesaikan persamaan pembezaan separa dan pecahan persamaan pembezaan separa. Teori terbitan dan kamiran daripada pecahan (bukan integer) peringkat itu telah bermula lebih 300 tahun yang lalu. Dalam tahun-tahun kebelakangan ini, kalkulus pecahan telah menarik dalam pelbagai penyelidikan disebabkan oleh penggunaan yang luas dalam bidang kejuruteraan dan sains. Kami menyelesaikan masalah ini dengan menggunakan kaedah siri tepat Fourier dengan diubahsuai terbitan Riemann-Liouville. Terbitan dalam pecahan dinyatakan dalam erti kata Riemann-Liouville.

Bagi PDE kes, kita membandingkan keputusan dengan kaedah siri Fourier klasik dan penyelesaian yang tepat. Terdapat beberapa kes kaedah siri Fourier klasik tidak boleh menyelesaikan masalah pada titik tertentu. Sementara itu, kaedah siri tepat Fourier memberikan penyelesaian pada ketika itu. Bagi kes yang tidak dapat diselesaikan

dengan menggunakan kaedah siri Fourier klasik, kita dapat menyelesaikan dengan menggunakan kaedah siri tepat Fourier. Bagi PDE pecahan, tidak ada penyelesaian yang tepat untuk terbitan α sebagai nombor bukan integer. Oleh itu, kita berbanding keputusan dengan kaedah lain yang merupakan kaedah variasi lelaran (KVL) dan kaedah homotopi. Perisian Maple digunakan untuk semua pengiraan dalam kajian ini.

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The Most Merciful First and foremost

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Thank you very much.

I certify that a Thesis Examination Committee has met on 26 November 2014 to conduct the final examination of Nor Hafizah binti Zainal on her thesis entitled "**Study on Solving the Partial Differential Equations and Fractional Partial Differential Equations by using Corrected Fourier series**" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master degree of Science.

Members of the Thesis Examination Committee were as follows:

MOHAMAD RUSHDAN B MD SAID, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

SITI HASANA BT SAPAR, PhD
Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

NORAZAK BIN SENU, PhD
Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

M. MURSALEEN, PhD
Professor
Department of Mathematics
Aligarh Muslim University
202002 Aligarh
India
(External Examiner)

NORITAH OMAR, PhD
Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Master of Science**.** The members of the Supervisory Committee were as follows:

Adem Kilicman, PhD.
Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Dato' Mohamed Bin Suleiman, PhD.
Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

BUJANG KIM HUAT, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

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Name and Matric No.: Nor Hafizah binti Zainal, GS29257

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This is to confirm that:

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Signature:_______________________
Name of
Chairman of
Supervisory
Committee : Prof. Dr. Adem Kilicman

Signature:_______________________
Name of
Chairman of
Supervisory
Committee : Prof. Dato' Dr. Mohamed bin Suleiman

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LIST OF ABREVIATIONS

CFS	Corrected Fourier series
DE	Differential Equation
FS	Fourier series
FPDE	Fractional Partial Differential Equation
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
VIM	Variational Iteration Method

CHAPTER 1

INTRODUCTION

1.1 Introduction

It is well known that differential equations are the most important mathematical tools in the real-world problems. In differential equation (DE), the quantity that being differentiated is called the dependent variable and the quantity with respect to that dependent variable are called independent variable. In physics, we often encounter equations containing second, third and higher order derivatives with respect to the independent variable. These are called second order DEs, third order DEs, and so on, where the order of the equations is refer to the order of the highest order of its derivative with the independent variable that appeared explicitly.

The DE that involves one independent variable is called ordinary differential equation (ODE). ODE is an equation containing a function of one independent variable and its derivative. An ODE is said to be order n, if n is the higher order derivative occurring in the equation. Meanwhile, a partial differential equation (PDE) is an equation containing two or more independent variables and its partial derivatives. In this study, we are focusing on solving PDEs in two independent variables case. As we know, partial differential equations are more difficult to solve than ordinary differential equations.

Many phenomena in sciences and engineering are depends on more than one independent variable. PDEs are used to characterize engineering systems where the behavior of a physical quantity is expressed in terms of its rate of change with respect to two or more independent variables. For example is the heated plate. The boundaries of the plate are held at different temperatures. Because the heat flows from regions of high to low temperature, the boundary conditions set up a potential that leads to heat flow from the hot to cool boundaries. In sufficient time elapse, such a system will eventually reach the stable or steady-state distribution of temperature. Then to determine this contribution, the Laplace equation, along with appropriate boundary conditions is provided (Chapra and Canale, 2006). We expressed the equation in the form

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{1.1}$$

In the Laplace equation (1.1), this is indicated by the absence of a time derivative. For the case where there are source distributions, the equation is represented as

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y) \tag{1.2}$$

where $f(x, y)$ is a function describes a heat source distribution and it is called the Poisson equation.

In addition, other examples of PDEs of two independent variables are

$$\frac{\partial u}{\partial t} - c\frac{\partial^2 u}{\partial x^2} = f(x,t) \tag{1.3}$$

$$\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = f(x,t) \tag{1.4}$$

(1.3) is called the heat equation that can be used to describe the heat transfer process along a thin rod, where the coefficient c being determined by the thermal conductivity, the specific heat and the density. Meanwhile, (1.4) is called the wave equation that can be used to model the vibration of a string, with the coefficient c^2 depending on the string tension and density. The quantity c can be interpreted as the speed of wave propagation, (Atkinson and Han, 2004). PDEs can be classified into elliptic, parabolic, and hyperbolic equations. The Poisson equations is in the class of elliptic equation, the heat equations is in the class of parabolic equation and the wave equations is in the class of hyperbolic equation.

Furthermore, we also interested in solving the fractional partial differential equations. There has been a growing interest in the field of fractional derivatives. Fractional derivatives are generalization of the derivative to a non-integer order. Nowadays, phenomena in life such as advancement of calculus of variations and optimal control to fractional dynamic systems, analytic and numerical tools and techniques, bioengineering and biomedical applications have been modeled by fractional partial differential equations.

The fractional derivative of order α of function $f(t)$ with respect to t is written as $D_t^{\alpha} f(t)$ or can also be written as $\frac{d^{\alpha} f(t)}{dt^{\alpha}}$ where α is non-integer. Meanwhile, for fractional partial derivative of function $u(x,t)$ of order α with respect to t is written as $\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}}$. To be more cleared, let we recalled the derivative of a function f with respect to x and it is defined as

$$D^1 f(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$D^2 f(x) = \lim_{h\to 0} \frac{f(x+2h)-2f(x+h)+f(x)}{h^2}$$

$$D^3 f(x) = \lim_{h\to 0} \frac{f(x+3h)-3f(x+2h)+3f(x+h)-f(x)}{h^3}$$

$$\vdots$$

$$D^n f(x) = \lim_{h\to 0} \frac{\sum_{m=0}^{n} (-1)^m \binom{n}{m} f\left(x+(n-m)h\right)}{h^n}, \text{ where } \binom{n}{m} = \frac{n!}{m!(n-m)!}. \quad (1.5)$$

For n to be non-integer values, we use the Euler's Gamma function, Γ property, which is $n! = \Gamma(n+1)$.

Thus,

$$\binom{n}{m} = \frac{n!}{\text{m!(n - m)!}} = \frac{\Gamma(n+1)}{\Gamma(m+1)\Gamma(n-m+1)}. \quad (1.6)$$

Let say, we have polynomial function $f(x) = x^m$, if we differentiate $f(x)$ with respect to x for n times, when $n \in \mathbb{Z}$, then we get

$$\frac{d^n}{dx^n}(x^m) = \frac{m!}{(m-n)!} x^{m-n}. \quad (1.7)$$

Example 1: Find first, second and third derivatives of polynomial function $f(x)=x^3$ with respect to x.

Solution:

$$\frac{d}{dx}\left(x^3\right)=\frac{3!}{(3-1)!}x^{3-1}=\frac{6}{2}x^2=3x^2$$
$$\frac{d^2}{dx^2}\left(x^3\right)=\frac{3!}{(3-2)!}x^{3-2}=6x$$
$$\frac{d^3}{dx^3}\left(x^3\right)=\frac{3!}{(3-3)!}x^{3-3}=\frac{3!}{0!}x^0=6$$

Example 2: Find fourth derivative of polynomial function $f(x)=\frac{1}{5}x^{10}$ with respect to x by using (1.7).

Solution: By using (1.7), we have

$$\frac{d^4}{dx^4}\left(\frac{1}{5}x^{10}\right)=\frac{1}{5}\frac{10!}{(10-4)!}x^{10-4}=\frac{1}{5}5040x^6=1008x^6$$

Now, we check the answer whether it is correct or not.

$$\left(\frac{1}{5}x^{10}\right)\xrightarrow{\text{1st derivative}}2x^9\xrightarrow{\text{2nd derivative}}18x^8\xrightarrow{\text{3rd derivative}}144x^7\xrightarrow{\text{4th derivative}}1008x^6$$

We can see here, it gives the same answer when we use (1.7).

For $n\notin\mathbb{Z}$, we use the Euler's Gamma function, Γ property. Thus we have

$$\frac{d^n}{dx^n}\left(x^m\right)=\frac{\Gamma(m+1)}{\Gamma(m-n+1)}x^{m-n} \text{ where } m,n\in\mathbb{Z}. \tag{1.8}$$

Example 3: Find fractional derivative of polynomial function $f(x)=x^3$ with respect to x with order $\frac{1}{2}$ and $\frac{1}{5}$.

Solution: By using (1.8), we have

i. $\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}\left(x^3\right)=\frac{\Gamma(3+1)}{\Gamma\left(3-\frac{1}{2}+1\right)}x^{3-\frac{1}{2}}=\frac{\Gamma(4)}{\Gamma\left(\frac{7}{2}\right)}x^{\frac{5}{2}}=\frac{16}{5\sqrt{\pi}}x^{\frac{5}{2}}$ or $1.805406667x^{\frac{5}{2}}$

ii. $\frac{d^{\frac{1}{5}}}{dx^{\frac{1}{5}}}\left(x^3\right)=\frac{\Gamma(3+1)}{\Gamma\left(3-\frac{1}{5}+1\right)}x^{3-\frac{1}{5}}=\frac{\Gamma(4)}{\Gamma\left(\frac{19}{5}\right)}x^{\frac{14}{5}}=1.278180088x^{\frac{14}{5}}$

When we go back more than three centuries, in 1695 the derivative of order $\alpha=\frac{1}{2}$ was described by Leibniz, where in a letter to L'Hopital, Leibniz discussed the differentiation of product functions to differentiation of order $\frac{1}{2}$, (as cited in Miller and Ross, 1993). After that, there a few different forms of fractional operator have been introduced such as Riemann-Liouville, and Caputo. In Riemann-Liouville fractional derivative, we take the fractional integral of order α first then take a first derivative, $D_t^{\alpha}f(t)=\frac{d}{dt}\left(J^{\alpha}f(t)\right)$. Meanwhile, in Caputo fractional derivative, we take first derivative and then follow up with fractional integral of order α, $D_t^{\alpha}f(t)=J^{\alpha}\left(\frac{d}{dt}f(t)\right)$.

In our study, we are going to use the corrected Fourier series (CFS) that have been introduced by Zhang, Chen, and Qu in 2005. Galerkin method with corrected Fourier series as its basis functions will be proposed. Gibbs oscillation or spurious oscillation is oscillation that occurs when using the truncated Fourier series. It always appeared near the discontinuities and aperiodic endpoints. In this research, for mth order PDEs, the m linearly independent solutions are uniformly convergent until their derivatives, no Gibbs oscillations in the solution themselves and in their derivatives until mth order over the PDEs entire interval. Since the corrected Fourier series is uniformly convergent, we will apply it to solve the problems of linear PDEs. Corrected Fourier series is a combination of uniformly convergent Fourier series and a correction function which consists of algebraic polynomials and Heaviside step functions.

$$f(x)=\sum_{|n|<\infty}A_n e^{i\alpha_n x}+\sum_{l=1}^{m+1}a_l\frac{x^l}{l!}+\sum_j b_j\frac{\left(x-x_j\right)^m}{m!}H\left(x-x_j\right) \quad (1.9)$$

The first part of (1.9) is an mth uniformly convergent Fourier series, the second part is a polynomial no-more-than $(m+1)$th order, and the last part is an mth integral of the Heaviside-step function at the discontinuities.

Definition 1 The Fourier projection of any function $\phi(x)$ on the basis function $e^{i(\alpha_n x)}$ in the interval $[0, x_0]$ is defined as

$$F_1\langle\phi(x)\rangle_n = \frac{1}{x_0}\int_0^{x_0} \phi(x)e^{-i(\alpha_n x)}dx\,,\ \alpha_n = \frac{2n\pi}{x_0}\,.$$

In the next chapter, we will discuss the literature review in the process of learning the field of study. A brief introduction on the Fourier and Corrected Fourier series also will be discussed in this chapter.

1.2 The objective of the Thesis

The main objectives of this thesis can be summarized as follows:

i. To solve partial differential equations by using corrected Fourier series.
ii. To solve fractional partial differential equations by using corrected Fourier series with modified Riemann-Liouville derivatives.

1.3 Outline of the Thesis

In Chapter 1, a basic theory of differential equations and a theory of fractional derivative were discussed. A brief introduction on the corrected Fourier series was also discussed in chapter 1. In this study, we are focusing in solving second order partial differential equation and fractional partial differential equations with non-integer order. In Chapter 2, we put some literature review in the process of learning the field of study.

In Chapter 3, we derived corrected Fourier series method for solving partial differential equations. We divided the derivation in three cases which is Heat problem, Wave Problem and Poisson Problem. In Chapter 4, we discussed a theory of Fractional partial differential equations. We introduced some definitions of fractional differential equations. Here, we derived corrected Fourier series method for solving fractional differential equation. We choose the alpha as between 0 to 1 and 1 to 2.

In Chapter 5, we test the problem related to the case by using corrected Fourier series method. For PDEs case, we compared the results with exact solution and classical Fourier method. For the case fractional PDEs, we don't have exact solution when the order alpha, α is non-integer. Thus, we compared the result with other approximation method which is Variational iteration method (VIM) and Homotopy method. Finally, in the last chapter, the summary of the whole thesis, conclusions are given.

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BIODATA OF STUDENT

Nor Hafizah binti Zainal was born on the 19th August 1989 in Lumut, Perak. She started her primary education at Sekolah Kebangsaan Methodist ACS Lumut, Lumut, Perak and completed her secondary education at Sekolah Menengah Kebangsaan Seri Manjung, Seri Manjung, Perak. She pursued her Matriculation Program at Perak Matriculation College under science physical.

She obtained her Bachelor of Science (Hons.) degree major in Mathematics from Universiti Putra Malaysia under the Public Service Department of Malaysia Scholarship in June 2010. Currently, she is pursuing her Master Degree in Applied Mathematics at the Department of Mathematics, Universiti Putra Malaysia. She is working on the corrected Fourier series method for solving partial differential equations and fractional partial differential equations.

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1. Nor Hafizah binti Zainal, Adem Kilicman, (2011) *Solving the linear partial differential equations by using corrected Fourier series*, Proceeding of the Simposium Kebangsaan Sains Matematik (SKSM-19), 9-11 November 2011, Uitm Pulau Pinang.

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