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MULTI-CRITERIA DIVISIBLE LOAD SCHEDULING IN BINARY TREE NETWORK

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MULTI-CRITERIA DIVISIBLE LOAD SCHEDULING IN BINARY TREE NETWORK

By

SHAMSOLLAH GHANBARI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

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DEDICATION

To:

My father who is a root for me on the earth
My mother (her soul) who released me like a sapling from the dimness of the earth
My wife who turned me into a stalwart tree, and adorned me with the green leaves of life
My children (Amir and Ilia) who magnified the beauty of my life a hundred times like a fruit
All my professors and teachers who taught me the way of inflorescence and the customs of true life

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

MULTI-CRITERIA DIVISIBLE LOAD SCHEDULING IN BINARY TREE NETWORK

By

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January 2016

Chair : Professor Mohamed Othman, PhD
Institute : Mathematical Research

The Divisible Load Theory (DLT) is a paradigm in the area of parallel and distributed computing. Based on the divisible load theory, the computation and communication can be divided into some arbitrary independent parts, in which each part can be processed independently by a processor.

A class of the scheduling methods which is defined based on the DLT is called the Divisible Load Scheduling (DLS).

The traditional divisible load scheduling assumes that the processors report their true computation and communication rates, i.e., they do not cheat the algorithm. In the real applications, the processors may cheat the algorithm, which means, the processors might not report their true computation or communication rates. However, the problem that the processors may not report their true computation rates is called computation rate-cheating problem. The same definition can be considered for the communication rate-cheating problem. However, this problem was investigated by Thomas E. Carroll and D. Grosu in their research publications. The results of their research indicate that the computation rate-cheating reduces the performance of the divisible load scheduling.

This thesis focuses on the computation and communication rate-cheating problems aiming to reduce the effects of computation and communication rate-cheating on the performance of the divisible load scheduling model.

We adopt a multi-criteria approach to the problem. We propose three different multi-criteria based methods in order to improve the performance of the divisible load scheduling. The first method is a multi-objective divisible load scheduling method. The results show that this method is able to considerably improve the performance of the divisible load scheduling when the processors cheat their computation rates. The

experimental results indicate that the proposed method is able to reduce the finish time by approximately 66% in the best case. The limitation of the proposed multi-objective method is that this method slightly increases the start-up time. In order to reduce the limitation of the multi-objective method, a second method has been proposed, which is an Analytical Hierarchy Process (AHP) method. It is briefly called AHP-based method.

The experimental results show that the AHP-based method is able to improve the performance of divisible load scheduling. In addition, it has a lower start-up time com- paring the multi-objective method.

In the third proposed method, it is assumed that both the communication and computation might not be reported at the true rates; hence, we have a new approach to the communication and computation rate-cheating problems. We propose a priority-based divisible load scheduling method for the first time. The results show that this method is able to allocate the optimal load when the processors cheat their computation and communication rates. The proposed priority-based divisible load scheduling method is a novel effort in the area of divisible load scheduling over the past two decades.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

MULTI-KRITERIA BAGI PENJADUALAN BEBAN BERBAGI DALAM RANGKAIAN POKOK DEDUA

Oleh

SHAMSOLLAH GHANBARI

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Teori Beban Berbagi (DBB) adalah paradigma dalam bidang pengkomputeraan selari dan teragih. Berdasarkan teori beban berbagi, pengiraan dan komunikasi boleh dibahagikan kepada sebarang bahagian yang bebas, dimana setiap bahagian boleh diproses secara berasingan oleh pemproses.

Terdapat satu kelas di dalam cara penjadualan yang ditakrifkan dalam konteks DBB yang dinamakan sebagai Penjadualan Beban Berbagi (PBB).

Penjadualan Beban Berbagi (PBB) tradisional menganggap bahawa pemproses-pemproses memberi laporan yang betul bagi kadar pengiraan dan kadar komunikasi, iaitu tidak berlaku penipuan algoritma. Di dalam aplikasi sebenar, pemproses mungkin menipu algoritma, bermakna pemproses-pemproses mungkin tidak melaporkan kadar pengiraan atau kadar komunikasi yang betul. Walau bagaimanapun, masalah dimana pemproses mungkin tidak memberikan laporan kadar pengiraan yang betul dinamakan sebagai masalah penipuan kadar pengiraan. Takrif yang sama diberikan kepada masalah penipuan kadar komunikasi. Namun, masalah ini telah dikaji oleh Thomas E. Caroll dan D. Grosu di dalam penerbitan penyelidikan mereka. Hasil penyelidikan mereka menunjukkan bahawa penipuan kadar pengiraan menyebabkan pengurangan prestasi bagi penjadualan beban berbagi.

Tesis ini memberi tumpuan kepada masalah penipuan kadar pengiraan dan komunikasi dalam mengurangkan kesan oleh penipuan kadar pengiraan dan kadar komunikasi ke atas prestasi model penjadualan beban berbagi.

Kami mempunyai pendekatan multi-kriteria kepada masalah tersebut. Kami mencadangkan tiga multi-kriteria berdasarkan kaedah dalam meningkatkan prestasi penjadualan beban berbagi.

Kaedah yang pertama adalah kaedah objektif berganda penjadualan beban berbagi. Keputusan menunjukkan bahawa kaedah ini mampu meningkatkan prestasi penjadualan beban berbagi apabila berlaku penipuan kadar pengiraan oleh pemproses. Keputusan eksperimen telah menunjukkan bahawa kaedah yang dicadangkan mampu untuk mengurangkan masa selesai kira-kira 66% dalam kes yang terbaik. Kaedah objektif berganda yang dicadangkan adalah terhad kerana kaedah ini meningkatkan sedikit masa permulaan.

Bagi mengurangkan had bagi kaedah objektif berganda, kaedah yang kedua telah dicadangkan, iaitu kaedah Proses Analisis Hirarki (PAH) yang dikenali sebagai kaedah PAH-asas.

Keputusan eksperimen menunjukkan bahawa kaedah ini juga dapat meningkatkan prestasi penjadualan beban berbagi. Di samping itu, kaedah ini telah dapat mengurangkan masa permulaan berbanding dengan kaedah objektif berganda.

Dalam kaedah ketiga yang dicadangkan, ia diandaikan bahawa kedua-dua komunikasi dan pengiraan mungkin tidak dilaporkan pada kadar benar; maka, kami memperkenalkan pendekatan baru kepada masalah penipuan kadar pengiraan dan komunikasi. Kami mencadangkan satu kaedah keutamaan-asas bagi kaedah penjadualan beban berbagi untuk kali pertama. Keputusan menunjukkan bahawa kaedah ini mampu memberi peruntukkan beban yang besar apabila berlaku penipuan kadar pengiraan dan komunikasi oleh pemproses. Kaedah keutamaan asas penjadualan beban berbagi yang dicadangkan adalah satu usaha murni di dalam bidang penjadualan beban berbagi sejak dua dekad yang lalu.

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I certify that a Thesis Examination Committee has met on 12 January 2016 to conduct the final examination of Shamsollah Ghanbari on his thesis entitled "Multi-Criteria Divisible Load Scheduling in Binary Tree Network" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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Declaration by graduate student

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LIST OF ABBREVIATIONS

AHP	Analytical Hierarchy Process
CI	Consistency Index
CR	Consistency Rate
DLS	Divisible Load Scheduling
DLT	Divisible Load Theory
EDF	Empirical Distribution Function
GA	Genetic Algorithm
IDLT	Iterative Divisible Load Theory
MADM	Multi-Attribute Decision Making
MCDM	Multi-Criteria Decision Making
NFT	Normal Form Table
NP	Non Polynomial
PCD	Probing and Continuous Distribution Strategy
PDD	Probing and Delayed Distribution Strategy
PSD	Probing and Selective Distribution Strategy
RI	Random Index
RUMR	Robust-UMR
UMR	Uniform Multi-Round

CHAPTER 1

INTRODUCTION

1.1 Background

The problem of how to effectively distribute the jobs among resources to improve the performance so that some jobs do not suffer unbounded delays is called job scheduling. It is very important to assign appropriate resources to the jobs. Through a good scheduling algorithm, the system can perform better and applications can avoid unnecessary delays.

A class of scheduling techniques that is suitable for using in the area of parallel and distributed computing with big data is called Divisible Load Scheduling (DLS). The first article concerning the Divisible Load Theory (DLT) was proposed by Yuan-Chieh and Robertazzi (1988). In the same time Agrawal and Jagadish (1988) proposed another divisible load scheduling algorithm in a separate paper. The DLT is based on the fact that the load can be divided into various parts, in which each part can be executed by an independent processor. Over the past two decades, the DLT has found a wide variety of applications in the area of parallel processing. Moreover, the DLT has been applied to a wide variety of interconnection topologies. Basically, it was applied to the bus and single level tree network topologies. Bataineh et al. (1994) used the DLT for a multi level tree and daisy chain. After that Blazewicz and Drozdowski (1995) developed the DLT for hypercubes. Later, the DLT was applied to the two-dimensional, three-dimensional and k-dimensional meshes by Blazewicz et al. (1999a), Drozdowski and Glazek (1999) and Li (2003) respectively. It has also been applied in homogeneous and heterogeneous platforms by Blazewicz et al. (1999b) and Beaumont et al. (2003) recpectively. More recently, the DLT was used for scheduling in grid and cloud environments by Robertazzi (2007) and Suresh et al. (2015) respectively.

In general, the divisible load scheduling assumes that the initial amount V of the load is held by the originator denoted by p_0. A common assumption is that the originator does not do any computation. It only distributes the load into parts α_1, α_2, ..., α_m to be processed on the worker processors denoted by p_1, p_2, ..., p_m. According to Sohn and Robertazzi (1993), the condition for the optimal solution is that, all of the processors stop processing at the same time; otherwise, the load could be transferred from busy to idle processors to improve the solution time. Assume that w_0, w_1, ..., w_m are the inverse computing speeds (computation rates) of the processors. It is also assumed that z_0, z_1, ..., z_m are the inverse transmission speeds (communication rates) of the processors. In this situation we assume that the transmission commences simultaneously on all links, and that computation follows the load reception on each processor.

The traditional divisible load scheduling assumes that the processors report their true computation and communication rates, i.e., they do not cheat the algorithm. In the real applications, the processors may cheat the algorithm. This means that the processors may not report their true computation or communication rates.

This issue was investigated by Carroll and Grosu (2008, 2012) in their research publications. The results of their research indicate that the computation rate-cheating, reduces the performance of the divisible load scheduling. In fact, the divisible load scheduling model only obtains its optimal performance if the processors report their true computation rates.

This thesis focuses on the computation and communication rate-cheating problems. We have a multi-criteria perspective to the problem. The goal is to improve the performance of the divisible load scheduling, where the processors cheat the algorithm.

1.2 Problem Statement

Based on the traditional divisible load theory, it is assumed that, the processors do not cheat the algorithm. This means that the traditional divisible load theory assumes that, the processors report their true computation and communication rates to the originator.

The divisible load scheduling has also been examined on the processors that cheat the algorithm. The main idea of the computation and communication rate-cheating problems refer to the misreporting and time varying problems, which were investigated in respect of the divisible load scheduling by Jeeho and Robertazzi (1998). Subsequently, Carroll and Grosu (2008, 2012) focused on the application case of misreporting in the divisible load scheduling. They proposed a strategy-proof mechanism for the divisible load scheduling under the bus and single-level network topologies. A few years later, they investigated the effects of computation rate-cheating on the multi-level tree network topology. Finally, they proposed an incentive-based mechanism for the divisible load scheduling under the multi-level tree network topology.

However, the computation rate-cheating problem may occur, if the processors execute their fraction of loads with different rates. Suppose that, the originator allocates α =(α_0, α_1, ..., α_m) fraction of load to the processors. This allocation is based on the assumption that, the actual computation rate of p_j is equal to w_j. It is also assumed that the actual communication rate of p_j is equal to z_j. In fact, the originator (p_0) learns the actual computation rate of the worker processors once they complete execution of their fraction of the load. According to Carroll and Grosu (2008, 2012), if the processors cheat the algorithm the divisible load scheduling model will fail to achieve its optimal performance. As a result, the total finish time will be increased on this occasion.

This thesis focuses on the computation and communication rate-cheating problems with the aim of reducing the effects of rate-cheating on the performance of the divisible load scheduling.

1.3 Research Objectives

The goal of this research is to present scheduling based on the divisible load theory in order to obtain the best possible response time when the processors cheat the system.

The objectives of this research are to

- propose a multi-objective divisible load scheduling method in a binary tree network.
- propose an analytical hierarchy process divisible load scheduling method in a binary tree network.
- propose a priority-based divisible load scheduling in a tree network.

The mentioned objectives have the following effects on the existing research in the area of the divisible load scheduling.

- This research improves the performance of the divisible load scheduling when the processors cheat the algorithm.
- It secures the divisible load scheduling model against the computation rate-cheating problem.
- It also reduces the limitations of the previous works which was proposed by Carroll and Grosu (2008, 2012).
- Furthermore, this research reduces the effects of communication and computation rate-cheating on the finish time in a divisible load scheduling model.

1.4 Research Scope

This study is concerned with scheduling in the area of parallel and distributed computing. It focuses on the divisible load scheduling in the binary tree network topology. It is assumed that the worker processors may cheat the algorithm. The thesis also concerns the multi-criteria problem solving as a technique for improving the performance of the system under the computation and communication rate-cheating problems. It mainly uses the multi-objective and multi-attribute methods. We also focus on the analytical hierarchy process as a multi-attribute technique for handling the computation and communication rate-cheating problems. The scope of research is shown in Fig. 1.1. As the figure shows, by using probing process, the divisible load model is able to produce some information about the behaviour of the processors. The gathered information helps the algorithm to anticipate the actual computation and communication rates of the processors. For this purpose we use the multi-criteria decision making models.

1.5 Research Significance

Generally, this thesis focuses on the divisible load scheduling in the multi-level tree and single-level tree network topologies. It discusses the divisible load scheduling

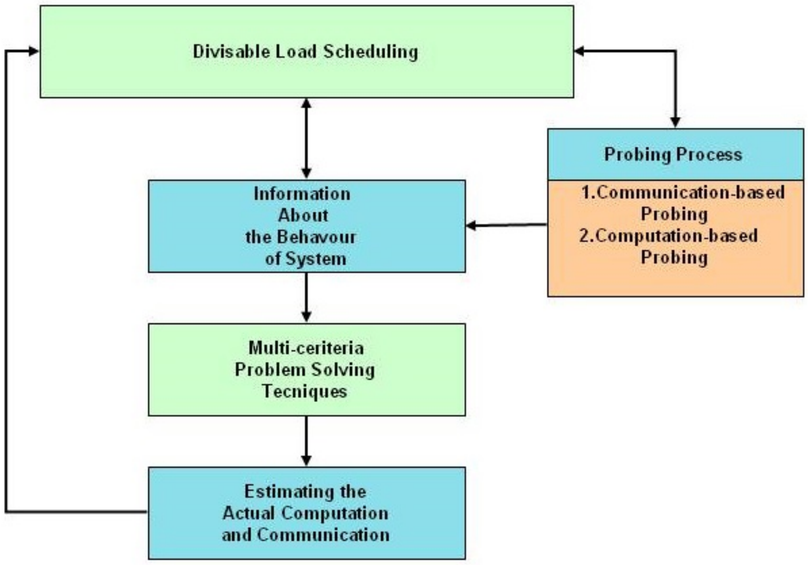

Figure 1.1: The scope of research.

under the communication and computation rate-cheating problem. The computation and communication rate-cheating problems are the important issues in the area of distributed and parallel computing. According to the recent research the rate-cheating decreases the performance of the divisible load scheduling model. This research improves the performance of divisible load scheduling model when the processors cheat their communication and computation rates.

1.6 Research Contributions

The following list demonstrates the contributions of the research:

- This thesis proposed a multi-objective divisible load scheduling method.
  - The proposed multi-objective method enhanced the performance of the divisible load scheduling when the processors cheat the algorithm.
  - It improved the total finish time, payment and utility.
  - It also reduced the finish time by approximately 66% in the best case.
  - Moreover, we proposed a new method of computation-based probing which is useful for predicting the behaviour of divisible load scheduling.
- This research formulated the divisible load scheduling model as a multi-attribute decision-making problem for the first time.
  - We formulated the divisible load scheduling as an analytical hierarchy process problem.

- It improved the total finish time, payment and utility almost the same as the multi-objective method.
- It also considerably reduced the start-up time.

- This research proposed a priority-based method for the divisible load scheduling for the first time.
  - The priority-based approach is a novel effort in the area of divisible load scheduling over the past two decades.
  - We proposed a new method of communication-based probing which is useful for predicting the behaviour of divisible load scheduling.
  - We also found a relationship between the comparison matrices and the processors. Since, the divisible load theory assumes that, the processing must be executed by the independent processors, the relationship between the comparison matrices and the processors helps us to predict the effects of each processor on the other processors.

1.7 Thesis Organization

The other chapters of this thesis are organized as follows. Chapter 2 presents a review of the literature concerning the divisible load theory. Chapter 3 generally describes the methodology used in this thesis. It also briefly explains the proposed methods. Three different methods are proposed in this thesis. The first method is a multi-objective divisible load scheduling which is explained in chapter 4. The second proposed method is an AHP-based divisible load scheduling method which is explained in chapter 5. The third proposed method is a priority-based divisible load scheduling which is explained in chapter 6. Lastly, the conclusion and future works are presented in chapter 7.

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