UNIVERSITI PUTRA MALAYSIA

INFLUENCE OF CUTOUT SHAPES ON POSTBUCKLING OF FUNCTIONALLY GRADED MATERIAL PLATE UNDER INPLANE-COMPRESSIVE LOAD

HAMED JAMALI

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By

HAMED JAMALI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Master of Science

September 2014

Dedicated to

My dearest parents

For *their endless love, support and encouragement*

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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September 2014

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Faculty: Engineering

Functionally graded materials (FGM) are composite materials with microstructure gradation optimized for the functioning of engineering components. For the case of fibrous composites, the fiber density is varied spatially, leading to variable material properties tailored to specific optimization requirements. There is an increasing demand for the use of such intelligent materials in space and aircraft industries. The current preferred methods to study engineering components made of FGM are mainly modeling particularly those that are finite element (FE) based on rectangular and circular plate/shell with constant thickness and study on the structures with nonlinearity on shape have not yet sufficiently matured. Hence this thesis reports the research study on postbuckling and stability analyzing of FGM plate/shell with four different cut-out shapes including circular, square, horizontal elliptical and vertical-elliptical cutout shape under different boundary conditions. Comparison for FE model performed in two stages, including FGM plate and Postbuckling of FGM plate, to verify the model and Postbuckling of model. The verified model was used in current research study for analyzing the FGM plate with different cutout shapes. Validation of the results of this study has been done by verifying the results with Postbuckling results of aluminum plate. The pustbuckling analysis of FGM Plate subjected to edge loading (edge shortening and edge compression) was performed through finite element software ABAQUS.

The structures considered in this study are functionally graded in a single direction and elastic region was assumed for materials which were used within all analysis stages in this research. This study has shown that tightening the boundary of plate increases the stability of the plate and the plate with elliptical shape cutout has the highest Postbuckling load which means the plate with vertical elliptical cutout shape has the higher stability among the others. The elliptical cutout shape is the best shape for having higher mechanical stability and Postbuckling resilience.

Abstrak tesis yan dikemukakan kepada Senat Universiti Putra Malaysia
Sebagai memenuhi keperluan untuk ijazah Master Sains

PENGARUH BENTUK POTONG PADA PLAT PASCALENGKUNGAN DI BAWAH BEBAN INPLAN-MAMPATAN

Oleh

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Bahan Fungsi Bergred (FGM) adalah merupakan bahan rencam dengan penggredan stuktur mikro yang dioptimumkan untuk berfungsi dengan komponen-komponen kejuruteraan. Bagi kes bahan rencam bergentian, ketumpatan gentian berubah mengikut ruang, membawa kepada perubahan sifat bahan yang boleh dibentuk untuk pengoptimuman penggunaan yang khusus. Terdapat peningkatan permintaan terhadap penggunaan bahan-bahan pintar seperti ini di dalam industri aeroangkasa dan pesawat terbang. Kaedah pilihan terkini untuk mengkaji komponen-komponen kejuruteraan yang dibuat daripada FGM adalah menggunakan pemodelan khusus unsur terhingga (FE) berdasarkan plat/kelompang segiempat dan bulat dengan ketebalan yang malar dan kajian struktur dengan ketaklelurusan untuk bentuk yang masih belum cukup matang. Oleh itu tesis ini melaporkan kajian penyelidikan pada pascalengkungan dan analisis kestabilan plat/kelompang FGM dengan empat bentuk bahagian dipotong yang berbeza termasuk bentuk bulat, empat segi, eliptik mendatar dan eliptik menegak di bawah keadaan sempadan yang berbeza. Perbandingan untuk model FE dilaksanakan dalam dua peringkat, termasuk plat FGM dan pascalengkungan plat FGM, untuk mengesahkan model dan model pascalengkungan. Model yang telah disahkan akan digunakan dalam kajian penyelidikan semasa untuk analisis empat bentuk bahagian dipotong plat FGM. Kesahan hasil keputusan kajian ini telah dilakukan dengan mengesahkan keputusan pascalengkungan plat aluminium. Analisis pascalengkungan plat FGM tertakluk kepada bebanan hujung (pemendekan hujung dan pemampatan hujung) telah dilakukan menggunakan perisian unsur terhingga ABAQUS.

Sruktur yang dipertimbangkan di dalam kajian ini adalah gred fungsi dalam satu arah dan rantau elastik dengan andaian untuk bahan-bahan yang telah digunakan dalam semua peringkat analisis di dalam kajian ini. Kajian ini telah menunjukkan bahawa dengan mengetatkan sempadan plat akan meningkatkan kestabilan plat dengan bentuk potong eliptik mempunyai beban pascalengkungan tertinggi yang bermaksud plat dengan bentuk potong eliptik menegak mempunyai kestabilan yang lebih tinggi

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I certify that a Thesis Examination Committee has met on 11 September 2014 to conduct the final examination of Hamed Jamali on his thesis entitled "Influence of Cutout Shapes on Postbuckling of Functionally Graded Material Plate under Inplane-Compressive Load" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U. (A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

1.1 Background

Composite materials have fully established themselves as workable engineering materials and are now relatively commonplace around the world, particularly for structural purposes. Early military applications of polymer matrix composites during World War II led to large-scale commercial exploitation, especially in the marine industry, during the late 1940s and early 1950s. Today, the aircraft, automobile, leisure, electronic and medical industries are quite dependent on fibre-reinforced plastics, and these composites, namely particulate or mineral filled plastics, are also widely used in industry because of the associated cost reduction [1].

In the continuing quest for improved performance, which may be specified by various criteria including less weight, more strength and lower cost, traditional materials frequently reach the limit of their usefulness. Hence, material scientists, engineers are always striving to produce either improved traditional composite materials or new materials such as functionally graded materials (FGMs). The FGM concept was originated in Japan in 1984 during the space-plane project, in the form of a proposed thermal barrier material capable of withstanding a surface temperature of 2000 K and a temperature gradient of 1000 K across a cross section less than 10 mm. Since 1984, FGM thin films have been comprehensively researched, and are almost a commercial reality. The FGMs were first developed by Japanese scientists in the 1980s. Since that time, the FGMs have been used in several branches and are still being broadened [1]. FGMs are composite materials with microstructure gradation optimized for the functioning of the engineering component. For the case of fibrous composites, the fibre density is varied spatially, leading to variable material properties tailored to specific optimization requirements.

1.1.1 FGM Applications

FGMs offer great promise in applications where the operating conditions are severe. For instance, wear-resistant linings for handling large heavy abrasive ore particles, rocket heat shields, heat exchanger tubes, thermoelectric generators, heat-engine components, plasma facings for fusion reactors, and electrically insulating metal/ceramic joints. They are also ideal for minimizing thermo-mechanical mismatch in metal-ceramic bonding.

Koch and Gunter [2] carried out a research on a new generation of cutting tools based on functionally graded sialons for solving the machining problems of the 21st century. It was proposed that new ceramic tool materials on the basis of Silicon Nitride/Oxide ("Sialons") with a tough core would be developed, to allow a significantly higher performance in machining, in particular for "heavy-to-machine-parts". It was also proposed that the output would enable the European machining industry to increase and speed up the production combined with saving of resources and should benefit for health and environment.

The Swedish Defence Research Agency [3] presented an article in their annual report on armour for future combat vehicle. In order to meet the conflicting demands of lower vehicle weight and much improved protection, future generations of fighting vehicles will need new types of armour. A promising passive armour concept, studied at Swedish Defence Research Agency, is designed to cause interface defeat of the projectile. Interface defeat of tungsten kinetic energy long-rod projectiles has been demonstrated at velocities close to 2000 m/s using today's ceramics and armour technologies. Spark Plasma Sintering is an interesting technology to produce FGMs. FGMs has the potential to be very efficient armour materials. Swedish Defence Research Agency [3] has been conducting initial experiments to produce an FGM with a hard outer surface of TiB_2 and a strong, ductile inner surface of titanium. There are still some difficulties to overcome but the results so far are encouraging. Swedish Defence Research Agency [3] also conducts research into active protection systems against KE-projectiles and electromagnetic armour.

Siegmund [4] describes a program to develop low-cost, functionally graded (FG) carbon-carbon composites for use in a wide range of new applications including automotive structural and heat transfer components, orthopedic implants, friction materials for the specialty automotive, truck and aerospace industries. C-C composites have been the material of choice for high-end high temperature applications for commercial and military aircraft. However, their high cost has limited their application to other significant markets. The research team (Purdue University, University of Notre Dame, Indiana University, Honeywell Aircraft Landing Systems, and National Composite Centre) proposes to change this by introducing a new class of C-C composites with significantly lower cost. The program merges the related expertise of the team members and proposes the development of new technologies to make FG-C-C composites with $10/lb a reality. The program uniquely integrates a robotic manufacturing process, a novel process chemistry approach, materials design and structural analysis with an industrial-scale operation [1].

Bey [5] investigated functionally graded metallic foams as an alternative thermal protection system for space transportation vehicles. An integrated thermal-structural concept in which the load bearing structure has insulating capability and has potential for significant weight savings over current thermal protection systems (TPS). Current TPS do not have a structural function so they are parasitic from a structural viewpoint. Current TPS include coated ceramic tiles or blankets of fibrous insulation affixed to the vehicle surface and metallic panels in which fibrous insulation is encapsulated in foil and placed between an outer metal surface and the vehicle structure. A multifunctional TPS concept is based on metallic panels that are continuously graded in composition and porosity.

U.S. Department of Defense [6] researched on F135 engine and PW J52 engine and they applied functional graded thermal barrier coatings on turbine components, which will increase component life under severe environment and reduce the down-time for the repair of components and enhance readiness of the fleet. These were run in two engine tests for qualification: (i) F402 engine (AV-8B) test as test engine for insertion in F135 engine (JSF) and (ii) PW J52 (EA-6B) engine test.

1.2 Research Motivation

Since prehistory, materials have played a crucial role in the development of our society and culture. In the twentieth century, the exploitation of base elements from the periodic table into various inorganic and organic compounds has made way for the development of advanced polymers, the engineering alloys, and advanced structural ceramics (figure 1.1). Furthermore, FGMs have been developed by combining advanced engineering materials in the form of particulates, fibers, whiskers, or platelets. In the continuous drive to improve structural performance, engineers and scientists currently seek to tailor the material architecture at microscopic scales to optimize certain functional properties of a structure.

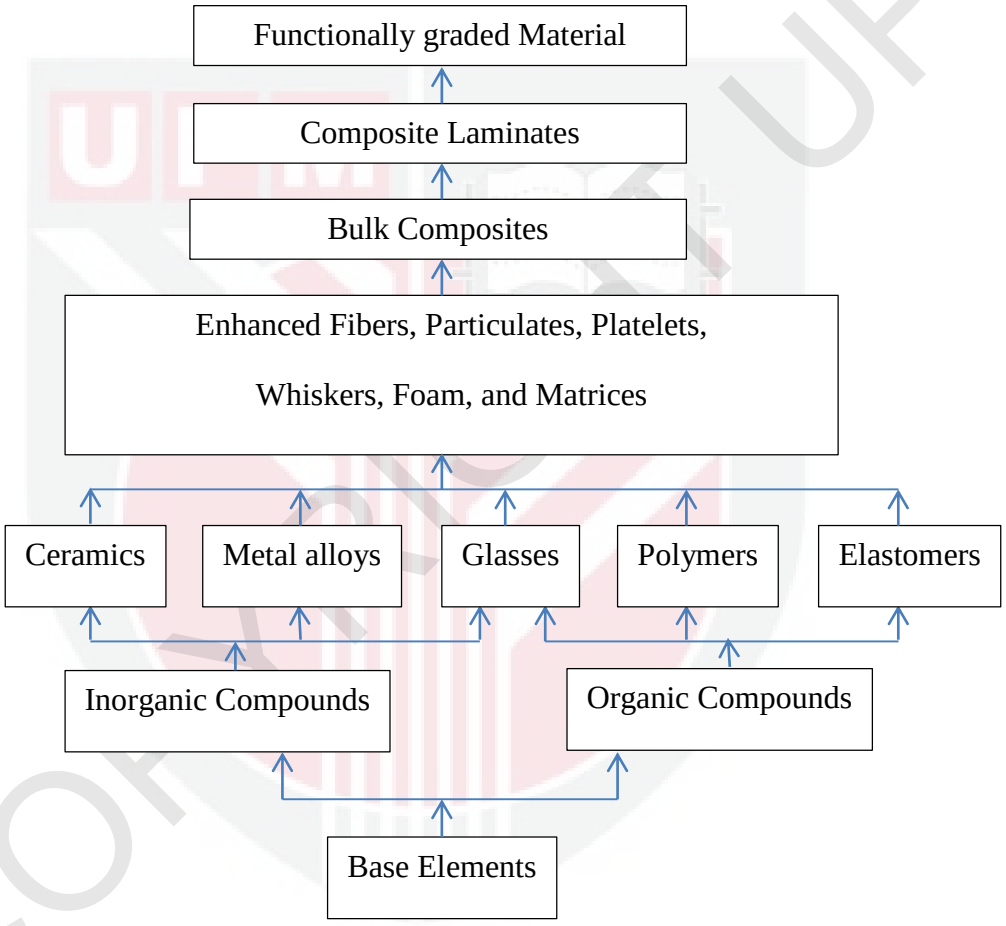

Figure 1.1. Illustration of the modern material hierarchy [7]

1.3 Problem statement

Composite materials are often used in different engineering fields, especially in the aerospace field. The main advantage of composite materials is the high stiffness-to-weight and strength-to-weight ratios. Some of limitations of composite materials are: the weakness of interfaces between layers may lead to delamination, extreme thermal loads may lead to debonding between matrix and fibre due to mismatch of mechanical properties, and residual stresses may be present due to difference in coefficients of thermal expansion of the fibre and the matrix. To overcome these limitations, functionally graded materials (FGMs) have recently been proposed.

The FGMs are made in such a way that the volume fractions of two or more materials are varied continuously along a certain dimension. The FGMs can be made as required for application, for example, thermal barrier plate structures can be made from a mixture of ceramic and metal for high temperature application. The advantage of the FGM plate is that its material properties vary continuously from one surface to the other, thus it avoids the interface problem that exists in homogeneous composites [1].

Regarding to advantages and application of FGMs and also exist of an increasing demand for the use of such intelligent materials in space and aircraft industries. The technical challenges associated with the development of military and civilian aerospace vehicles for the 21st century have identified several key areas that need further development. One important area is the design technology and analysis of large scale FGM structures. The high performance requirements of these structures have led to a search for ways to exploit their tailor ability to meet specific mission goals such as increasing the mechanical stability of structures, and buckling and Postbuckling resistance are the factors which is considered as a mechanical stability. To increase the safety of structures and have a safer operation, it is critically important to design and hire a material/structure which can undergoes more amount of loads (energy) after it buckles and within postbuckling range before it completely fails.

An important structural component used in practically all aerospace vehicles is the rectangular plate with cutout. Cutouts commonly appear in plates as access ports for mechanical and electrical systems in turbo engines or are made to reduce the structural weight in components such as wing ribs and spars. Often during flight, these members experience compression loads, which causes buckling and post buckling phenomenon in those structure under operation time and flight. It is highly important to find a way to increase the stability of structures before they fail thus their buckling and Postbuckling behavior (as a mechanical stability) are important factors that must be considered in their design.

In this research study, the Postbuckling of rectangular FGM plate with 4 different cutout shapes, which are commonly used in designing in space and aircraft industries, have been considered.

1.4 Research Scope

Research works on functionally graded materials, have been covered topics such as mechanical and thermal, and also thermo-mechanical Postbuckling of rectangular and circular FGM plate/shell without any hole. With Regards to the literature review, there is no research work on Postbuckling analysis of FGM plate with cutout. With considering to previous studies which have been carried out by other researchers on buckling and Postbuckling of FGMs, in this thesis influence of cutout shape and size of cutout on Postbuckling of rectangular FGM plate will be investigated to cover the objectives of this thesis. During this study elastic region will be assumed for materials, which are used in this research, in all steps of analysis. This study will focused on influence of cut-out shape and size on Postbuckling loads of FGM plate, which is undergoing in plane mechanical loads in elastic region.

1.5 Objectives

The objective of this research is to study the post buckling of plate made of FGMs, which has central hole and undergoes in plane-load. This research aims at find out best cutout shape which could make the most Postbuckling resistance while the FGM plate undergoes inplane load, under two different boundary conditions. In this study elastic region is assumed for the all cases and in all stages of analysis. The basic aspects of the research are as follows:

I) To study the post buckling of FGM plate with square /circular shape cut-out under mechanical load and different boundary conditions, so as to be able to predict accurately all required material properties from those of the constituents,

II) To analyze the post buckling of FGM plate with horizontal/vertical elliptical cut-out shape under mechanical load and different boundary conditions, so as to be able to predict accurately all required material properties from those of the constituents.

III) To Validate the Postbuckling results with Postbuckling of Aluminum plate.

1.6 Thesis Layout

This thesis consists of five chapters:

Chapter 1 describes the advent of FGMs, and presents the research objectives. It also reviews the application of FGMs which helps to justify the research objectives.

Chapter 2 reviews publications on composites and FGMs. Then it reviews publications on the application of FEM on FGMs. Main consideration of literature review is on postbuckling of FGM plate under mechanical and thermal loads. Latter on some review would be made on type of cutout shapes and dimension of hole which is already done by researcher in Postbuckling and buckling of other types of material but on the same structure.

Chapter 3 is methodology that describes all steps and materials that are used in this research. Explanation for all FE models and boundary conditions, which are used in validation and comparison and also current research, discusses in this chapter. The comparison and validation exercise are carried out in three stages. The first stage involves the comparison of the post buckling of the FGM plate under mechanical load which would verify the accuracy of Postbuckling method and FE tools itself. The second stage involves the validation of FGM plat under thermal load which proofs the material composition of FGM model. The third stage is performed to insure the precision of the Postbuckling of FGM plate.

Chapter 4 is the finite element modeling process in ABAQUS. This chapter shows the steps and sequences for preparing appropriate model on ABAQUS to compute the Postbuckling forces/deflections in elastic region for rectangular FGM plate.

Chapter 5 consists of the results of validations and comparisons and the results and discussion of the current research. The results and discussion that shows the results of post buckling of the plate with cut-out under mechanical load and different boundary conditions. The effects of cut-out shapes and boundary conditions on mechanical stability of such these structures will discuss.

Chapter 6 is the final part, which summarizes the current work and contains conclusion of the current research and gives some recommendation for future works.

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