



**UNIVERSITI PUTRA MALAYSIA**

***DENOISING OF DIGITAL IMAGES USING SECOND GENERATION  
WAVELET TRANSFORMS-HIDDEN MARKOV MODEL***

**ASEM IB MOHAMED KHMAG**

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WAVELET TRANSFORMS-HIDDEN MARKOV MODEL**

**By**

**ASEM IB MOHAMED KHMAG**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,  
in Fulfillment of the Requirements for the Degree of Doctor of Philosophy**

**June 2016**

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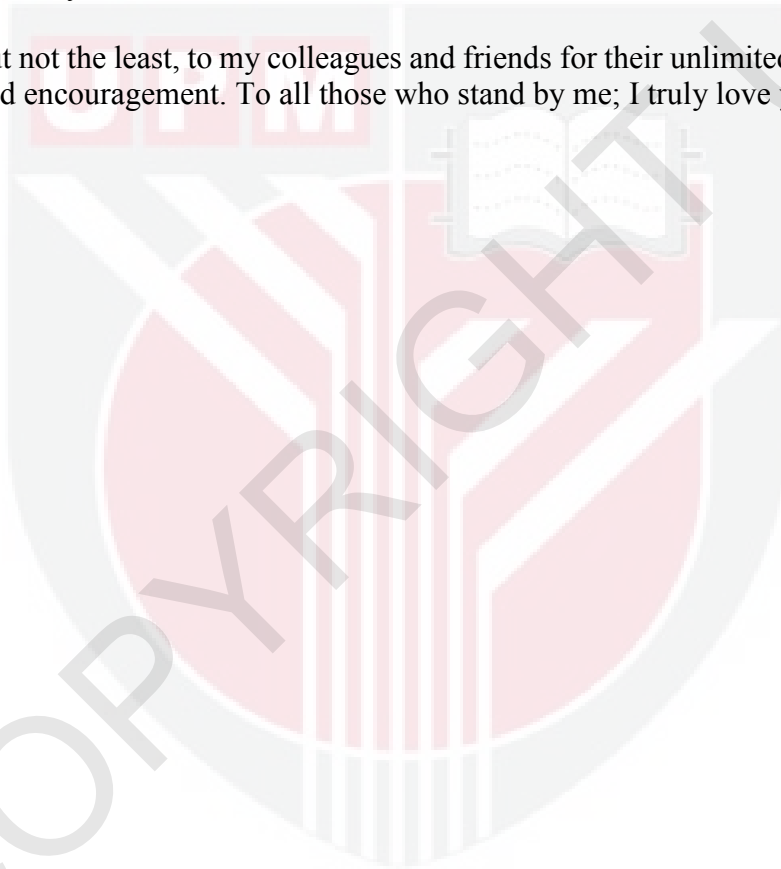
## **DEDICATION**

Firstly, everything would not be possible without the support of the higher education ministry in state of Libya for the financial support of my PhD study in Universiti Putra Malaysia. So, thanks to them and my great country Libya.

Secondly, to my beloved parent, I know I wouldn't finish the PhD without her non-stopping love, encouragement and support.

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Last but not the least, to my colleagues and friends for their unlimited support, endless care and encouragement. To all those who stand by me; I truly love you all.



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

## **DENOISING OF DIGITAL IMAGES USING SECOND GENERATION WAVELET TRANSFORMS-HIDDEN MARKOV MODEL**

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**June 2016**

**Chairman : Abd Rahman Ramli, PhD**  
**Faculty : Engineering**

The digital images are defined as digital signals come across with many kinds of difficult scenarios during transmission and acquisition. One of the main problems in the scientific digital world is the noise. For example, amendments due to additive white Gaussian noise (AWGN) or the multiplicative (speckle) in some cases that can be resulted from poor quality image acquisition. The main aim of an image denoising technique is to minimize the level of noise, and protect the fine details of the image as much as possible. Wavelet denoising has the superior reputation to remove the additive noise while preserving the signal features, nevertheless of its frequency content. In this regard, the second-generation wavelets (SGWs) that employ the properties of sparsity and multiresolution from discrete wavelet transformation (DWT) is used in noise reduction. The aim of this thesis is to restore a high quality image from the noisy counterpart where the natural images mostly suffered from AWGN. In this issue, a newly developed algorithm based on second-generation wavelet transformation using semi-soft thresholding is introduced. In order to increase the robustness of the proposed algorithm, the level of noise in digital image can be estimated if the noise standard deviation  $\sigma$  is unknown. This estimation can be done by exploiting one of the features of SGWs. Moreover, to capture the dependency between a pixel and its neighbours on the wavelet transform, hidden Markov model (HMM) is used. The HMM also allows the hidden states to connect to each other to capture the dependencies among the coefficients in wavelet domain. Due to the lack of translation invariance in the wavelet basis function, some artifacts may appear after applying the denoising algorithm. Cycle spinning idea (for range of shift operations) is implemented in order to enhance the quality of the denoised estimates, and minimize the Gibbs phenomenon disturbing artifacts that are often existing in wavelet-based image reconstruction and denoising.

The main steps in the proposed denoised algorithm are: firstly, apply the second-generation wavelet transformation on the noisy image. Then perform hierarchically, point-wise adaptive thresholding on the wavelet coefficients. Once the wavelet coefficients are modified, the estimation of the wavelet coefficients using HMM is

determined. Finally, the inverse process can be applied to the wavelet coefficients to restore it back to the original form and attain the denoised image. After applying the denoising algorithm on the contaminated image, the cycling spinning algorithm is applied to increase the visual quality of the restored image. Furthermore, to evaluate the suggested algorithm, two kinds of assessment scales are conducted; subjective and objective scales. Firstly, quantitative comparison that represents the objective scale is used in the proposed algorithm to evaluate the denoised images. It contains assessment measurements such as: peak signal to noise ratio (PSNR), mean squared error (MSE), structure similarity index (SSIM), and finally the image quality index (Q-index). Secondly, subjective scale, good measureable results do not assurance high visual quality of the denoised images. So in real applications, the visual quality is still an important metric. According to that, the visual comparison in subjective analysis is used. The denoised images subjected to a poll where people were asked to pick the two least noisy images, and rank them as first and second choice. The images that are chosen to be tested in this study are: Lena, Barbara, Baboon, Boat, F16 and Peppers. The suggested algorithm (SGWs-HMM) outperformed the best state-of-the-art denoising algorithms in terms of quantitative measurements and design simplicity in most of the time. Mathematically, in PSNR, the improvement margin of SGWs-HMM was in range from 0.6dB up to 5.6dB compared with different denoising algorithms under investigation, and in SSIM it showed (~0.5-0.72) higher than HMM and Block matching 3D (BM3D) algorithms in different tested images. As a conclusion, the digital image denoising technique that is proposed in this study can enhance and improve the noisy images both qualitatively and quantitatively. The proposed algorithm is designed in order to tackle many limitations of the existing algorithms such as complexity load, inability to be universal, vulnerability to severe image degradations especially in high noise levels, etc.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**NYAH HINGARAN UNTUK IMEJ DIGITAL MENGGUNAKAN  
PENJELMAAN WAVELET GENERASI KEDUA- MODEL TERSEMBUNYI  
MARKOV**

Oleh

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**Jun 2016**

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Imej digital dianggap sebagai isyarat digital yang melalui pelbagai jenis senario cabaran sepanjang proses penyampaian dan penerimaan. Salah satu masalah utama dalam dunia digital saintifik ini adalah hingar. Sebagai contoh, herotan akibat dari tambahan Gaussian hingar putih (AWGN) dan pada kes tertentu pendaraban (belu) yang disebabkan oleh perolehan imej berkualiti rendah. Matlamat utama algoritma bagi nyah hingaran imej adalah untuk mengurangkan tahap bunyi bising, di samping memelihara imej butiran halus sebanyak mana yang boleh. Nyah hingaran wavelet mempunyai reputasi yang lebih tinggi untuk mengeluarkan hingaran tambahan pada imej di samping memelihara ciri-ciri isyarat, tanpa mengira kandungan frekuensinya. Dalam hal ini, wavelet generasi kedua (SGWs) yang menggunakan sifat-sifat kejarangan dan pelbagai resolusi dari transformasi wavelet diskret (DWT) digunakan dalam pengurangan hingar. Tujuan tesis ini adalah untuk memulihkan imej yang berkualiti tinggi daripada gangguan-gangguan hingar di mana imej semula jadi kebanyakannya mengalami AWGN. Dalam hal ini, algoritma baru dibangunkan berdasarkan wavelet transformasi generasi kedua menggunakan teknik thresholding separa lembut. Dalam usaha untuk meningkatkan keteguhan algoritma yang dicadangkan, tahap bunyi di dalam imej semula jadi boleh dianggarkan jika sisihan  $\sigma$  hingar standard tidak diketahui. Anggaran ini boleh dilakukan dengan mengeksploitasi salah satu daripada ciri-ciri SGWs. Selain itu, untuk mengenalpasti kebergantungan antara piksel dan sekelilingnya iaitu pada domain wavelet, model Markov tersembunyi (HMM) digunakan. HMM juga membolehkan nod-nod yang tersembunyi untuk berhubung antara satu sama lain untuk menangkap kebergantungan antara pekali wavelet. Oleh kerana kekurangan varians terjemahan dalam fungsi asas wavelet, beberapa artifak boleh muncul selepas menggunakan algoritma denoising itu. Idea kitaran berputar (untuk pelbagai operasi peralihan) dilaksanakan untuk meningkatkan kualiti anggaran nyah hingar, dan meminimumkan artifak fenomena Gibbs yang mengganggu dan sentiasa ada dalam berdasarkan wavelet pembinaan semula imej dan nyah hingaran.

Langkah-langkah utama dalam nyah hingaran algoritma yang dicadangkan adalah: pertama, mengaplikasikan kedua transformasi –generasi wavelet pada imej yang hingar. Kemudian melaksanakan secara hirarki, pengembangan penyesuaian titik –bijak pada wavelet pekali. Setelah wavelet pekali diubah suai, anggaran wavelet pekali menggunakan HMM ditentukan. Akhir sekali, proses songsang boleh digunakan untuk wavelet pekali bagi mengembalikannya kembali kepada bentuk sedia ada dan mencapai imej nyah hingaran. Selepas menggunakan algoritma nyah hingaran pada imej yang tercemar, algoritma berputar digunakan untuk meningkatkan kualiti visual imej yang telah dibina semula. Tambahan pula, untuk menilai algoritma yang dicadangkan itu, dua jenis skala penilaian dijalankan; skala yang subjektif dan objektif. Pertama, perbandingan kuantitatif yang mewakili skala objektif digunakan dalam algoritma yang dicadangkan untuk menilai imej yang telah dinyah hingar. Ia mengandungi ukuran penilaian seperti: isyarat puncak kepada nisbah hingar (PSNR), min ralat kuasa dua (MSE), indeks struktur persamaan (SSIM), dan akhirnya indeks kualiti imej (Q-indeks). Kedua, skala subjektif, keputusan kuantitatif yang baik melakukan kualiti visual tanpa gerenti imej baik yang dibina semula. Jadi dalam aplikasi sebenar, kualiti visual masih merupakan matriks utama. Oleh itu, perbandingan visual dalam analisis subjektif digunakan. Imej-imej yang dinyah hingar tertakluk kepada satu tinjauan di mana orang telah diminta untuk mengambil kedua-dua imej yang kurang bising, dan kedudukan mereka sebagai pilihan pertama dan kedua. Kumpulan imej yang telah dipilih bagi diuji dalam kajian studi ini adalah: Lena, Barbara, Baboon, Boat, F16 and Peppers. Cadangan algoritma (SGWs-HMM) mengatasi prestasi terbaik teknik terkini algoritma nyah hingaran dari segi ukuran kuantitatif dan kesederhanaan pengiraan dalam kebanyakan masa. Secara matematik, dalam PSNR, SGWs-HMM peningkatan margin adalah dalam julat daripada 0.6dB sehingga 5.6dB daripada algoritma berlainan dengan nyah hingaran yang disiasat, dan di dalam SSIM ia menunjukkan (~0,5-0,72) lebih tinggi daripada HMM dan Blok hampir sama 3D (BM3D) algoritma dalam imej berbeza diuji. Kesimpulannya, imej semula jadi teknik nyah hingar yang dicadangkan dalam kajian ini dapat meningkatkan dan memperbaiki imej hingar secara kualitatif dan kuantitatif. Algoritma yang dicadangkan direka bagi menangani banyak batasan algorithm sedia ada seperti beban kerumitan, tidak bergerak secara universal, pendedahan kepada kualiti penurunan imej yang teruk terutamanya di tahap hingar yang tinggi, dan lain-lain.



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Asem Ib Mohamed Khmag



I certify that a Thesis Examination Committee has met on 21 June 2016 to conduct the final examination of Asem Ib Mohamed Khmag on his thesis entitled "Denoising of Digital Images using Second Generation Wavelet Transforms-Hidden Markov Model" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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## LIST OF ABBREVIATIONS

AWGN	Additive White Gaussian Noise
BLS-GSM	Bayes Least Squares-Gaussian Scale Mixture
BM3D	Block Matching-3D
Bior	BiorSplines
CWT	Complex Wavelet Transforms
Coif	Coiflet
Db	Daubechies
DCT	Discrete Cosine Transform
DFT	Discrete Fourier Transform
DT-CWT	Dual Tree Complex Wavelet Transform
DWT	Discrete Wavelet Transform
EEG	Electroencephalogram
EM	Expectation Maximization
FFT	Fast Fourier Transform
FWT	Fast Wavelet Transform
GGD	Generalized Gaussian Distribution
GIS	Geographic Information System
GMM	Gaussian Mixture Model
GP	Genetic Programming
GSM	Gaussian Scale Mixture
GUI	Graphic User Interface
HDP-HMT	Hierarchical Dirichlet Process Hidden Markov Tree
HMM	Hidden Markova Model
HMT	Hidden Markov Tree
ICM	Iterated Conditional Modes
IID	Independent, Identically Distributed
IM	Independent Mixture
IQI	Image Quality Index
ISGWs	Inverse Second-generation Wavelets
LGMM	Local Gaussian Mixture Model
MAP	Maximum A Posteriori



ML	Maximum Likelihood
MMSE	Minimum Mean Square Estimation
MSE	Mean Squared Error
NLM	Non-local Mean
OWT	Orthonormal Wavelet
PCA	Principle Component Analysis
PDF	Probability Density Function
PMC	Parallel Model Combination
PSNR	Peak Signal-to-Noise Ratio
RMSE	Root Mean Squared Error
SGWs	Second-generation Wavelets
SGWs-HMM	Second-generation Wavelets- Hidden Markova Model
SNR	Signal-to-Noise Ratio
SSIM	Structure Similarity Index
SURE	Stein's Unbiased Risk Estimate
SVM	Support Vector Machine
SVR	Support Vector Regression
Sym	Symmlet
TV	Total Variation
UINFGP	Universal Impulse Noise Filtering using Genetic Programing
UDWT	Undecimated Discrete Wavelet Transform
WD-HMM	Wavelet Domain- Hidden Markov Model

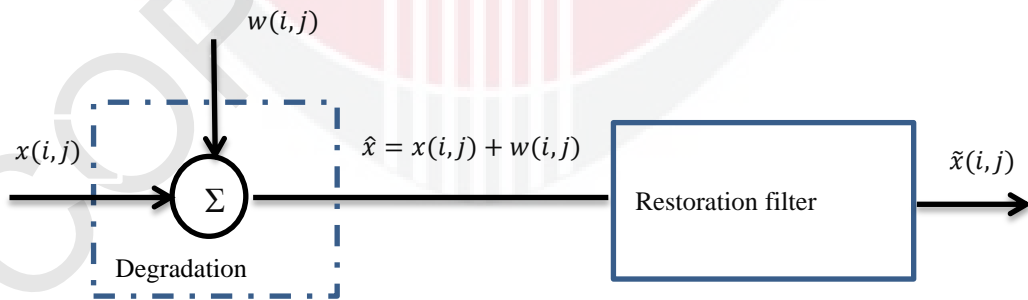
## CHAPTER 1

### INTRODUCTION

#### 1.1 Background

During the past several decades, considerable research has been done on the image restoration. Several techniques were used based on the noise models and kind of the image itself. Most natural images are presumed to be corrupted by random additive noise, which usually is modeled as Gaussian noise. The noise considered to be white if it has the same power at all its flat frequencies i.e. the noise has the same power spectrum in the whole image (Nath, 2013) .

As depicted in Figure 1.1, the procedure of degradation process without blaring is shown as an additive noise,  $w(i, j)$ , which works on an input signal (image),  $x(i, j)$ , to yield a corrupted image  $\hat{x}(i, j)$ . Given this space of noise observation, along with some knowledge of the additive noise, the restoration system produces an estimate,  $\tilde{x}(i, j)$ , of the noise-free image. The main aim is to attain a reconstructed image as close as possible to the original image using a desired denoising estimation. In many fields such as astronomy, medical imaging, and computer vision, the collected data are often noisy as a result of data acquisition processes or due to natural phenomena such as atmospheric disturbances. Even acquiring an image with the usage of a digital camera can cause corruption to the image of the scene with the noise generated by the capturing tools such as the Charge-Coupled Device (CCD) sensors. Moreover, in some cases the blurs in the image are presented by atmospheric turbulence, aberrations in the visual structure and relative motion among ground and camera.



**Figure 1.1: The degradation and restoration model for the process of additive noise**

Thus, noise is added to the data when it is transmitted over transmission channels. The corrupting noise might result in degradation of the visual quality of the images and may also mask and prevent the appearance of the important image details. Even if the perceived images do not show noise degradation due to the masking effects of the human visual system, many image analysis tasks, such as segmentation and

registration, might suffer in the presence of noise (Liu and Allebach, 2014). Thus, it becomes imperative that the level of the noise present in digital images be reduced prior to any further processing.

In the regard of additive noise, there are many approaches to deal with it, especially in natural images. Those approaches have benefited from the improved modeling of digital images. Methods such as spatial domain, transform domain and learning based show a superior performance in this issue (Shao et al., 2014). Although linear filters are useful in a wide variety of applications, still suffering from demerits in some situations which are become inadequate choice as a filter type. For example, linear filters do not take into account any structure in images. Therefore, linear filters tend to blur sharp ridges, destroy curves and other small details of the image, and it has poor performance when the image contains many repetitive patterns (e.g., squares). However, non-linear filters can be successfully applied to achieve detail preserving noise reduction since they adopt the local features of an image. In addition, non-linear filters have the capability to deal with non-uniform smoothing which can easily be adapted locally to the features of the image, such as eliminating impulsive, multiplicative and heavy tailed noise (Shao et al., 2008).

Furthermore, non-linear spatial filters use a low pass filtering on groups of local pixels with the hypothesis that more noise occupies mainly the higher frequency spectrum region. Low pass filters will not only smooth away unwanted noise but also blur sharp edges and ridges in images whereas the high pass filters can make the sharp edges even sharper and increase the local resolution, but at same time will also enlarge the noise source.

Generally, image restoration imposes conciliation between noise elimination and preserving the main features of the original image and its fine details. In order to attain a high performance in this regard, a denoising technique has to adjust with the most distinctive features in an image, (i.e., edges, ridges, etc.). In the past several decades, various algorithms have been developed that improve on spatial filters by taking out the noise more effectively while protecting the delicate details in the image. Spatial domain filters try to utilize the correlations, which exist in most digital images (Li et al., 2010; Bini and Bhat, 2014). In addition, some of these algorithms borrow concepts from partial differential equations and computational fluid dynamics such as level set approaches (Malladi and Sethian, 1996; Sethian, 1999), total variation methods (Chambolle et al., 1998; Chan and Zhou, 2000), non-linear isotropic and anisotropic diffusion (Black et al., 1998; Weickert et al., 1998) and essentially non-oscillatory (Semenov) schemes (Zhou and Zhou, 1999). Other techniques involve impulse removal filters with local adaptive filtering in the transform domain to remove not only white and mixed noise, but also their mixtures (Egiazarian et al., 1999). In the same regard, the transform domain based method considers transforming images into further domains, in which similarities of transformed coefficients are employed (Mallat, 2008). A different class of methods exploits the decomposition of the image data into the wavelet domain (Chang et al., 2000; DeVore and Lucier, 1992; Donoho and Johnstone, 1994; Donoho and Johnstone, 1995; Vidakovic, 2009; Weaver et al., 1991). Wavelet-based denoising techniques have a wide range discussion due to its

popularity (Dabov et al., 2009; Portilla et al., 2003; Luisier et al., 2007; Zhang et al., 2010). Those methods achieve better performance (Luisier et al., 2010) with comparing to spatial domain methods, because they have superior features such as multiresolution and sparsity (Pizurica et al., 2006). The wavelet representation naturally provides a useful tool in the construction of spatially adaptive algorithms that can preserve high frequency components such as edges in an image. It compresses the essential information in a signal into a few, large coefficients which represent image signal details at different resolution scales and facilitates the removal of the corrupting noise. This sparse representation of the data in the wavelet domain also makes them ideal for the purpose of data compression.

However, as the complexity of the wavelet coefficient statistical models increases, the denoising performance is not improved as much as it is expected. Furthermore, the time and computational cost of building and training these statistical models are increased as well. Recently, many researchers introduced artificial intelligence to wavelet based denoising techniques since some soft tools in computing approaches, such as Neural Network and Fuzzy Logic, have the abilities of learning, labeling and describing uncertainties. Although some new approaches have been proposed (Puvanathan and Bizheva, 2007; Bai, 2008), the advantages of artificial intelligence have not been fully utilized.

Self-organizing maps and feed forward neural networks were suggested to detect impulse noise (Turkmen, 2014). Genetic Programming (GP) has recently gained attention in solving many image processing problems. GP approaches have also been used for the removal of impulse noise. A two-stage GP detector for the detection of salt & pepper and uniform impulse noise is reported in Universal Impulse Noise Filtering using GP (UINFGP) (Petrović and Crnojević, 2008).

This research presents a wavelet based approaches that used semi-soft thresholding method. The proposed denoising algorithm exploits attractive features of the second-generation wavelets (SGWs) and the dependency between the coefficients that can be captured by HMM, provides a robustness by using the over-complete representation algorithm to digital images with different image structures and textures, and finally, guarantees a suitable trade-off between detail preservation and noise suppression.

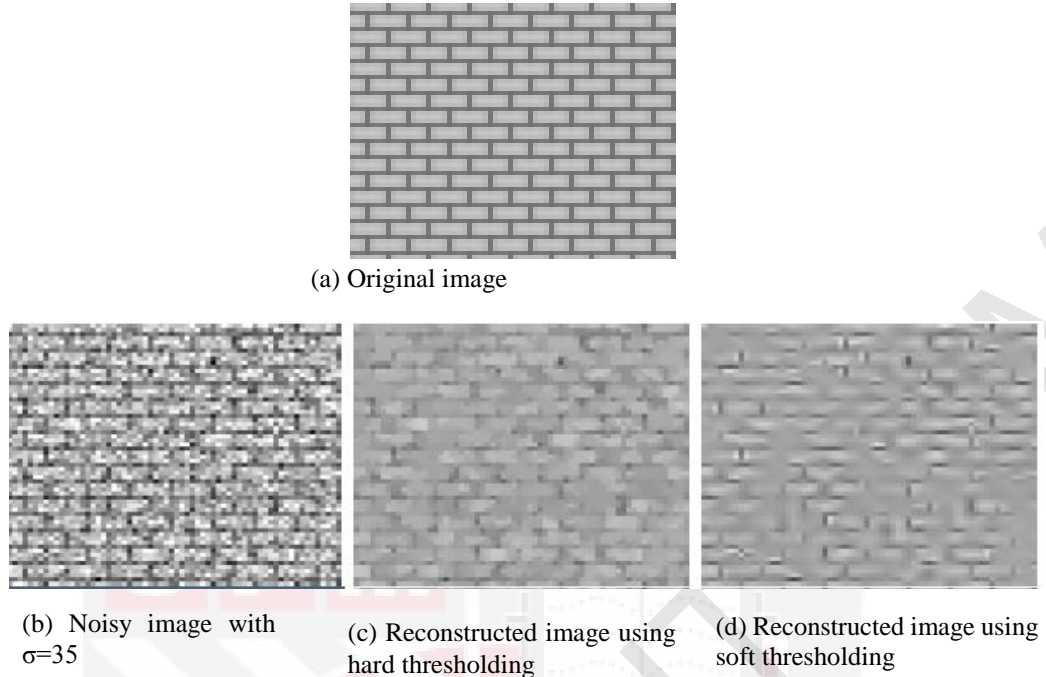
Image denoising as one of image restoration fields truly has no limited applications, forensic and legal investigations, as well as defense and border security, and the area of video surveillance and security for analysis and restoration are the main applications that motivated the researches to go deeply in the field of image denoising. In addition, the demand for image restoration and high quality appearance applications comes from various arenas, namely, video-commerce, multimedia industry, robotics, forensics, airports, smart homes, office environments, and law courts, etc.

## 1.2 Problem Statements

Image denoising is the procedure of reconstructing the original digital image by eliminating the noise from a degraded image. It is implemented in order to suppress the noise and preserve as much image textures and fine details as possible (Hong et al., 2009). Image restoration process can be modeled as obtaining an optimal estimate of the unknown noise-free image from the available noise-contaminated image. Since the image denoising is seminal field of study, a considerable amount of scientific literatures have emphasized on image denoising in the last decade and up until now there is still a wide range of interest in this subject. Despite various algorithms and tools that have been proposed, derived and improved in the field of the image restoration, the problem is that many denoising techniques are always prone to have over-smoothing and extra blurring in the crucial image features as well as introducing artifacts. It is due to the use of only one threshold value for all decomposition levels, it is called the universal threshold (UT) as it is clear in the earliest algorithm of VisuShrink (Moulin and Liu, 1999; Yuan and Buckles, 2004). Moreover, methods such as spatial domain, transform domain and dictionary learning have suffered from different demerits: very vulnerable to severe image degradations (high noise levels), computation complexity burden, and difficulties in characterizing natural images with various patterns (repeated textures and symmetric patterns). Furthermore, Image restoration try to recover the original image from degraded with prior knowledge of degradation process. However, practically, the noisy image in some cases does not provide the specific information about the noise details. Thus, the searching for an efficient image restoration method is still a challenging task. Besides, the amount of noise usually depends on the signal intensity. Practitioners often attribute it to the statistical distribution that it is loyal to, especially when the dependency between the contaminated coefficients is measured. For instance, Additive white Gaussian noise pursues Gaussian distribution, Speckle noise follows Gamma distribution and Brownian noise pursues Browning noise distribution, etc. (Nath, 2013).

Generally, when the magnitude of the measured signal is sufficiently high, the noise is supposed to be independent of the original image that it corrupts, and modeled as an additive Gaussian random variable. Although there are large amounts of research in the area of image denoising, but they did not reach the level of applicability in the reconstructed image, especially when images that are rich in periodic patterns, repeated textures or self-similarity textures (lines, squares, etc.) are considered to be denoised. Figure 1.2 shows an example that addressed this issue.





**Figure 1.2: Example of a periodic image (Buades et al., 2005)**

### 1.3 Objectives

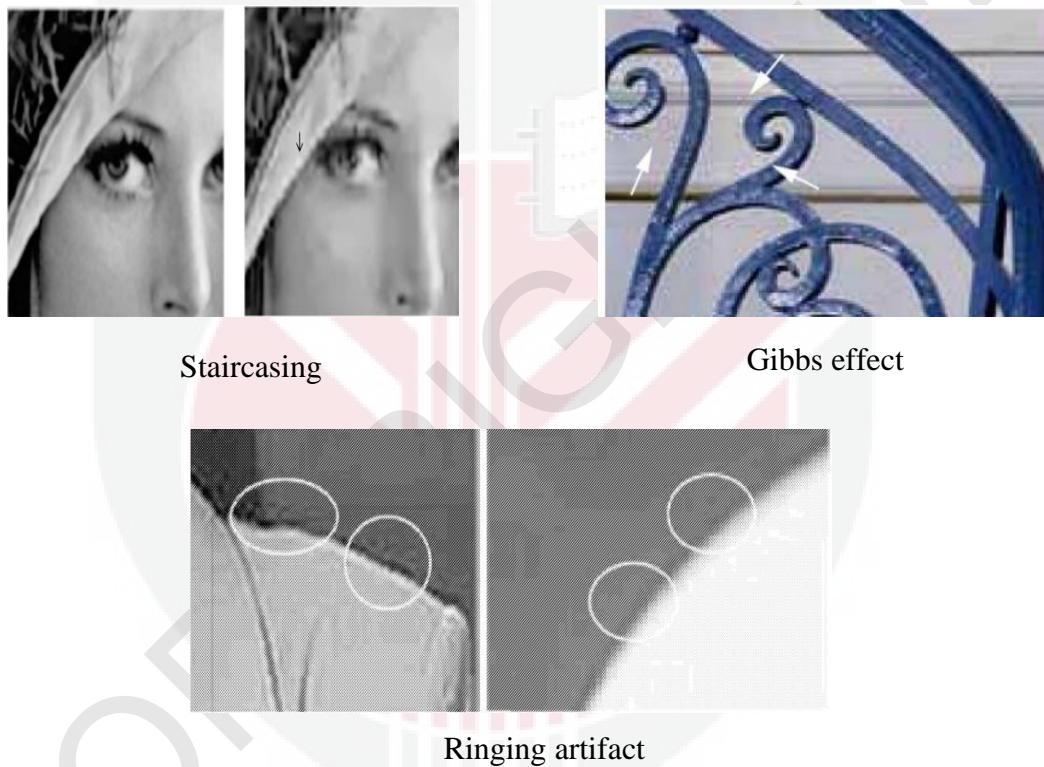
In this thesis, the image denoising problem is addressed with focus on the removal of additive Gaussian noise that is mainly considered as an issue in digital images. The main aim is to design digital image noise (AWGN) removal based on second-generation wavelet and catch the dependency among wavelet coefficients using HMM. Shortly, the objectives of this thesis will be presented; it is consisting of the following:

1. To design a non-linear thresholding filter by using adaptive semi-soft thresholding in second-generation wavelet transform and use the developed statistical hidden Markov model in wavelet domain, in order to smooth the image and reduce the noise prior to thresholding.
2. To reduce the reconstruction effects that might appear on the resulted image such as staircasing, Gibbs effect, wavelet outlines and ringing artifacts by performing an over-complete expansion (cycle spinning) which yields even fewer visual artifacts and better image quality and it gives competitive results in the subjective and objective assessments and it is computationally less expensive
3. To evaluate the denoising algorithm, mathematically by computing the objective assessments such as PSNR, MSE, SSIM and image quality index (IQI), and subjectively using the visual appearance and visual quality questionnaire (voting approach).

Figure 1.3 shows an example that addressed the issue of different wavelet outliers. Therefore, the general goal of this research is to design and implement an efficient digital image denoising method for AWGN type, which can fulfill the following requirements:

**a. Competitive performance**

The proposed algorithm should be competitive with best state-of-the-art denoising methods according to certain objective measurements (objectively), such as peak signal to noise ratio (PSNR). In addition, it should also satisfy human visual assessments, as a subjective estimation.



**Figure 1.3: Main outliers that resulted from wavelet denoising approaches (Chen et al., 2014)**

**b. Least human interaction**

The human interaction should be minimized during the denoising processes when applying the proposed algorithm, by other meaning; the entire denoising steps should be totally automatic.

### **c. Low computational burden**

With regard of computing capacity and time execution, the proposed algorithm should consider those issues. The proposed algorithm should work under moderate environment and specifications of computer systems. Moreover, it should satisfy the hardware requirements and be qualified to accomplish the whole denoising process in a short time period.

### **d. Sufficient reliability**

The proposed algorithm should validate consistent and repeatable experimental results regardless of the sources of images, amount of noise and how many times the denoising algorithm is implemented.

### **e. Wide range of application (universal)**

The main application of the algorithm is to restore digital images contaminated by AWGN. In order to have more robustness, it should also be applicable to the denoising of images corrupted by other types of noise, such as Multiplicative and Poisson noise, which are normally attained in biomedical and astronomy images.

To conclude, most of denoising techniques in wavelet domain are based on either hard or soft thresholding, the visual quality is affected by the manner of choosing the threshold function and its value. The four common wavelet families (Daubechies, Symmlet, Coiflet and BiorSplines) are used in this study; these wavelets are common use in image restoration field. Time complexity of the denoising algorithms and the visual quality in terms of objective and subjective assessment are yet to be obtained. In addition to the mathematical design of the denoising algorithm, this thesis will compare the proposed algorithm to state-of-the-art denoising techniques.

## **1.4 Thesis Scope**

In this study, non-linear thresholding filtering based on second-generation discrete wavelet transform will be designed. The second-generation of wavelets, which is planned based on the lifting scheme approach, is considered as new version of wavelets, and it has various applications such as remote sensing, astronomy, etc. (Ebadi et al., 2013). This procedure exploits the superior properties of the wavelets such as sparsity, multiresolution and fast features of the wavelet transform. In addition, it utilizes the point that the wavelets transform maps white noise in the signal domain into white noise in the transform domain. On the other hand, signal energy becomes more focused into very fewer coefficients in the transform domain; energy of the noise does not. It has pivotal merit in the issue of separating the original signal from noise.



Since the main focus on the digital images, the effects, impacts and the techniques that have done in AWGN are studied extensively in the literature, and primarily consider this kind of noise in this thesis. In transform domain, the process in which small coefficients are detached while others are left undamaged is known as hard thresholding. Unfortunately, this procedure causes spurious blips, it is known as artifacts, in the images as a result of eliminating the smaller coefficients that have the higher frequency parts in the noisy image which are related to image main details and noise coefficients. On the other hand, in the soft thresholding method, small coefficients are removed while others are modified based on some criteria. This approach has also its demerits where the shortcoming of the optimal soft thresholding is that it smooths the high frequency components such as edges and ridges of the image that will affect the visual appearance of the reconstructed image. In order to conquer the drawbacks of hard and soft thresholding, an algorithm using proposed semi-soft thresholding will be used. In addition, to capture the dependency between the child and parent coefficients in the decomposition levels, hidden Markov model is investigated. One of the important advantages of using HMM in the proposed algorithm is its ability to model non-stationary signals or events where the additive noises in natural images are considered as non-stationary signal. Moreover, in order to suppress the visual artifacts that may appear after applying the denoising algorithm, over-complete algorithm is presented to remove the ringing artifact and the oversmoothed patches, and to improve the visual quality of the reconstructed image. The images that are used in the experimental purposes are all standard gray-scale and natural testing images. These gray-scale images contain 8 bit data which means the brightness levels are from 0-255. The images that are chosen to be tested in this study are: Lena, Barbara, Baboon, Boat, F16 and Peppers. These images selected from a popular image database, the USC-SIPI Image Database (University of Southern California)<sup>1</sup>.

## 1.5 Thesis Organization

Remainder of this thesis is prepared as follows:

Chapter 2 of this thesis provides information about the thresholding and shrinkage methods, general overview about wavelet analysis, hidden Markov model (HMM), image and noise models, classification of denoising methods, image quality evaluation. Firstly, thresholding methods and its merits and demerits are discussed. Secondly, a brief exploration on shrinkage techniques and different methods in image restoration, problems and solutions are explained. Furthermore, this chapter discusses the various parts in wavelet transformation, analysis, types and its features among different transform domain approaches are stated. Statistical analysis technique presented by HMM is explained in details in this chapter as well. Lastly, a basic theory in noise models, classification of denoising methods, related and previous works in the field of image denoising techniques are explained.

<sup>1</sup> <http://sipi.usc.edu/database/>

Chapter 3 presents a second-generation wavelets hidden Markov model (SGWs-HMM) approach based on second-generation wavelet transform in order to reduce and overcome the AWGN that appear in natural images with more details in four subsections. The first subsection discusses the denoising based wavelet principle. The second subsection describes the noise variance estimation and how it can be calculated in the noisy images properly. The third subsection presents the second-generation wavelet transform and shows the main framework of the proposed denoising algorithm. Furthermore, several advantages of the proposed SGWs-HMM technique are explained. This subsection also directed the full block diagram of SGWs-HMM with detailed explanation of each part in the proposed design. Finally the usage of the GUI and the full programming package steps are presented in the end of this chapter in order to facilitate the dealing with the software part of the proposed algorithm. The advantages, disadvantages, and evaluation of the developed denoising algorithm also described.

Chapter 4 provides the results and discussion of the proposed algorithm. In this chapter, firstly, the proposed image restoration algorithm performance on different benchmark digital images is discussed. Secondly, the results and discussion on the developed algorithm and the image quality assessment are presented. Thirdly, the results of the six several benchmark images with detailed explanation are stated in six different subsections. Fourthly, the proposed image denoising algorithm is compared in terms of subjective (mathematically) and objective (visually) assessments with best state-of-the-art denoising techniques. Finally, an overall discussion on the results is included in this chapter.

Lastly, Chapter 5 summarizes the contributions of this thesis, provides conclusion for this thesis, and suggests future works. The Appendices provide additional experimental results, definitions and proofs of some theories.

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