UNIVERSITI PUTRA MALAYSIA

WAVELET METHODS FOR SOLVING LINEAR AND NONLINEAR SINGULAR BOUNDARY VALUE PROBLEMS

ALIASGHAR KAZEMI NASAB

FS 2014 64

WAVELET METHODS FOR SOLVING LINEAR AND NONLINEAR SINGULAR BOUNDARY VALUE PROBLEMS

By

ALIASGHAR KAZEMI NASAB

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

September 2014

COPYRIGHT

DEDICATIONS

To all my family members specially my wife, my daughter and my parents for their love and continuous support
also
to all my teachers

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

WAVELET METHODS FOR SOLVING LINEAR AND NONLINEAR SINGULAR BOUNDARY VALUE PROBLEMS

By

ALIASGHAR KAZEMI NASAB

September 2014

Chair: Prof. Adem Kılıçman, PhD

Faculty: Science

In this thesis, wavelet analysis method is proposed for solving singular boundary value problems. Operational matrix of differentiation is introduced. Furthermore, product operational matrix is also presented. Many different examples are solved using Chebyshev wavelet analysis method to confirm the accuracy and the efficiency of wavelet analysis method.

An efficient and accurate method based on hybrid of Chebyshev wavelets and finite difference methods is introduced for solving linear and nonlinear singular ordinary differential equations such as Lane-Emden equations, boundary value problems of fractional order and singular and nonsingular systems of boundary and initial value problems. High-order multi-point boundary value problems are also solved. The useful properties of Chebyshev wavelets and finite difference method make it a computationally efficient method to approximate the solution of nonlinear equations in a semi-infinite interval. The given problem is converted into a system of algebraic equations using collocation points. The main advantage of this method is the ability to represent smooth and especially piecewise smooth functions properly. It is also clarified that increasing the number of subintervals or the degree of the Chebyshev polynomials in a proper way leads to improvement of the accuracy. Moreover, this method is applicable for solving problems on large interval. Several examples will be provided to demonstrate the powerfulness of the proposed method. A comparison is made among this method, some other well-known approaches and exact solution which confirms that the introduced method are more accurate and efficient. For future studies, some problems are proposed at the end of this thesis.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

KAEDAH ANAK GELOMBANG UNTUK MENYELESAIKAN LINEAR DAN TAK LELURUS SINGULAR MASALAH NILAI SEMPADAN

Oleh

ALIASGHAR KAZEMI NASAB

September 2014

Pengerusi: Prof. Adem Kılıçman, PhD

Fakulti: Sains

Dalam tesis ini, kaedah analisis gelombang telah dicadangkan untuk menyelesaikan masalah nilai sempadan tunggal. Matriks operasi bagi pembezaan telah diperkenalkan. Selain itu, produk operasi matriks juga dibentangkan. Banyak contoh yang berbeza telah diselesaikan dengan menggunakan kaedah analisis gelombang Chebyshev untuk mengesahkan ketepatan dan kecekapan kaedah analisis anak gelombang.

Satu kaedah yang cekap dan tepat berdasarkan hibrid gelombang Chebyshev dan kaedah perbezaan terhingga telah diperkenalkan untuk menyelesaikan persamaan pembezaan biasa tunggal linear tak linear serta masalah-masalah nilai sempadan dalam perintah pecahan dan singular dan tak singular sistems sempadan dan nilai awal masalah. Ciri-ciri berguna gelombang Chebyshev dan kaedah perbezaan terhingga menyebabkan ia menjadi satu kaedah pengiraan yang cekap bagi menganggar penyelesaian persamaan linear dalam jarak semi-tak terhingga. Masalah tersebut akan ditukarkan ke dalam sistem persamaan algebra dengan menggunakan titik gabungan. Kelebihan utama kaedah ini adalah keupayaan untuk mewakili ciri-ciri licin terutamanya fungsi cebis demi cebis yang licin dengan baik. Ia juga menunjukkan bahawa ketepatan ini boleh dipertinkatkan sama ada dengan menambah bilangan subselang atau meningkatkan bilangan titk gabungan dalam subselang. Selain itu, kaedah ini adalah sah untuk pengiraan yang domainnya besar. Beberapa contoh akan disediakan untuk menunjukkan kekuasaan kaedah yang telah dicadangkan. Perbandingan telah dibuat di antara kaedah ini, dengan beberapa pendekatan lain yang terkenal dan juga penyelesaian yang tepat telah mengesahkan bahawa kaedah yang diperkenalkan adalah lebih tepat dan berkesan. Untuk kajian masa depan, beberapa masalah telah dicadangkan dalam akhir tesis ini.

ACKNOWLEDGEMENTS

First of all, I am most thankful to Allah for his blessings and unfailing answering of my prayers. His divine presence has been an abundant spiritual reservoir that constantly supports me with the strength of mind and body to ride through many bumpy chapters along my life journey. I would not have been able to carry out my research work without the support of many people during the past several years.

I am really thankful to my supervisor, Prof. Dr. Adem Kılıçman, for his useful conversations and friendly behaviors. I appreciate his constant efforts to find any possible funding sources to support my research studies. I am also grateful to him for giving me an opportunity to work as a Teaching Assistant for bachelor courses.

My sincere acknowledgements to my co-supervisors, Assoc. Prof. Dr. Zainidin K. Eshkuvatov, Assoc. Prof. Dr. Wah June Leong and Prof. Dr. Saeid Abbasbandy for their useful suggestions and answering my queries.

I will always remember my friends that I have made during my stay in Malaysia, and with whom I have cherished some joyous moments and refreshing exchanges.

I wish to extend my utmost thanks to my relatives in Iran, especially my parents, and parents-in-law for their love and continuous support. Finally, my thesis would have never been in this shape without lovely efforts from my wife Zohreh and my daughter Mahsa. Their invaluable companionship, warmth, strong faith in my capabilities has always helped me to be assertive in difficult times. Their optimistic and enlightening boosts have made this involved research task a pleasant journey.

I certify that a Thesis Examination Committee has met on 11 September 2014 to conduct the final examination of Aliasghar Kazemi Nasab on his thesis entitled "Wavelet Methods for Solving Linear and Nonlinear Singular Boundary Value Problems" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Degree of Doctor of Philosophy.

Members of the Thesis Examination Committee were as follows:

Isamiddin S.Rakhimov, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Chairperson)

Zanariah binti Abdul Majid, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Ibragimov Gafurjan, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Jeremy Levesley, PhD
Professor
Faculty of Science
University of Leicester
United Kingdom
(External Examiner)

ZULKARNAIN ZAINAL, PhD
Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 9 December 2014

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

Adem Kılıçman, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Chairperson)

Zainidin K. Eshkuvatov, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

Wah June Leong, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

Saeid Abbasbandy, PhD
Professor
Faculty of Science
Imam Khomeini International University of Iran
(Member)

BUJANG BIN KIM HUAT, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 19 January 2014

Declaration by Graduate Student

I hereby confirm that:

- this thesis is my original work;
- quotations, illustrations and citations have been duly referenced;
- this thesis has not been submitted previously or concurrently for any other degree at any other institutions;
- intellectual property from the thesis and copyright of thesis are fully-owned by Universiti Putra Malaysia, as according to the Universiti Putra Malaysia (Research) Rules 2012;
- written permission must be obtained from supervisor and the office of Deputy Vice-Chancellor (Research and Innovation) before thesis is published (in the form of written, printed or in electronic form) including books, journals, modules, proceedings, popular writings, seminar papers, manuscripts, posters, reports, lecture notes, learning modules or any other materials as stated in the Universiti Putra Malaysia (Research) Rules 2012;
- there is no plagiarism or data falsification/fabrication in the thesis, and scholarly integrity is upheld as according to the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) and the Universiti Putra Malaysia (Research) Rules 2012. The thesis has undergone plagiarism detection software.

Signature: ____________________ Date: ____________________

Name and Matric No.: ________________________________

Declaration by Members of Supervisory Committee

This is to confirm that:
- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) are adheared to.

Signature: ____________________
Name of
Chairman of
Supervisory
Committee ____________________

Signature: ____________________
Name of
Member of
Supervisory
Committee ____________________

Signature: ____________________
Name of
Member of
Supervisory
Committee ____________________

Signature: ____________________
Name of
Member of
Supervisory
Committee ____________________

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

ODEs	Ordinary Differential Equations
PDEs	Partial Differential Equations
FDM	Finite Difference Method
FVM	Finite Volume Method
FEM	Finite Element Method
VIM	Variational Iteration Method
BVPs	Boundary Value Problems
CWT	Continuous Wavelet Transform
DWT	Discrete Wavelet Transform
OMD	Operational Matrix of Derivative
POM	Product Operation Matrix
HPM	Homotopy Perturbation Method
HAM	Homotopy Analysis Method
NPSM	Non-Polynomial Spline Method
LGSM	Lie-Group Shooting Method
CWFD	Chebyshev Wavelet Finite Difference
HFC	Hermite Functions Collocation
ADM	Adomian Decomposition Method
LOM	Legendre Operational Matrix
ChW	Chebyshev Wavelet
db4	Daubechies Wavelet of order 4
RMSE	Root mean square error
RKM	Reproducing kernel method

CHAPTER 1

INTRODUCTION

1.1 Background

Change is an intrinsic nature of the universe; the world is a moving entity. In order to understand and predict change we often need to create models reflecting rates of change. The information of rates should be translated into the language of mathematics by setting up an equation containing a derivative which is called a differential equation. Many phenomena in physics, chemistry, engineering, medicine, biology, astronomy and economics can be modelled using differential equations. In spite of many analytical techniques developed for their solution, many differential equations can not be solved analytically. Therefore, many attempts have been made to propose numerical methods. When it comes to numerical methods, the frequent questions often are: how close is the numerical solution to the exact solution? and how possible to measure this closeness? to what extent the numerical solution preserves the physical quality of the exact solution? (Asadzadeh, 2012).

A differential equation is a relation between an unknown function y of one or several variables and its derivatives of various orders. Differential equations can be classified either as ordinary or as partial. If the function y depends on only one variable ($x \in \mathbb{R}$), then the equation is called an ordinary differential equation (ODE). A partial differential equation (PDE) is a differential equation in which the function of interest depends on two or more variables. The order of the differential equation is determined by the order of the highest derivative of the function y that appears in the equation.

Numerical methods used for numerical solution of physical problems can be categorized into the following classes (Mehra et al., 2009):

Finite difference methods (FDM):
The unknowns that appear in equation are defined by their values on discrete grid and differential operators are replaced by difference operators using neighboring points.

Finite volume methods (FVM):
Finite volume methods are similar to the finite difference method, values are calculated at discrete places on a meshed geometry. "Finite volume" refers to the small volume surrounding each node point on a mesh.

Finite elements methods (FEM):
In this approach, the unknown solution is approximated by a linear combination of a set of linearly independent test functions, which are piecewise continuous and non vanishing only on the finite number of elements in the domain.

Spectral methods:
These methods use basis functions which are infinitely differentiable and non vanishing on the entire domain (global support).

Wavelet methods:
In wavelet methods, basis functions employed which are differentiable (according to the requirement) and non vanishing on the compact support.

In FDM and FVM, the differential equation is approximated while in other methods its solution is approximated. In spectral methods, bases functions are infinitely differentiable with global support, while in finite difference or finite element methods, bases functions have small compact support but poor continuity properties. So spectral methods have high accuracy, but poor spatial localization, while FDM, FVM and FEM have good spatial localization but low accuracy. On the other hand, wavelet methods seem to have the advantage of other methods simultaneously (high spectral accuracy as well as good localization).

1.2 Motivation and problem statement

In recent decades, many researchers have been attempting to answer the three main questions that arise in the study of singular boundary value problems: existence and uniqueness of solutions, behaviour of the solution in the neighbourhood of the singular points and its numerical approximation.

Many nonlinear phenomena in physics, chemistry, engineering and other sciences can be modeled as a singular two-point boundary value problems (BVPs). The Thomas-Fermi differential equation, the Ginzburg-Landau equation, the Lane-Emden equation, the Bratu and Troesch equations take the form of singular boundary value problems of second order. The singularity typically occurring at an end of the interval of integration. ODEs with singularities arise also in numerous applications which are of interest in modern applied mathematics.

The existence of singularities makes the approximate solution lose its accuracy in the neighbourhood of the singular points. Even for local methods, such as the finite difference or finite element methods, spurious oscillations appearing near the singularity may distort the solution in the whole domain. This phenomenon is even more critical for global solution methods, such as the Chebyshev method, whose accuracy depends on the regularity of the solution. For a solution with a low regularity, the "infinite accuracy" commonly associated to spectral methods is lost and such methods show little advantages over local approximation methods. Therefore, a suitable treatment of the singularities is necessary for preserving, as far as possible, the high accuracy of spectral methods (Botella and Peyret, 2001).

Spectral methods can be applied for solving differential equations. The solution function is expanded as a finite series of very smooth basis functions as follows,

$$y(t) \cong y^N(t) = \sum_{i=0}^{N-1} a_i \psi_i(t) \tag{1.1}$$

in which, the best choice of ψ_i, are the eigenfunctions of a singular Sturm-Liouville problem. The most important characteristic of this method is that it reduces the given problem to those of solving a system of algebraic equations which can be solved easily. If the function y belongs to $C^\infty[a,b]$, the produced error of approximation (1.1), when N tends to infinity, approaches zero with exponential rate (Canuto et al., 1988). This phenomenon is usually referred to as "spectral accuracy" (Gottlieb and Orszag, 1977). The accuracy of derivatives obtained by direct, term by term, differentiation of such truncated expansion naturally deteriorates (Canuto et al., 1988), but for low order derivatives and sufficiently high-order truncations this deterioration is negligible. So, if solution function and coefficient functions are analytic on $[a,b]$, spectral methods will be very efficient and suitable.

The Troesch's problem comes from the investigation of the confinement of a plasma column under radiation pressure, while the Bratu problem is used in a different variety of applications such as the fuel ignition of the thermal combustion theory, the model thermal reaction process, the Chandrasekhar model of the expansion of the Universe, chemical reaction theory, radiative heat transfer and nanotechnology (Wazwaz, 2005b; Syam and Hamdan, 2006; Buckmire, 2004; Mcgough, 1998; Mounim and de Dormale, 2006; Li and Liao, 2005; Liao and Tan, 2007).

The application of the Lane-Emden equation in astrophysics, its importance in the kinetics of combustion and the Landau-Ginzburg critical phenomena motivates physicists to pay considerable attention to solve it (Dixon and Tuszyski, 1990; Fermi, 1927; Fowler, 1930; Frank-Kameneëtìskiæ, 1969; Chandrasekhar and Chandrasekar, 1958; Eddington, 1988; Spitzer Jr, 1942). On the other hand, its nonlinearity and singular behaviour at the origin makes it fascinating for mathematicians to consider it as a prototype for testing new methods for solving nonlinear differential equations.

Fractional calculus has received much attention from scientists and engineers in recent years. Many researchers in various fields found that derivatives of non-integer order are useful for the description of some natural physics phenomena and dynamic system processes such as damping laws, diffusion process, etc. (Ciesielski and Leszczynski, 2003; Metzler and Klafter, 2000). In general, it is difficult to solve fractional differential equations analytically. Therefore, it is necessary to introduce some reliable and efficient numerical algorithms to solve them. During the past decades, an increasing number of numerical methods have been developed.

Chebyshev polynomials which are the eigenfunctions of a singular Sturm-Liouville problem have many advantages. They can be considered as a good representation of smooth functions by finite Chebyshev expansions provided that the function is infinitely differentiable. The Chebyshev expansion coefficients converge faster than any finite power of $\frac{1}{n}$ as n goes to infinity for problems with smooth solutions. The numerical differentiation and integration can be performed. Moreover, they have been applied to solve different kinds of boundary value problems (Canuto et al., 1988; Voigt et al., 1984; Fox and Parker, 1968). Marzban and Hoseini (2013) combined Chebyshev polynomials with block pulse functions to construct a composite Chebyshev finite difference method for solving linear optimal control problems with time delay.

In recent years, wavelets have received considerable attention by researchers in different fields of science and engineering. The main characteristic of wavelet analysis is the ability to perform local analysis (Misiti et al., 2000). Wavelet analysis is able to reveal signal aspects that other analysis method miss, such as trends, breakdown points, discontinuities, etc. In contrast with other orthogonal functions, multiresolution analysis aspect of wavelets permit the accurate representation of a variety of functions and operators. In other words, we can change M and k simultaneously to get more accurate solution. Another benefit of wavelet method for solving equations is that after discreting the coefficients matrix of algebraic equations is sparse. So it is computationally efficient the use of wavelet methods for solving equations. In addition, the solution is convergent (Adibi and Assari, 2010).

1.3 Objectives of the Research

The objectives of this research are to find out some accurate and efficient numerical algorithms which can be applied for solving singular ordinary differential equations and boundary value problems of fractional order. Wavelet analysis and Chebyshev wavelet finite difference methods are proposed to obtain more accurate solutions.

The specific objectives to be addressed are:

- to propose Chebyshev wavelet analysis method for solving both linear and nonlinear singular initial and boundary value problems,
- Chebyshev wavelet analysis method is also proposed for solving Troesch's and Bratu's equations,
- to employ Chebyshev wavelet finite difference method for obtaining more accurate and efficient solution to singular initial and boundary value problems such as Lane-Emden type equations,
- to solve high-order multi-point boundary value problems,

- solving fractional ordinary differential equations by using Chebyshev wavelet finite difference method,
- to apply Chebyshev wavelet finite difference method for solving singular and non-singular system of boundary and initial value problems.

1.4 Outline of the Thesis

This thesis is structured as follows. In Chapter 1, a brief introduction to the research topic is given. The problems under consideration for solving in the succeeding chapters are stated and the main objectives of the thesis are summarized. Chapter 2 includes some notations, definitions and preliminary facts that will be used further in this research work. Some main concepts like approximation theory, orthogonal functions, multiresolution analysis and wavelet analysis method are explained in this chapter.

In Chapter 3, we introduce Chebyshev wavelet analysis method and is then employed to get an accurate and efficient solution to linear and nonlinear singular boundary value problems.

In Chapter 4, we solve Bratu's and Troesch's equations by using Chebyshev wavelet analysis method.

In Chapter 5, Chebyshev wavelet finite difference method is presented. we employ Chebyshev wavelet finite difference method for solving linear and nonlinear singular initial and boundary value problems such as Lane-Emden equations.

In Chapter 6, high-order multi-point boundary value problems are solved.

In Chapter 7, ordinary differential equations of fractional order are considered to solve.

In Chapter 8, we apply Chebyshev wavelet finite difference method for solving singular and non-singular system of boundary and initial value problems.

Finally, in Chapter 9, we conclude with a brief discussion of the work carried out and the main results drawn from this research. A number of possible extensions to this work in further studies are outlined.

BIBLIOGRAPHY

Abbasbandy, S., Hashemi, M. and Liu, C. S. 2011. The Lie-group shooting method for solving the Bratu equation. *Communications in Nonlinear Science and Numerical Simulation* 16 (11): 4238–4249.

Abdulaziz, O., Hashim, I. and Momani, S. 2008. Solving systems of fractional differential equations by homotopy-perturbation method. *Physics Letters A* 372 (4): 451–459.

Abramowitz, M. and Stegun, I. . A. 1972. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*.

Adibi, H. and Assari, P. 2010. Chebyshev Wavelet Method for Numerical Solution of Fredholm Integral Equations of the First Kind. *Mathematical Problems in Engineering,* Article ID 138408.

Adibi, H. and Rismani, A. 2010. On using a modified Legendre-spectral method for solving singular IVPs of Lane-Emden type. *Computers and Mathematics with Applications* 60 (7): 2126–2130.

Agarwal, R. P. and O'Regan, D. 2007. Second order initial value problems of Lane–Emden type. *Applied Mathematics Letters* 20 (12): 1198–1205.

Agarwal, R. P., Wong, F. H. and Lian, W. C. 1999. Positive solutions for nonlinear singular boundary value problems. *Applied Mathematics Letters* 12 (2): 115–120.

Agarwall, R. P. and Akrivis, G. 1982. Boundary value problems occurring in plate deflection theory. *Journal of Computational and Applied Mathematics* 8 (3): 145–154.

Al-Mdallal, Q. M., Syam, M. I. and Anwar, M. 2010. A collocation-shooting method for solving fractional boundary value problems. *Communications in Nonlinear Science and Numerical Simulation* 15 (12): 3814–3822.

Aregbesola, Y. A. S. 2003. Numerical solution of Bratu problem using the method of weighted residual. *Electron J South Afr Math Sci* 3 (1): 1–7.

Asadzadeh, M. 2012, An Introduction to the Finite Element Method (FEM) for Differential Equations.

Aslanov, A. 2008a. A generalization of the Lane-Emden equation. *International Journal of Computer Mathematics* 85 (11): 1709–1725.

Aslanov, A. 2008b. Determination of convergence intervals of the series solutions of Emden-Fowler equations using polytropes and isothermal spheres. *Physics Letters A* 372 (20): 3555–3561.

Aslanov, A. 2014. A Singular Initial-Value Problem for Second-Order Differential Equations. *Abstract and Applied Analysis, Article ID 526549* Article ID 526549.

Aziz, T. and Kumar, M. 2001. A fourth-order finite-difference method based on non-uniform mesh for a class of singular two-point boundary value problems. *Journal of Computational and Applied Mathematics* 136 (1-2): 337–342.

Babolian, E., Bromilow, M., England, R. and Saravi, M. 2007. A modification of pseudo-spectral method for solving a linear ODEs with singularity. *Applied Mathematics and Computation* 188 (2): 1260–1266.

Babolian, E. and Fattahzadeh, F. 2007. Numerical solution of differential equations by using Chebyshev wavelet operational matrix of integration. *Applied Mathematics and Computation* 188 (1): 417–426.

Bataineh, A. S., Noorani, M. and Hashim, I. 2009. Modified homotopy analysis method for solving systems of second-order BVPs. *Communications in Nonlinear Science and Numerical Simulation* 14 (2): 430–442.

Batiha, B. 2010. Numerical solution of Bratu-type equations by the variational iteration method. *Hacet. J. Math. Stat* 39 (1): 23–29.

Bender, C. M., Milton, K. A., Pinsky, S. S. and Simmons, L. M. 1989. A new perturbative approach to nonlinear problems. *Journal of Mathematical Physics* 30 (7): 1447.

Bhrawy, A. and Alofi, A. 2012. A Jacobi-Gauss collocation method for solving nonlinear Lane-Emden type equations. *Communications in Nonlinear Science and Numerical Simulation* 17 (1): 62–70.

Biles, D. C., Robinson, M. P. and Spraker, J. S. 2002. A generalization of the Lane–Emden equation. *Journal of mathematical analysis and applications* 273 (2): 654–666.

Botella, O. and Peyret, R. 2001. Computing singular solutions of the Navier-Stokes equations with the Chebyshev-collocation method. *International Journal for Numerical Methods in Fluids* 36 (2): 125–163.

Bougoffa, L. and Al-Khadhi, M. A. 2009. New Explicit Solutions for Troeschs Boundary Value Problem. *Applied Mathematics and Information Sciences* 3 (1): 89–96.

Bruce K. Driver. 2003. *Analysis Tools with Applications*. Springer.

Buckmire, R. 2004. Application of a Mickens finite-difference scheme to the cylindrical Bratu-Gelfand problem. *Numerical Methods for Partial Differential Equations* 20 (3): 327 – 337.

Caglar, H., Caglar, N., Özer, M., Valarstos, A. and Anagnostopoulos, A. N. 2010. B-spline method for solving Bratu's problem. *International Journal of Computer Mathematics* 87 (8): 1885–1891.

Canuto, C., Hussaini, M., Quarteroni, A. and Zang, T. 1988. *Spectral methods in Fluid Dynamics*. Springer-Verlag New York, Inc.

Çaglar, H., Çaglar, S. and Twizell, E. 1999. The numerical solution of fifth-order boundary value problems with sixth-degree B-spline functions. *Applied Mathematics Letters* 12 (5): 25–30.

Chandrasekhar, S. and Chandrasekar, S. 1958. *An introduction to the study of stellar structure*. , vol. 2. Dover Publications Inc.

Chang, R. Y. and Wang, M. L. 1983. Shifted Legendre direct method for variational problems. *Journal of Optimization Theory and Applications* 39 (2): 299–307.

Chang, S. 2010. A variational iteration method for solving Troeschs problem. *Journal of Computational and Applied Mathematics* 234 (10): 3043–3047.

Chang, S. and Chang, I.-L. 2008. A new algorithm for calculating one-dimensional differential transform of nonlinear functions. *Applied Mathematics and Computation* 195 (2): 799–808.

Chawla, M. M. and Katti, C. P. 1979. Finite difference methods for two-point boundary value problems involving high order differential equations. *BIT Numerical Mathematics* 19 (1): 27–33.

Chen, C. and Hsiao, C. 1975. A walsh series direct method for solving variational problems. *Journal of the Franklin Institute* 300 (4): 265–280.

Cheng, C., Tsay, Y. and Wu, T. 1977. Walsh Operational Matrices for Fractional Calculus and Their Application to Distributed Systems. *Journal of the Franklin Institute* 303 (3): 267–284.

Chowdhury, M. and Hashim, I. 2009. Solutions of EmdenFowler equations by homotopy-perturbation method. *Nonlinear Analysis: Real World Applications* 10 (1): 104–115.

Chui, C. K. C. 1992. *An introduction to wavelets*. Academic Press.

Ciesielski, M. and Leszczynski, J. 2003. Numerical simulations of anomalous diffusion. *CMM - 2003 Computer Methods in Mechanics* .

Coddington, E. A. and Levinson, N. 1955. *Theory of ordinary differential equations*. Tata McGraw-Hill Education.

Daftardar Gejji, V. and Jafari, H. 2007. Solving a multi-order fractional differential equation using adomian decomposition. *Applied Mathematics and Computation* 189 (1): 541–548.

Daubechies, I. 1988. Orthonormal bases of compactly supported wavelets. *Comm. Pure Appl. Math.* 41: 909–996.

Daubechies, I. and Others. 1992. *Ten lectures on wavelets*. , vol. 61. SIAM.

Davis, H. T. 1962. *Introduction to nonlinear differential and integral equations*. Courier Dover Publications.

Davis, P. J. and Rabinowitz, P. 1975. *Methods of numerical integration. Computer science and applied mathematics*, vol. 17. Academic Press.

Deeba, E., Khuri, S. and Xie, S. 2000. An Algorithm for Solving Boundary Value Problems. *Journal of Computational Physics* 159 (2): 125–138.

Derili, H. and Sohrabi, S. 2008. Numerical solution of singular integral equations using orthogonal functions. *Mathematical Sci.(QJMS)* 2 (3): 261.

Diethelm, K. and Walz, G. 1997. Numerical solution of fractional order differential equations by extrapolation. *Numerical Algorithms* 16 (3-4): 231–253.

Diethelm, K., Ford, N. J., Freed, A. D. 2002. A Predictor-Corrector Approach for the Numerical Solution of Fractional Differential Equations. *Nonlinear Dynamics* 29 (1-4): 3–22.

Dixon, J. and Tuszyski, J. 1990. Solutions of a generalized Emden equation and their physical significance. *Physical Review A* 41 (8): 4166–4173.

Doha, E., Abd- Elhameed, W., Youssri, Y. and Elhameed, W. A. 2013. Second kind Chebyshev operational matrix algorithm for solving differential equations of LaneEmden type. *New Astronomy* 23-24: 113–117.

Ebaid, A. 2011. A new analytical and numerical treatment for singular two-point boundary value problems via the Adomian decomposition method. *Journal of Computational and Applied Mathematics* 235 (8): 1914–1924.

Eddington, A. S. 1988. *The internal constitution of the stars*. Cambridge University Press.

El-Gamel, M. 2007. Comparison of the solutions obtained by Adomian decomposition and wavelet-Galerkin methods of boundary-value problems. *Applied Mathematics and Computation* .

El-Mesiry, A., El-Sayed, A. and El-Saka, H. 2005. Numerical methods for multi-term fractional (arbitrary) orders differential equations. *Applied Mathematics and Computation* 160 (3): 683–699.

Elbarbary, E. M. 2005. Chebyshev finite difference method for the solution of boundary-layer equations. *Applied Mathematics and Computation* 160 (2): 487–498.

Elbarbary, E. M. E. and El-Kady, M. 2003. Chebyshev finite difference approximation for the boundary value problems. *Applied Mathematics and Computation* 139 (2-3): 513–523.

Erdelyi, Arthur and Magnus, Wilhelm and Oberhettinger, Fritz and Tricomi, Francesco G and Bateman, H. 1953. *Higher transcendental functions*. New York McGraw-Hill.

Feng, X., Mei, L. and He, G. 2007. An efficient algorithm for solving Troeschs problem. *Applied Mathematics and Computation* 189 (1): 500–507.

Fermi, E. 1927. Un Metodo Statistico per la determinazione di Alcune priorieta dell' atome. *Rend. Accad. Naz. Lincei* 6.

Fowler, R. H. 1930. Emden's equation: The solutions of Emden's and similar differential equations. *Monthly Notices of the Royal Astronomical Society* 91: 63.

Fox, L. and Parker, I. B. 1968. *Chebyshev polynomials in numerical analysis*. Oxford U.P.

Frank-Kameneëtìskiæ, D. A. 1969. *Diffusion and heat transfer in chemical kinetics*. , vol. 2. Plenum Press (New York).

Gottlieb, D. and Orszag, S. A. 1977. *Numerical analysis of spectral methods : theory and applications*. , vol. 2. SIAM.

Grossmann, A. and Morlet, J. 1984. Decomposition of Hardy functions into square integrable wavelets of constant shape. *SIAMJ. Math. Anal.* 15 (4): 723–736.

Haar, A. 1910. Zur theorie der orthogonalen funktionen systeme. *Math. Ann.* 69: 331–371.

Hashim, I., Abdulaziz, O. and Momani, S. 2009. Homotopy analysis method for fractional IVPs. *Communications in Nonlinear Science and Numerical Simulation* 14 (3): 674–684.

Hassan, H. N. and El-Tawil, M. A. 2011. An efficient analytic approach for solving two-point nonlinear boundary value problems by homotopy analysis method. *Mathematical Methods in the Applied Sciences* 34 (8): 977–989.

Hassan, I. H. A. H. and Ertürk, V. S. 2007. Applying Differential Transformation Method to. *Int. J. Contemp. Math. Sciences* 2 (30): 1493–1504.

He, J. 2003. Variational approach to the Thomas-Fermi equation. *Applied Mathematics and Computation* 143 (2-3): 533–535.

Hilfer R. 2000. *Applications Of Fractional Calculus In Physics*. World Scientific.

Hong, D., Wang, J. and Gardner, R. 2004. *Real Analysis with an Introduction to Wavelets and Applications*. Elsevier.

Horedt, G. P. 1986. Seven-digit tables of Lane-Emden functions. *Astrophysics and Space Science* 126 (2): 357–408.

Horedt, G. P. 2004. *Polytropes: Applications in Astrophysics and Related Fields*. Springer.

Horng, I. R. and Chou, J. H. 1985. Shifted Chebyshev direct method for solving variational problems. *International Journal of Systems Science* 16 (7): 855–861.

Hosseini, S. G. and Mohammadi, F. 2011. A New Operational Matrix of Derivative for Chebyshev Wavelets and its Applications in Solving Ordinary Differential Equations with Non analytic Solution. *Applied Mathematical Sciences* 5 (51): 2537–2548.

Hwang, C. and Shih, Y. 1982. Solution of integral equations via Laguerre polynomials. *Computers and Electrical Engineering* 9 (3-4): 123–129.

Irena, R., Svatoslav, S., Ewa, W., Michael, Z. and Others. 2009. Limit properties of solutions of singular second-order differential equations. *Boundary Value Problems* 2009.

Jafari, H., Yousefi, S., Firoozjaee, M., Momani, S. and Khalique, C. 2011. Application of Legendre wavelets for solving fractional differential equations. *Computers and Mathematics with Applications* 62 (3): 1038–1045.

Jalilian, R. 2010. Non-polynomial spline method for solving Bratu's problem. *Computer Physics Communications* 181 (11): 1868–1872.

Jamet, P. 1970. On the convergence of finite-difference approximations to one-dimensional singular boundary-value problems. *Numerische Mathematik* 14 (4): 355–378.

Karimi Vanani, S. and Aminataei, A. 2010. On the numerical solution of differential equations of Lane-Emden type. *Computers and Mathematics with Applications* 59 (8): 2815–2820.

Khalifa, A. K., Elbarbary, E. M. and Abd Elrazek, M. A. 2003. Chebyshev expansion method for solving second and fourth-order elliptic equations. *Applied Mathematics and Computation* 135 (2-3): 307–318.

Khuri, S. 2004. A new approach to Bratus problem. *Applied Mathematics and Computation* 147 (1): 131–136.

Khuri, S. and Sayfy, A. 2011. Troeschs problem: A B-spline collocation approach. *Mathematical and Computer Modelling* 54 (9-10): 1907–1918.

Khuri, S. A. 2003. A Numerical Algorithm For Solving Troesch'S Problem. *International Journal of Computer Mathematics* 80 (4): 493–498.

Kiguradze, T. 2009. On solvability and unique solvability of two-point singular boundary value problems. *Nonlinear Analysis: Theory, Methods and Applications* 71 (3): 789–798.

Kilbas, A., Srivastava, H. M. and Trujillo, J. 2006. *Theory and Applications of Fractional Differential Equations, 204*. Elsevier Science.

Kumar, M. 2002. A three-point finite difference method for a class of singular two-point boundary value problems. *Journal of Computational and Applied Mathematics* 145 (1): 89–97.

Kumar, P. and Agrawal, O. P. 2006. An approximate method for numerical solution of fractional differential equations. *Signal Processing* 86 (10): 2602–2610.

Lakestani, M., Dehghan, M. and Irandoust-pakchin, S. 2012. The construction of operational matrix of fractional derivatives using B-spline functions. *Communications in Nonlinear Science and Numerical Simulation* 17 (3): 1149–1162.

Li, S. and Liao, S. 2005. An analytic approach to solve multiple solutions of a strongly nonlinear problem. *Applied Mathematics and Computation* 169 (2): 854–865.

Li, X. and Wu, B. 2011. A novel method for nonlinear singular fourth order four-point boundary value problems. *Computers and Mathematics with Applications* 62 (1): 27–31.

Li, Y. 2010. Solving a nonlinear fractional differential equation using Chebyshev wavelets. *Communications in Nonlinear Science and Numerical Simulation* 15 (9): 2284–2292.

Li, Y. and Sun, N. 2011. Numerical solution of fractional differential equations using the generalized block pulse operational matrix. *Computers and Mathematics with Applications* 62 (3): 1046–1054.

Li, Y. and Zhao, W. 2010. Haar wavelet operational matrix of fractional order integration and its applications in solving the fractional order differential equations. *Applied Mathematics and Computation* 216 (8): 2276–2285.

Liao, S. 2003. A new analytic algorithm of Lane-Emden type equations. *Applied Mathematics and Computation* 142 (1): 1–16.

Liao, S. and Tan, Y. 2007. A General Approach to Obtain Series Solutions of Nonlinear Differential Equations. *Studies in Applied Mathematics* 119 (4): 297–354.

Lu, J. 2007. Variational iteration method for solving a nonlinear system of second-order boundary value problems. *Computers and Mathematics with Applications* 54 (7-8): 1133–1138.

Lubich, C. 1985. Fractional linear multistep methods for Abel-Volterra integral equations of the second kind. *Mathematics of computation* 45 (172): 463–469.

Machado, J. T., Kiryakova, V. and Mainardi, F. 2011. Recent history of fractional calculus. *Communications in Nonlinear Science and Numerical Simulation* 16 (3): 1140–1153.

Mallat, S. G. 1989. Multiresolution approximations and wavelet orthonormal bases of Lˆ2 (R). *Transactions of the American Mathematical Society* 315 (1): 69–87.

Mandelzweig, V. B. and Tabakin, F. 2001. Quasilinearization approach to nonlinear problems in physics with application to nonlinear ODEs. *Computer Physics Communications* 141 (2): 268–281.

Marzban, H., Razzaghi, M. and Shafiee, M. 2008. Analysis of time-varying singular bilinear systems by hybrid functions. *International Journal of Systems Science* 39 (3): 229–235.

Marzban, H. R. and Hoseini, S. M. 2013. Solution of linear optimal control problems with time delay using a composite Chebyshev finite difference method. *Optimal Control Applications and Methods* 34 (3): 253–274.

Mcgough, J. S. 1998. Numerical continuation and the Gelfand problem. *Applied Mathematics and Computation* 89 (1-3): 225–239.

Mehra, M., Siddiqi, A. H., Brokate, M. and Gupta, A. K. 2009. Wavelets and Differential Equations-A short review. *AIP Conference Proceedings* 241–252.

Metzler, R. and Klafter, J. 2000. The random walk's guide to anomalous diffusion: a fractional dynamics approach. *Physics reports* 339 (1): 1–77.

Meyer, Y. 1989. Wavelets and operators. *Analysis at Urbana* 1: 256–365.

Miller, K. and Ross, B. 1993. *An Introduction to the Fractional Calculus and Fractional Differential Equations*. John Wiley and Sons.

Mirmoradi, S. H., Hosseinpour, I., Ghanbarpour, S. and Barari, A. 2009. Application of an approximate analytical method to nonlinear Troeschs problem. *Applied Mathematical Sciences* 3 (32): 1579–1585.

Misiti, M., Misiti, Y., Oppenheim, G. and Poggi, J.-M. 2000. *Wavelet Toolbox 4 Users Guide*.

Mohammadi, F., Hosseini, M. M. and Mohyud-din, S. T. 2011. A new operational matrix for Legendre wavelets and its applications for solving fractional order boundary values problems. *International Journal of the Physical Sciences* 6 (32): 7371–7378.

Mohanty, R. K. 2000. A fourth-order finite difference method for the general one-dimensional nonlinear biharmonic problems of first kind. *Journal of computational and applied mathematics* 114 (2): 275–290.

Mohanty, R. K. 2005. A family of variable mesh methods for the estimates of (du/dr) and solution of non-linear two point boundary value problems with singularity. *Journal of computational and applied mathematics* 182 (1): 173–187.

Momani, S., Abuasad, S. and Odibat, Z. 2006. Variational iteration method for solving nonlinear boundary value problems. *Applied Mathematics and Computation* 183 (2): 1351–1358.

Momani, S. and Noor, M. A. 2007. Numerical comparison of methods for solving a special fourth-order boundary value problem. *Applied Mathematics and Computation* 191 (1): 218–224.

Momani, S. and Odibat, Z. 2006. Analytical solution of a time-fractional Navier-Stokes equation by Adomian decomposition method. *Applied Mathematics and Computation* 177 (2): 488–494.

Momani, S. and Odibat, Z. 2008a. A novel method for nonlinear fractional partial differential equations: Combination of DTM and generalized Taylor's formula. *Journal of Computational and Applied Mathematics* 220 (1-2): 85–95.

Momani, S. and Odibat, Z. 2008b. Numerical methods for nonlinear partial differential equations of fractional order. *Applied Mathematical Modelling* 32 (1): 28–39.

Mounim, A. S. and de Dormale, B. 2006. From the fitting techniques to accurate schemes for the Liouville-Bratu-Gelfand problem. *Numerical Methods for Partial Differential Equations* 22 (4): 761–775.

Noor, M. A. and Mohyud-Din, S. T. 2008. Homotopy perturbation method for solving sixth-order boundary value problems. *Computers and Mathematics with Applications* 55 (12): 2953–2972.

Odibat, Z. 2006. Approximations of fractional integrals and Caputo fractional derivatives. *Applied Mathematics and Computation* 178 (2): 527–533.

Odibat, Z., Momani, S. and Erturk, V. S. 2008. Generalized differential transform method: Application to differential equations of fractional order. *Applied Mathematics and Computation* 197 (2): 467–477.

Odibat, Z. M. 2009. Computational algorithms for computing the fractional derivatives of functions. *Mathematics and Computers in Simulation* 79 (7): 2013–2020.

Odibat, Z. M. and Momani, S. 2006. Application of variational iteration method to nonlinear differential equations of fractional order. *International Journal of Nonlinear Sciences and Numerical Simulation* 7 (1): 27–34.

O'Neil, P. V. 1987. *Advanced Engineering Mathematics*. Wadsworth Pub. Co.

O'Regan, D. 1990. Existence of positive solutions to some singular and nonsingular second order boundary value problems. *Journal of Differential Equations* 84 (2): 228–251.

Othmar Koch, E. B. W., Koch, O. and Weinmüller, E. B. 2001. Iterated Defect Correction for the Solution of Singular Initial Value Problems. *SIAM Journal on Numerical Analysis* 38 (6): 1784–1799.

Pandey, R. K. 1996a. On a class of weakly regular singular two point boundary value problems I. *Nonlinear Analysis: Theory, Methods and Applications* 27 (1): 1–12.

Pandey, R. K. 1996b. On a class of weakly regular singular two-point boundary value problemsII. *Journal of differential equations* 127 (1): 110–123.

Pandey, R. K., Kumar, N., Bhardwaj, A. and Dutta, G. 2012. Solution of Lane-Emden type equations using Legendre operational matrix of differentiation. *Applied Mathematics and Computation* 218 (14): 7629–7637.

Parand, K., Dehghan, M. and Pirkhedri, A. 2010a. The use of Sinc-collocation method for solving Falkner Skan boundary-layer equation. *International Journal for numerical methods in fluids* 68 (1): 36–47.

Parand, K., Dehghan, M., Rezaei, A. and Ghaderi, S. 2010b. An approximation algorithm for the solution of the nonlinear Lane-Emden type equations arising in astrophysics using Hermite functions collocation method. *Computer Physics Communications* 181 (6): 1096–1108.

Parand, K. and Pirkhedri, A. 2010. Sinc-Collocation method for solving astrophysics equations. *New Astronomy* 15 (6): 533–537.

Parand, K., Rezaei, A. and Taghavi, A. 2010c. Lagrangian method for solving Lane-Emden type equation arising in astrophysics on semi-infinite domains. *Acta Astronautica* 67 (7-8): 673–680.

Paraskevopoulos, P. N., Sklavounos, P. and Georgiou, G. 1990. The Operation Matrix of Integration for Bessel Functions. *Journal of the Franklin Institute* 327: 329–341.

Peano, G. 1886. Sull'integrabilita delle equazioni differenziali del primo ordine. *Atti. della Reale Accad. dell Scienze di Torino* 21: 677–685.

Podlubny, I. 1998. *Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, Some Methods of Their Solution and Some of Their Applications*. New York: Academic Press.

Rachunkova, I., Spielauer, A., Stanek, S. and Weinmuller, E. B. 2013. Positive solutions of nonlinear Dirichlet BVPs in ODEs with time and space singularities. *Boundary Value Problems* 2013 (1): 6.

Rachunkova, I., Stanek, S. and Tvrdy, M. 2009. *Solvability of nonlinear singular problems for ordinary differential equations*. , vol. 5. Hindawi Publishing Corporation, ISBN 978-977-454-040-0.

Rachuunkova, I., Stanek, S., Weinmuller, E. and Zenz, M. 2010. Neumann problems with time singularities. *Computers and Mathematics with Applications* 60 (3): 722–733.

Ramos, J. 2003. Linearization methods in classical and quantum mechanics. *Computer Physics Communications* 153 (2): 199–208.

Ramos, J. 2005. Linearization techniques for singular initial-value problems of ordinary differential equations. *Applied Mathematics and Computation* 161 (2): 525–542.

Ramos, J. 2008. Series approach to the Lane-Emden equation and comparison with the homotopy perturbation method. *Chaos, Solitons and Fractals* 38 (2): 400–408.

Ramos, J. 2009. Piecewise-adaptive decomposition methods. *Chaos, Solitons and Fractals* 40 (4): 1623–1636.

Rashidinia, J. and Ghasemi, M. 2011. B-spline collocation for solution of two-point boundary value problems. *Journal of Computational and Applied Mathematics* 235 (8): 2325–2342.

Rashidinia, J., Jalilian, R. and Farajeyan, K. 2009. Spline approximate solution of eighth-order boundary-value problems. *International Journal of Computer Mathematics* 86 (8): 1319–1333.

Razzaghi, M. 1988. Fourier series direct method for variational problems. *International Journal of Control* 48 (3): 887–895.

Razzaghi, M. and Yousefi, S. . 2001. Legendre wavelets method for the solution of nonlinear problems in the calculus of variations. *Mathematical and Computer Modelling* 7177 (01): 45–54.

Rehman, M. and Ali Khan, R. 2011. The Legendre wavelet method for solving fractional differential equations. *Communications in Nonlinear Science and Numerical Simulation* 16 (11): 4163–4173.

Roberts, S. and Shipman, J. 1972. Solution of Troesch's two-point boundary value problem by a combination of techniques. *Journal of Computational Physics* 10 (2): 232–241.

Roberts, S. M. and Shipman, J. S. 1976. On the closed form solution of Troesch's problem. *Journal of Computational Physics* 21 (3): 291–304.

Rudd, M. and Tisdell, C. C. 2007. On the solvability of two-point, second-order boundary value problems. *Applied Mathematics Letters* 20 (7): 824–828.

Saadatmandi, A. and Dehghan, M. 2010. A new operational matrix for solving fractional-order differential equations. *Computers and Mathematics with Applications* 59 (3): 1326–1336.

Saadatmandi, A., Dehghan, M. and Eftekhari, A. 2009. Application of He's homotopy perturbation method for non-linear system of second-order boundary value problems. *Nonlinear Analysis: Real World Applications* 10 (3): 1912–1922.

Saeedi, H., Moghadam, M. M., Mollahasani, N. and Chuev, G. 2011. A CAS wavelet method for solving nonlinear Fredholm integro-differential equations of fractional order. *Communications in Nonlinear Science and Numerical Simulation* 16 (3): 1154–1163.

Samko, S. G., Kilbas, A. A. and Marichev, O. I. 1993. Fractional Integrals and Derivatives: Theory and Applications. *Gordon and Breach, Yverdon* .

Scott, M. R. 1974, Conversion of boundary-value problems into stable initial-value problems via several invariant imbedding algorithms, Tech. rep., Sandia Labs., Albuquerque, N. Mex.(USA).

Shang, X. and Yuan, Y. 2011. Homotopy perturbation method based on Green function for solving non-linear singular boundary value problems. In *2011 International Conference on Machine Learning and Cybernetics*, 851–855. IEEE.

Shawagfeh, N. T. 1993. Nonperturbative approximate solution for Lane-Emden equation. *Journal of Mathematical Physics* 34 (9): 4364.

Shen, J. 1995. Efficient Spectral-Galerkin Method II. Direct Solvers of Second- and Fourth-Order Equations Using Chebyshev Polynomials. *SIAM Journal on Scientific Computing* 16 (1): 74–87.

Singh, O. P., Pandey, R. K. and Singh, V. K. 2009. An analytic algorithm of Lane-Emden type equations arising in astrophysics using modified Homotopy analysis method. *Computer Physics Communications* 180 (7): 1116–1124.

Spitzer Jr, L. 1942. The Dynamics of the Interstellar Medium. III. Galactic Distribution. *The Astrophysical Journal* 95: 329.

Strang, G. and Nguyen, T. 1996. *Wavelets and Filter Banks*. Wellesley College.

Stromberg, J. 1981. A modified Franklin system and higher-order spline on R, as unconditional basis for Hardy spaces. In *Harmonic Analysis*, 475–494.

Syam, M. I. and Hamdan, A. 2006. An efficient method for solving Bratu equations. *Applied Mathematics and Computation* 176 (2): 704–713.

Tenreiro Machado, J. 2009. Fractional derivatives: Probability interpretation and frequency response of rational approximations. *Communications in Nonlinear Science and Numerical Simulation* 14 (9-10): 3492–3497.

Timoshenko, S., Woinowsky-Krieger, S. and Woinowsky-Krieger, S. 1959. *Theory of plates and shells*. , vol. 2. McGraw-hill New York.

Timoshenko, S. P. 1961. Theory of Elastic Stability. *Me Graw-Hill, New York* .

Troesch, B. A. 1960. Intrinsic difficulties in the numerical solution of a boundary value problem. *Space Tech. Labs, Tech. Note NN* 142: 1339–1565.

Twizell, E. H. and Tirmizi, S. I. A. 1988. Multiderivative methods for nonlinear beam problems. *Communications in Applied Numerical Methods* 4 (1): 43–50.

Van Gorder, R. and Vajravelu, K. 2009. On the selection of auxiliary functions, operators, and convergence control parameters in the application of the Homotopy Analysis Method to nonlinear differential equations: A general approach. *Communications in Nonlinear Science and Numerical Simulation* 14 (12): 4078–4089.

Voigt, R. G., Hussaini, M. Y., Streett, C. L. and Zang, T. A. 1984. Spectral methods for partial differential equations. In *Transactions of the First Army Conference on Applied Mathematics and Computing, ARO Report*, 883–925. DTIC Document, Society for Industrial and Applied.

Walter, G. G. 1994. *Wavelets and Other Orthogonal Systems with Applications*. CRC Press.

Wang, C. C., Lee, Z.-Y. and Kuo, Y. 2008. Application of residual correction method in calculating upper and lower approximate solutions of fifth-order boundary-value problems. *Applied Mathematics and Computation* 199 (2): 677–690.

Wang, Y. and Fan, Q. 2012. The second kind Chebyshev wavelet method for solving fractional differential equations. *Applied Mathematics and Computation* 218 (17): 8592–8601.

Wang, Y., Song, H.-F. and Li, D. 2009. Solving two-point boundary value problems using combined homotopy perturbation method and Greens function method. *Applied Mathematics and Computation* 212 (2): 366–376.

Wazwaz, A. 2001a. A reliable algorithm for obtaining positive solutions for nonlinear boundary value problems. *Computers and Mathematics with Applications* 41 (10-11): 1237–1244.

Wazwaz, A. 2001b. The numerical solution of fifth-order boundary value problems by the decomposition method. *Journal of Computational and Applied Mathematics* 136 (10-11): 259–270.

Wazwaz, A. 2005a. Adomian decomposition method for a reliable treatment of the Bratu-type equations. *Applied Mathematics and Computation* 166 (3): 652–663.

Wazwaz, A. 2005b. Adomian decomposition method for a reliable treatment of the EmdenFowler equation. *Applied Mathematics and Computation* 161 (2): 543–560.

Wazwaz, A. 2006. The modified decomposition method for analytic treatment of differential equations. *Applied Mathematics and Computation* 173 (1): 165–176.

Weibel, E. S. 1959. On the confinement of a plasma by magnetostatic fields. *Physics of Fluids* 2 (52).

Wu, G. and Lee, E. 2010. Fractional variational iteration method and its application. *Physics Letters A* 374 (25): 2506–2509.

Yildirim, A. and Özis, T. 2009. Solutions of singular IVPs of Lane-Emden type by the variational iteration method. *Nonlinear Analysis: Theory, Methods and Applications* 70 (6): 2480–2484.

Yousefi, S. A. 2006. Legendre wavelets method for solving differential equations of Lane-Emden type. *Applied Mathematics and Computation* 181 (2): 1417–1422.

Yuste, S. 2006. Weighted average finite difference methods for fractional diffusion equations. *Journal of Computational Physics* 216 (1): 264–274.

Zarebnia, M. and Sajjadian, M. 2012. The sinc-Galerkin method for solving Troeschs problem. *Mathematical and Computer Modelling* 56 (9-10): 218–228.

Zhang, B., Wu, Q. and Luo, X. 2006. Experimentation with two-step Adomian decomposition method to solve evolution models. *Applied Mathematics and Computation* 175 (2): 1495–1502.

Zhang, J. 2009a. The numerical solution of fifth-order boundary value problems by the variational iteration method. *Computers and Mathematics with Applications* 58 (11-12): 2347–2350.

Zhang, Y. 2009b. A finite difference method for fractional partial differential equation. *Applied Mathematics and Computation* 215 (2): 524–529.