UNIVERSITI PUTRA MALAYSIA

# *IMPRECISE WEIGHTS AND NON-DISCRETIONARY FACTORS IN DATA ENVELOPMENT ANALYSIS*

MARYAM HEYDAR

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**IMPRECISE WEIGHTS AND NON-DISCRETIONARY FACTORS IN DATA ENVELOPMENT ANALYSIS**

By

**MARYAM HEYDAR**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirement for the Degree of Master of Science**

**March 2014**

# DEDICATION

**This thesis is especially dedicated to my lovely mother, dear husband and kind sister for unconditional patience, support and encouragement through this project and my life**

Abstract of thesis presented to the senate of Universiti Putra Malaysia in fulfilment of requirement of the degree of Master of Science

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**March 2014**

**Chairman: Mohd Rizam Abu Bakar, PhD**

**Faculty: Science**

The aim of this thesis is to investigate and identify the main effect of qualitative and imprecise data, such as knowledge, experience and human judgments, in efficiency evaluation of the real world problems on one hand, and non-discretionary data such as environmental factors in Data Envelopment analysis issues (DEA) on the other hand.

In the first part, a fuzzy weighed CCR model is represented to assess decision making units (DMUs) with normal data and fuzzy essence in weights of input and output factors. In this case, incorporation of fuzzy arithmetic and traditional method in DEA is helpful to solve and achieve the interval efficiency scores without additional restrictions for each data which prevents the model from becoming infeasible. The outcome of the model demonstrates the effect of data on the efficiency score.

This thesis then offers a non-discretionary fuzzy additive model for cases in efficiency analysis and benchmarking that, decision-maker is confronted with some exogenously fixed data with fuzzy essence. A detailed procedure on a method applying $\alpha$-cut concept is provided to linearize the model with the intention of achieving appropriate benchmarking for inefficient DMUs.

In conclusion, the thesis argues that impact of imprecise weights and non-discretionary factors in DEA is essential and requires new modified models. The models offer more informative and persuasive data to assist the manager to be aware of uncertain effects of factors on efficiency score as well as to evaluate and benchmark DMUs with fuzzy nature while non-discretionary factors are taken into account.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

**KESAN WAJARAN TIDAK TEPAT DAN FAKTOR LUAR KAWALAN DALAM ANALISIS PENGUMPULAN DATA**

Oleh

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Mac **2014**

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Tujuan tesis ini adalah untuk mengkaji dan mengenal pasti kesan utama kualitatif data dan data tidak tepat, seperti pengetahuan, pengalaman dan pertimbangan manusia. Ia juga mengkaji penilaian kecekapan suatu masalah sebenar. Selain itu, ia juga mengkaji data luar kawalan seperti faktor alam sekitar dalam analisis pengumpulan data.

Dalam bahagian pertama, suatu model CCR wajaran fuzzy diwakili untuk menilai unit membuat keputusan dengan data normal dan intipati fuzzy dalam wajaran faktor input dan output. Dalam kes ini, penggunaan aritmetik fuzzy dan kaedah tradisional dalam analisis pengumpulan data membantu untuk menyelesaikan dan mencapai skor selang kecekapan tanpa sekatan tambahan untuk setiap data yang menghalang model daripada menjadi tidak praktikal. Hasil cemerlang model ini mempamerkan kesan data ke atas skor kecekapan.

Seterusnya, tesis ini menawarkan model fuzzy luar kawalan tambahan bagi kes-kes dalam analisis kecekapan dan penanda aras, pembuat keputusan berhadapan dengan beberapa data luaran tetap dengan intipati fuzzy. Dalam prosedur terperinci tesis ini menyediakan kaedah menggunakan konsep $\alpha$ -cut untuk melinearkan model dengan niat untuk mencapai penanda aras yang sesuai bagi unit membuat keputusan yang tidak cekap.

Kesimpulannya, tesis ini berhujah bahawa kesan wajaran yang tidak tepat dan faktor-faktor luar kawalan dalam analisis pengumpulan data adalah penting dan memerlukan model diubahsuai. Model ini menawarkan lebih bermaklumat dan menyakinkan untuk membantu pengurus untuk menyedari kesan faktor tidak tentu ke atas skor kecekapan dan juga untuk menilai dan membuat keputusan penanda aras unit dengan alam semula jadi fuzzy manakala faktor-faktor luar kawalan juga diambil kira.

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I would like to thank my loved ones, who have supported me throughout entire process, all by keeping me from failure and helping me achieving my goals. Their memories and presence have made the life more beautiful for me. I will be grateful forever for your love.

I certify that a Thesis Examination Committee has met on 10 March 2014 to conduct the final examination of Maryam Heydar on her thesis entitled "Imprecise Weights and Non-Discretionary Factors in Data Envelopment Analysis" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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**Declaration by Members of Supervisory Committee**

This is to confirm that:

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# LIST OF ABBREVIATIONS

| Abbreviation | Description |
|---|---|
| ADD | Additive |
| ADD-S | Additive Score |
| AR | Assurance Region |
| BCC | Banker, Charnes, Cooper |
| BM | Banker and Morey |
| CCR | Charnes, Cooper, Rhodes |
| CSW | Common Set of Weights |
| DA | Discriminant Analysis |
| DEA | Data Envelopment Analysis |
| DEA-DA | FDEA-discriminant Analysis |
| DMU | Decision Making Unit |
| ERP | Enterprise Resource Planning |
| FDEA | Fuzzy Data Envelopment Analysis |
| FLP | Fuzzy Linear Programming |
| FMS | Flexible Manufacturing System |
| I-O | Input Orientation |
| LHS | Left Hand Side |
| LP | Linear Programming |
| ND | Non-Discretionary |
| NDFDEA | Non-Discretionary Fuzzy DEA |
| O-O | Output Orientation |
| PCM | Professional Construction Management |
| PPF | Production Possibility Frontier |
| PPS | production possibility set |
| RA | Regression Analysis |
| RHS | Right Hand Side |
| R-S | Reference-Set |
| RTS | Return to Scale |
| SBM | Slack-based Measure |
| VRS | Variable Returns to scale |

# CHAPTER 1

# INTRODUCTION

## 1.1 Basic Definitions and Knowledge

In deliberating this thesis matters it is of the utmost importance to begin by defining the specialized words we will be using.

### 1.1.1 Data Envelopment Analysis

Data Envelopment Analysis (DEA) is one of the most powerful mathematical tools which applies the effective use and allocation of resources, to assist management in achieving organizational aims by obtaining more profit. It provides a means by which managers will be able to benchmark their performance, based on the results, against their competitors to make better future decisions.

DEA is a standard tool with wide applications in studies of performance measurement and analysis. It measures the relative efficiency of units with the same inputs and outputs that are called Decision Making Units (DMUs). It evaluates the efficiency of a unit compared to other units and the result is expressed as a relative score. It offers strategies to manage the resources to achieve the expected outputs. Efficiency depends on the performance of a single unit in transition of inputs to outputs in comparison with other units in a specific area.

DEA was first introduced in 1978 by Charnes, Cooper and Rhodes (CCR). This method is based on the assumption of constant returns to scale and defines the relative efficiency of a DMU which results from weights given to the inputs and outputs of that DMU in contrast to other DMUs. DEA is established on linear programming and maximizes the proportion of the weighted sum of outputs to weighted sum of inputs. Some special properties and principal usages of DEA make it applicable in various fields.

Some of the key features of DEA are as follows:

- Easy comprehension and convenient application;
- Realistic assessment of all factors in the models;
- No need to present the weights and the production function;
- Figure out the best performance rather than ideal situation;
- Ability to import multiple inputs and outputs in models.

Potential applications of this technique include:

- Ranking decision making units;
- Introducing some units to compare the efficiencies;
- Providing proper ways to improve the efficiency;
- Assessing technical progression and regression of units;
- Allocating resources;

- Analysing inputs and outputs;
- Determining the functional potential of units.

The common definition of performance, which is explained below, is the foundation of DEA models.

performance of unit *k* = (output of unit *k*) / (input of unit *k*).

The definition above is appropriate for performance measurement of a single unit with an input and an output, but multiple units at the same time cannot be involved. To solve this problem, Farrell (1957) presented a new technique which is the basis of all DEA models. Farrell provided a model with multiple inputs and one output by taking the weights for inputs and output, but there was still a problem involving multiple outputs which was solved by Charnes et al. (1978).

DEA is sometimes known as the method of measuring technical efficiency which is trying to maximize outputs, using existing inputs. A standard DEA problem is applied to estimate the relative efficiency of DMUs with multiple inputs and outputs. Each DMU is allowed to choose its arbitrary weights whereas the outcome of the fraction of the weighted sum of outputs to weighted sum of inputs of all DMUs must be less than or equal to 1 (Charnes et al. 1978).

Consider a set of n DMUs, which consume changing amounts of m different inputs to produce s different outputs; assume that $x_{ij}$ and $y_{rj}$ represent the inputs and outputs respectively and $v_i$ for $i = 1,2,\ldots,m$ and $u_r$for $r = 1,2,\ldots,s$ are factor weights corresponding to inputs and outputs. If $DMU_o$ where $o = 1,2,\ldots,n$ indicates the DMU is under efficiency evaluation, then the following CCR fractional programming problem will be arise.

$$\begin{aligned} \max \quad & h_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ \text{s.t.} \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad j = 1,2,\ldots,\mathrm{n}, \qquad (1.1) \\ & u_r, v_i \ge 0. \end{aligned}$$

Maximizing the fractional weighted output factors to weighted input factors is the target of this optimization problem. The first constraint specifies that the weights of each DMU are not allowed to yield efficiency scores greater than 1 regarding other DMUs. In other words, the optimal value of efficiency score is not more than and only less than one. Another limitation states that the weights cannot be zero because if for two DMUs, with the same inputs and outputs, except in one case, consider zero weight for that distinct input or output; both units will have the same efficiency score (Charnes et al. 1978).

By applying Charnes et al. input oriented method (1978) (turn the denominator of the objective function into a constraint which should be equal to 1), the model 1.1 will be converted to the linear form. The following model is considered as the LP CCR model. The main purpose of solving this model is finding the value of the weights.

$$
\begin{aligned}
\max \quad & h_o = \sum_{r=1}^{s} u_r y_{ro} \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{io} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j = 1,2,\dots,n, \\
& u_r, v_i \ge 0.
\end{aligned}
\tag{1.2}
$$

This model is trying to reduce the inputs as much as possible by keeping outputs at their maximum level or combine the minimal inputs to reach the efficiency frontier.

Efficiency or inefficiency of a DMU depends on the status of that unit regarding production possibility set (PPS), which is formed from different combinations of inputs and outputs and each point on it represents a DMU. In each of the DMU models, a DMU will be effective when it is placed on the boundary of the PPS which is called Production Possibility Frontier (PPF), since this area displays the best combination of inputs and outputs. The value of the objective function of model 1.2 is smaller than or equal to 1. If this value is exactly 1 and at least a non-zero factor weight can be found for the inputs and outputs, then $DMU_o$ is efficient (Charnes et al.1978).

So far, many studies and investigations have been done on DEA and its applications. Easy understanding and implementing with high accuracy make wide use of DEA in various subjects. DEA, like other methods, has some major weaknesses which can be expressed in the following:

- Ignores the random disorders and environmental factors.
- Nonparametric nature of this approach sometimes produces inappropriate results.
- Sensitivity to the data distribution that can lead to undesirable results. Therefore, it is assumed that data dispersion is negligible.

In recent years, many efforts have been made to overcome some shortcomings of the DEA method, including the use of fuzzy logic and probability theory for incomplete and qualitative data. Fuzzy logic can be regarded as a comprehensive concept of probability theory, which has more extensive use due to its greater compatibility with the natural factors.

### 1.1.2 Fuzzy Sets

Fuzzy logic is a multi-valued logic which is much more in line with the needs of today's complex world in comparison with Aristotelian logic. By means of Fuzzy logic, we are allowed to use words like “It is quite true” or “It is partly true” and in contrast “It is almost impossible”, “Not much” and “Rarely”. Fuzzy set theory, as the prominent tool for managing imprecision or vagueness of real-world problems, has been developed to handle the concept of partial truth values that fluctuate from completely true to completely false.

Fuzzy management is the usage of classical management in fuzzy environment to assist managers in various tasks like decision making and planning. Using fuzzy concept, the new developed models are able to analyze qualitative factors. In addition to the flexibility of these models, incomplete, imprecise and linguistic data, such as knowledge, experience and human judgment can be imported to the models.

The concept of fuzzy was first expressed in Max Black's article called "ambiguity" in 1937. Zadeh (1965) published his paper entitled “Fuzzy Sets” about possible problems in modeling systems. According to Zadeh (1975), factual expression of a complex condition is very hard by formal quantities and it seems essential to use linguistic factors with the value of words or sentences. This approach has become popular due to its features which are easy to understand, high potential and low computational cost.

The membership of an element in a classical set is stated in binary terms. It is one, if an element belongs to the set and zero otherwise. By contrast, a gradual assessment of the membership is provided by a fuzzy set theory. Membership in a fuzzy set is not a deterministic or non-deterministic matter; each element has its own degree of membership. For any set $X$, a membership function on $X$ is any function from $X$ to the real unit interval [0,1]. For an element $x$ of $X$, the value $\mu_A(x)$ is called the membership degree of $x$ in the fuzzy set $A$. The grade of membership of the element $x$ to the fuzzy set $A$ is measured by the membership degree $\mu_A(x)$. The value one means that $x$ is a member of the set and zero means that $x$ is not a member. The values between 0 and 1 illustrate fuzzy members, which belong to the fuzzy set only partially.

Therefore, Fuzzy sets generalized classical sets or in other words membership in the traditional set theory (zero or one) is a particular case of fuzzy set theory. The fuzzy set expresses a gradual transition from membership ($\mu_A(x)=1$) to a non-membership ($\mu_A(x)=0$) and vice versa. Furthermore, a component can belong to various sets with different membership values while the summation of these values is considered to be one.

Two generally considered shapes for fuzzy numbers are triangular and trapezoidal. Triangular functions, express the theme of "close to a real number" and trapezoidal membership function defines a fuzzy interval. In this research we use triangular fuzzy numbers because the most typical fuzzy set membership function is triangular membership function (Figure 1.1), which can be indicated by three elements: left hand side element, middle element, and right hand side element.

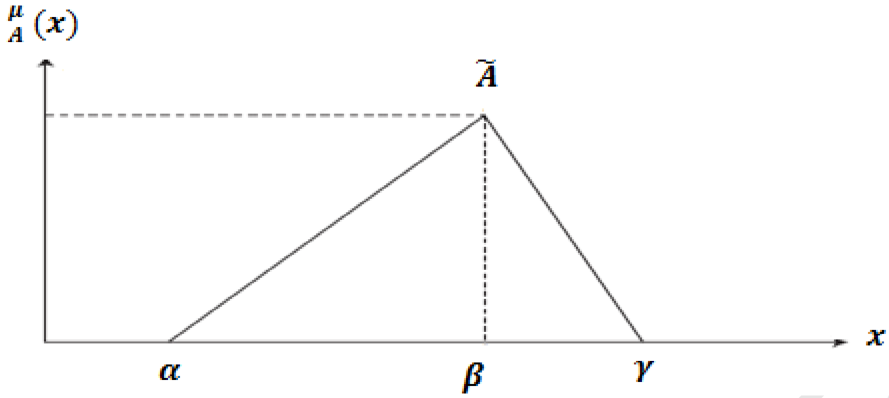

**Figure 1.1. Triangular membership function of $\tilde{A}$=($\alpha,\beta,\gamma$).**

Membership function of the fuzzy set *A* can be presented by a number between zero and 1. The crisp set $A_\alpha$ is the $\alpha$-cut of a fuzzy set *A*, which contains all members of *X*, whose membership degree in set *A* is greater than or equal to a certain value of.$\alpha$ This concept can be defined as below:

$$A_\alpha = \{x | \mu_A(x) \geq \alpha\}, \alpha \in [0,1].$$

Arithmetic operation on fuzzy numbers is based on two properties of fuzzy numbers:

- Any fuzzy number can be expressed completely and unique by $\alpha$-cut.
- All $\alpha$-cuts of each fuzzy number are closed intervals of real numbers.

So the arithmetic operations on fuzzy numbers will be defined like the arithmetic operations on closed intervals.

### 1.1.3 Fuzzy Data Envelopment Analysis

Decision making is very crucial in the current progressive world. Correct decisions are based on accurate data. Managers need to have information that would tell the truth and avoid the conjecture. Fuzzy Logic, which covers the possibilities and unknowns, obtains logical consequences according to the nature of the world, so we use the fuzzy concept to improve and overcome the shortcomings of DEA models in order to achieve precise results.

DEA as a non-parametric technique evaluates the performance of a set of homogeneous DMUs, which use similar inputs to produce similar outputs. Traditional DEA models commonly use crisp data, so it seems necessary to know the exact values of all inputs and outputs. Furthermore, inputs and outputs have been observed whose values are imprecise in many cases, so it is essential to take into account the existence of qualitative factors of inputs and outputs in a real evaluation problem. The first fuzzy mathematical programming approach was presented by Sengupta (1992) who combined fuzziness into the DEA model.

Issues including fuzzy data are the most common in fuzzy data envelopment analysis (FDEA), where the input and/or output factors have fuzzy nature. Relative efficiency evaluation with multiple inputs and outputs in fuzzy environment requires an FDEA model and it is more informative. It is however not necessary for all data to be fuzzy, even if only the inputs or only the outputs have fuzzy essence, FDEA models can be used.

Due to the fact that CCR is the basic and fundamental model in DEA, to clarify a typical FDEA model, fuzzy CCR model, which is the simplest FDEA model, can be considered. If we assume $\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^R)$ and $\tilde{y}_{rj} = (y_{rj}^L, y_{rj}^M, y_{rj}^R)$ when $i = 1,\dots,m,\ j = 1,\dots,n$ and $r = 1,\dots,s$ as the triangular fuzzy inputs and outputs respectively, also $u_r$ and $v_i$ be the weights of outputs and inputs, evaluating efficiency score of $DMU_o$ is possible by maximizing the value of $h_o = \frac{\sum_{r=1}^{s} u_r \tilde{y}_{ro}}{\sum_{i=1}^{m} v_i \tilde{x}_{io}}$ with respect to the conditions that for all $j$, $\frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}}$ is considered to be less than or equal to 1. Model 1.3 shows fuzzy CCR model, which is a crisp fractional model with ability to measure the relative performance of $DMU_o$ with fuzzy inputs and outputs:

$$\max \quad h_o = \frac{\sum_{r=1}^{s} u_r \tilde{y}_{ro}}{\sum_{i=1}^{m} v_i \tilde{x}_{io}}$$

$$\text{s.t.} \quad \frac{\sum_{r=1}^{s} u_r \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i \tilde{x}_{ij}} \le 1, \qquad j = 1,2,\dots,n, \tag{1.3}$$

$$u_r, v_i \ge 0.$$

In FDEA, we seek appropriate models for solving various types of FDEA problems including different fuzzy data; therefore perpending CCR models with different fuzzy essences and suitable methods for solving them is more applicable, while they assist us in solving other fuzzy models due to the point that fuzzy CCR is a fundamental FDEA model.

### 1.1.4 Non-Discretionary Factors

DEA helps the manager to improve the efficiency score of a DMU by decreasing inputs and/or increasing outputs, but in some cases, existence of some variables which are beyond the control of the management requires a technique to control the real effects of this non-discretionary variable. In other words, the values of these non-discretionary inputs or outputs are defined by forces exogenous in the organization under evaluation and the managers are not able to change the values of these fixed data, although in many real circumstances it is essential to take these non-discretionary factors into account in an efficiency analysis of an organization.

For instance, environmental factors such as snowfall or weather are considered as non-discretionary factors in evaluating the efficiency score of maintenance units. Likewise, in analyzing the performance of the branches of a restaurant chain, one of the effective elements is the number of competitors which is imposed on the problem without the control of the management. Similarly, in efficiency analysis of education centers, age of facilities is regarded as an exogenous factor that affects the performance of the education service provided.

By considering efficiency evaluation of DMUs with uncontrollable data, the non-discretionary (ND) model was introduced by Banker and Morey (1986). The model is capable of solving the problems with some ND data.

## 1.2 Problem Statement

Most real problems in DEA include imprecise data or factors with uncertain or uncontrollable values, which lead to having uncertain efficiency scores. FDEA models have improved in theory and practice due to the necessity of analysing these uncertainty problems. (See taxonomy of FDEA by Hatami-Marbani et al., 2011). In some cases, the previous FDEA models are not influential enough to cover many performance evaluation problems with specific properties due to their weakness in satisfying the objectives or constraints. It is essential to develop DEA models with fuzzy data or in fuzzy environment.

A FDEA problem typically refers to an efficiency evaluation issue with fuzzy inputs and outputs but there are some performance evaluation problems with normal data and fuzzy input and output weights. These issues refer to situations where the input and output factors are crisp data but with fuzzy essence. The main idea is derived from the question, “How do the input and output factor weights have effect on efficiency score?” Available FDEA models are not applicable for this problem and none of them can answer the question asked.

Since benchmarking inefficient DMUs is one of the most significant aims in DEA and the weighted additive (ADD) model is the most strong tool in this case for ordinary data (Khezrimotlagh et al., 2012), it seems necessary to develop the mentioned model to provide issues including uncontrollable factors with fuzzy aspects in order to benchmark inefficient DMUs in a proper way.

None of the preceding models provided by different FDEA researchers are utilizable for the applications and problems (efficiency evaluation problems with fuzzy weights and benchmarking problems with some ND factors in fuzzy area) we consider in the study. In addition, the crucial issue in these problems is that some of the FDEA models used additional restrictions for each fuzzy data to transform the fuzzy models into linear models, which can make infeasibility, so developing new methods with minimum possible restrictions added to the models are required.

## 1.3 Aim and Objectives

The aim of this research is to develop FDEA models related to problems with specific conditions that can be considered as fuzzy problems (fuzzy weights, fuzzy input and output problems with some ND factors), where the existing FDEA models are not capable. The objectives of this research can be represented briefly in the following statements:

- To represent a new and applicable FCCR model with scalar data and fuzzy weights and suggest a method for solving efficiency evaluation problems addressing the shortcomings of the existing FDEA models to clarify the main effect of each input and output weights in efficiency.

- To develop a comprehensive NDFADD model for FDEA problems with some uncontrollable data so that it leads to achieve the best bench marking for inefficient DMUs with respect to existing models.

## 1.4 Scope and limitations

The scope of this study is focused on analysing the efficiency of DMUs and the main effect of factors in fuzzy environment in two situations:

- Efficiency evaluation problems with normal crisp data and fuzzy essence in the weights of data: The model is based on CCR model and input and output weights are considered as triangular fuzzy numbers. The offered method is based on arithmetic of fuzzy numbers, particularly fuzzy division with interval efficiency scores as result. The idea of the method came from some recent works in FDEA of Saati et al. (2002), Liu (2008) and Wang et al. (2009).

- Benchmarking DMUs in regular FDEA problems including some uncontrollable factors: FADD model with triangular fuzzy input and output data is developed to accommodate ND factors with uncertain nature in order to obtain the best benchmarking for inefficient DMUs. The method is based on $\alpha$-cut concept and the most utilizable achievement from previous FDEA models, the Saati et al. (2002) model was very helpful in formulating the uncontrollable FDEA model.

## 1.5 Organisation of Chapters

The thesis is organised as follows: In Chapter 2, an overview of FDEA models is provided, while the $\alpha$-cut approach is being used for solving and analysing them as well as existing strategies for covering ND factors in FDEA models are reviewed. In Chapter 3, the proposed improved fuzzy weights CCR model is introduced and the procedure for applying it in efficiency evaluation problems with normal data and fuzzy input and output weights is explained. The process of presenting and applying the modified NDFADD model to overcome uncontrollable factors with fuzzy essence in DEA problems along with discussions for benchmarking inefficient DMUs are provided in Chapter 4. Additionally, to illustrate the capability of the proposed models, some numerical examples will be applied in the last part of Chapters 3 and 4 in order to show the relationship between the models and properties of the real problems.Finally, in Chapter 5, drawn conclusions and some directions for future research are presented.

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