UNIVERSITI PUTRA MALAYSIA

IDENTIFICATION OF SUITABLE EXPLANATORY VARIABLE IN GOLDFELD-QUANDT TEST AND ROBUST INFERENCE UNDER HETEROSCEDASTICITY AND HIGH LEVERAGE POINTS

ADAMU ADAMU MUHAMMADU

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By

ADAMU ADAMU MUHAMMADU

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DEDICATIONS

To Allah (Who knows all)

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

IDENTIFICATION OF SUITABLE EXPLANATORY VARIABLE IN GOLDFELD-QUANDT TEST AND ROBUST INFERENCE UNDER HETEROSCEDASTICITY AND HIGH LEVERAGE POINTS

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May 2016

Chairman : **Md. Sohel Rana, PhD**
Institute : **Institute For Mathematical Research**

Violation of the assumption of homogeneity of variance of the errors in the linear regression model, causes heteroscedasticity. In the presence of heteroscedastic errors, the ordinary least squares (OLS) estimates are unbiased and consistent, but their covariance matrix estimator is biased and not consistent. As a consequence, this problem negatively affects inference made with biased standard errors. As such, before making any inferences from the OLS, it is very important to check whether or not heteroscedasticity is present. It is now evident that Goldfeld-Quandt (GQ) test is a very powerful test of heteroscedasticity among its competitors. The GQ test requires ordering observations of one explanatory variable in increasing order such that arrangement of observations from the other explanatory variables and the dependent variable in the model follows. When the model involves more than one explanatory variables, identifying suitable variable to be used in the ordering becomes problem when there is no prior knowledge of which variable causes the heteroscedasticity problem. This study has developed an algorithm of identifying this variable prior to conducting the Goldfeld-Quandt test in multiple linear regression model.

To overcome the heteroscedasticity problem, many adjustment methods have been proposed in the literature to correct the biased covariance matrix estimator . These heteroscedasticity correcting estimators are known as heteroscedasticity-consistent covariance matrix estimators (HCCME) which include among others HC0, HC1, HC2, HC3, HC4, HC4m and HC5. However, HC4 and HC5 were designed to take into account, the combined problems of heteroscedasticity and high leverage points in a data. For the same purpose, Furno (1996) used a weighted least squares approach and Lima et al. (2009) extended the idea to HC4 and HC5. This study has modified the weighted HCCME used by Furno and Lima et al. to come out

with two new weighted HCCME that perform well in quasi t inference, in the presence of heteroscedasticity and high leverage points in small to moderate sample size.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

PENGECAMAN PEMBOLEHUBAH PENJELAS YANG SESUAI DALAM UJIAN GOLDFELD-QUANDT DAN INFERENS TEGUH BAGI HETEROSKEDASTISITI DAN TITIK TUASAN TINGGI

Oleh

ADAMU ADAMU MUHAMMADU

Mei 2016

Pengerusi : **Md. Sohel Rana, PhD**
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Penentangan terhadap andaian kehomogenan varians bagi ralat model regresi linear, menyebabkan heteroskedastisiti. Dengan kehadiran ralat berheteroskedastik, anggaran kuasadua terkecil biasa (OLS) adalah tidak pincang dan konsisten, tetapi penganggar kovarians matriknya pincang dan tidak konsisten. Akibatnya, ia menyebabkan inferens yang dibuat memberi kesan negatif dengan ralat piawai yang pincang. Oleh yang demikian, sebelum membuat sebarang inferens dari OLS , adalah sangat penting untuk menyemak kehadiran heteroskedastisiti. Ujian Goldfeld- Quandt (GQ) bagi menguji heteroskedastisiti telah terbukti sangat berkuasa diantara pesaing-pesaing yang lain. Ujian GQ memerlukan cerapan bagi satu pembolehubah penerang disusun secara menaik sedemikian hingga susunan bagi pembolehubah penerang yang lain dan pemboleh bersandar dalam model mengikuti susunan tersebut. Apabila model tersebut melibatkan lebih dari satu pembolehubah penerang, pengecaman pembolehubah yang sesuai untuk digunakan dalam penyusunan menjadi masalah apabila tidak ada maklumat awal tentang pembolehubah yang menyebabkan masalah heteroskedastisiti. Kajian ini telah membangunkan suatu tatacara bagi pengecaman pembolehubah tersebut sebelum menjalankan ujian GQ bagi model linear regresi.

Bagi mengatasi masalah heteroskedastisiti, banyak kaedah pelarasan telah dicadangkan dalam literatur untuk membetulkan penganggar kovarians matriks yang pincang. Penganggar pembetulan heteroskedastisiti ini dikenali sebagai penganggar kovarians matriks heteroskedastisiti konsisten (HCCME) yang melibatkan antara lain HC0, HC1, HC2, HC3, HC4, HC4m dan HC5. Walau bagaimanapun, HC4 dan HC5 telah direkabentuk untuk mengambil kira gabungan masalah heteroskedastisiti dan titik tuasan tinggi dalam data. Bagi tujuan yang sama, Furno (1996) menggunakan pendekatan kuasa dua terkecil berpemberat dan Lima et al. (2009) melanjutkan ide ini untuk HC4 dan HC5. Kajian ini telah mengubah

suai HCCME berpemberat yang digunakan oleh Furno dan Lima et al. bagi menghasilkan dua HCCME berpemberat yang baru yang dapat berfungsi dengan baik dalam inferens kuasi t, dengan kehadiran heteroskedastisiti dan titik tuasan tinggi bagi saiz sampel kecil hingga saiz sederhana.

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I certify that a Thesis Examination Committee has met on 04 May 2016 to conduct the final examination of Adamu Adamu Muhammadu on his thesis entitled "Identification of Suitable Explanatory Variable in Goldfeld-Quandt Test and Robust Inference under Heteroscedasticity and High Leverage Points" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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LIST OF ABBREVIATIONS

GQ	Goldfeld-Quandt
HCCM	Heteroscedasticity- Consistent Covariance Matrix
HCCME	Heteroscedasticity - Consistent Covariance Matrix estimators
$HCCME_w$	Weighted Heteroscedasticity-Consistent Covariance Matrix Estimators
HLP	High Leverage Point
OLS	Ordinary Least Squares
PDA	Percent detection abilities
WLS	Weighted Least Squares

CHAPTER 1

INTRODUCTION

1.1 Background of the study

One of the assumptions of classical least squares regression model is assumption of homoscedasticity which says the variance of the error term in the regression model is constant. In other words, population variance of each disturbance irrespective of the explanatory variables chosen in the regression model is same with one another (is some positive constant number equal to σ^2). If this assumption failed, implies the error terms do not come from populations with constant variance. The model in this case is said to be heteroscedastic regression model. In heteroscedastic regression model, the ordinary least squares (OLS) estimator of the parameters, though unbiased and consistent, but lost efficiency (it does not have the minimum variance in the class of unbiased estimators). To curtail this problem, quite a number of Heteroscedasticity - Consistent Covariance Matrix Estimators (HCCME) were proposed in the literature to be used as consistent estimate of OLS standard error in making inference in linear regression. These consistent standard errors were found useful under both heteroscedasticity and homoscedasticity. Heteroscedasticity is known, that is when for each y_i, there is a known σ^2 corresponding to it, thus each observation y_i in the regression stands as an average value of its corresponding other observations. In this case, method of correcting heteroscedasticity is weighted least squares approach.

Another issue regarding heteroscedasticity, is to know in the first place, whether or not it is present in your model under study. To this end, various methods of detecting the presence or otherwise of heteroscedasticity have been proposed. Most important, when heteroscedasticity is present in the model, one cannot go ahead to make an inference using the OLS standard error. Doing so will undermine the result of such inference. One will then resort to using the heteroscedasticity consistent standard error in making unbiased inference.

Various HCCME known as HC0, HC1, HC2, HC3, HC4, HC5 and HC4m were proposed in the literature. Furno (1996), suggested weighted least squares (WLS) (with a defined weight) to obtain the parameters and weighted HCCME to obtain the consistent standard errors in making inference under heteroscedasticity and leveraged data. Lima et. al. (2009) extended the application of Furno's weighting method on HC4 and HC5. We modified the weight due to Furno and the new approach is useful in making inference involving quasi-t test under heteroscedasticity and leveraged points in small sample to moderate sample size.

In the first phase of the research, this study has found a solution to one of the issues in Goldfeld-Quandt (GQ) test of heteroscedasticity. In this test, when involves multiple regression, there is problem of identifying the correct X variable with which causes the heteroscedasticity problem. Since the Goldfeld-Quandt test requires a single predictor to be identified for ordering, thus it is very important to identify the correct X variable which brings the heteroscedasticity problem. The study has come out with suggestion on how this variable can be identified in large samples and simulations study being made to prove the case.

1.2 Linear Regression Model and Heteroscedasticity - Consistent Covariance Matrix Estimators

The general linear regression model is of the form,

$$y = \beta_0 + \beta_1 x_1 + \ldots + \beta_{p-1} x_{p-1} + \varepsilon_i \tag{1.1}$$

which can also be written in matrix form as

$$Y = X\beta + \varepsilon \tag{1.2}$$

where Y is an $n \times 1$ column vector of the dependent variable; X is an $n \times p$ matrix of the independent variable (the first column takes values 1), p is the number of parameters (including the intercept β_0) and n is the sample size; β is a column vector of the unknown parameters (including the intercept β_0); ε is a column vector of unobservable random errors.

Under homoscedasticity (constant error variances), $E(\varepsilon_i^2) = \sigma^2$ for $i = 1,2,3,\ldots,n$. $\sigma^2 > 0$. $\mathrm{cov}(\beta) = \frac{\sigma^2}{X'X}$ which was estimated by $\frac{\hat{\sigma}^2}{X'X}$ where $\sigma^2 = e_i^{\frac{2}{n-p}}$, $i = 1,2,3,\ldots,n$ are residuals of the regression.

Under heteroscedasticity (non constant error variances), $E(\varepsilon_i^2) = \sigma_i^2$ for $i = 1,2,3,\ldots,n$. $\mathrm{cov}(\beta) = \frac{\sigma^2}{X'X}$ is biased and not consistent. To shed more light on this, consider a two-variable model,

$$y = \beta_1 + \beta_2 x_2 + \varepsilon_i^2 \tag{1.3}$$

$$var(\hat{\beta}_2) = \frac{\sum x_i^2 \sigma_i^2}{\left(\sum x_i^2\right)^2}$$

However, in heteroscedasticity of unknown form, the true values of σ^2 are not directly observable, White suggests using $\hat{u_i^2}$, the squared residual for each i, and estimates the var$(\hat{\beta}_2)$ as

$$\mathrm{var}(\hat{\beta}_2) = \frac{\sum x_i^2 \hat{u}_i^2}{\left(\sum x_i^2\right)^2} \tag{1.4}$$

White has shown that (1.5) is a consistent estimator of (1.4) that is, as the sample size increases indefinitely, (1.5) converges to (1.4) (see Gujarati, 2009).

Heteroscedasticity-Consistent Covariance Matrix (HCCM) is a regression covariance matrix consisting of covariances between the parameters of the regressors as its off diagonal elements and variances (corrected to suit heteroscedasticity) of the parameters as its diagonal elements. HCCM can be defined as

$$\mathrm{HCCM} = (X'X)^{-1} X' \hat{\Psi}_0 X (X'X)^{-1} \tag{1.5}$$

where X is $n \times k$ matrix of regressors (including the intercept which takes set of value 1 in its column). The definition of $\hat{\Psi}_0$ varies just as the different heteroscedasticity - consistent covariance matrix estimators vary. The common practice when heteroscedasticity is present, is to use the OLS parameters together with the consistent standard errors obtained from the HCCM in order to make a reliable inference.

1.3 Problem Statement

This study addresses two problems regarding heteroscedasticity, that is, its detection and its estimation. The first issue considered in this study is in GQ test for detecting heteroscedasticity. The test when involves more than one explanatory variables, requires one to choose and order one of the explanatory variables such that other variables in the model follow. If an investigator has no prior knowledge of this suitable variable, there is no straightforward method of identifying this variable prior to conducting the test. One may conduct the test on each variable in turn. This study addresses this issue by suggesting a method of identifying this variable prior to conducting the test when sample size is large.

The second issue considered estimation of heteroscedastic model. There are many consistent standard error estimation methods in the presence of heteroscedasticity in linear regression, nevertheless, some of these estimators are affected by the presence of high leverage points. HC4 and HC5, although were designed such that they take care of the high leverage points, yet their performance is negatively affected when there are many high leverage points if OLS is used. Lima et al. (2009) extended the application of Furno's WLS approach (Furno, 1996) to HC4 and HC5 and found them performing. We have discovered in this study, that there is still a need of some adjustment in the weighting procedure in order to improve the performance of HC4 and HC5 when the number of high leverage points is large and the level of heteroscedasticity is increases, in small to moderate sample size.

1.4 Objectives of the Study

This study has two main objectives:

1. To identify explanatory variable suitable for ordering observations, in Goldfeld-Quandt test of heteroscedasticity in multiple regression involving large samples.
2. To develop a robust heteroscedasticity - consistent covariance matrix estimator, when there are combined problems of heteroscedasticity and high leverage points, in small to moderate sample sizes.

1.5 Structure of the Thesis

Chapter Two reviews some literature on high leverage point (HLP) and its identification, some heteroscedasticity-consistent covariance matrix Estimators and weighted heteroscedasticity-consistent covariance matrix Estimators ($HCCME_w$) under leverage points. It briefly explains weighted least squares (WLS) method under heteroscedasticity, heteroscedasticity test and its graphical method of detection. It mentions some formal tests of heteroscedasticity and briefly talks on two of the methods.

Chapter Three explains the GQ test of heteroscedasticity and the proposed procedure of identifying the suitable explanatory variable with which to arrange the observations, when the test involve multiple regression. It discusses the simulation technique used in order to get the numerical confirmation of the said proposed method, presents the simulation results and summarily discusses the results. It also assesses the new technique based on real data example and finally displays some pictorial results.

Chapter Four discusses how the inference in this study based on HCCME and $HCCME_w$ is being done. It explains the proposed weighting procedure, the simulation techniques used in generating the required data and other things relating to the numerical assessment of the proposed procedure in comparison with other methods. It presents the numerical results of the simulation and summarily explains the results. Chapter Five summarises the outcomes of the study, gives some concluding notes and include the future research.

BIBLIOGRAPHY

Ahmed, M., Aslam, M. and Pasha, G.R. (2011). Inference under heteroscedasticity of unknown form using an adaptive estimator. *Communications in Statistics Theory and Methods* 40: 4431-4457.

Aslam, M. (2014) Using heteroscedasticity consistent standard errors in the linear regression model with correlated regressors. *Communications in Statistics-Simulation and Computation* 43: 2353-2373.

Bagheri, A., Midi, H. and Imon, A.H.M.R. (2009). Two-step robust diagnostic method for identification of multiple high leverage points. *Journal of Mathematics and Statistics* 5: 97-106.

Breusch, T.S. and Pagan, A.R. (1979). A simple test for heteroscedasticity and random coefficient variation. *Econometrica* 47: 1287-1294.

Cai, L. and Hayes, A.F. (2008). A new test of linear hypothesis in OLS regression under heteroscedasticity of unknown form. *Journal of Educational and Behavioral Statistics* 33: 21-40.

Chatterjee, S. and Hadi, A.S. (2006). *Regression Analysis by Example,* 4th Edition. New York: Wiley.

Cheng, T.C. (2012). On simultaneously identifying outliers and heteroscedasticity without specific form. *Computational Statistics and Data Analysis* 56: 2258-2272.

Chesher, A. and Jewitt, I. (1987). The bias of a heteroskedasticity consistent covariance matrix estimator. *Econometrica* 55: 1217-1222.

Cribari-Neto, F. (2004). Asymptotic inference under heteroskedasticity of unknown form. *Computational Statistics & Data Analysis* 45: 215-233.

Cribari-Neto, F. and da Silva, W.B. (2011). A new heteroscedasticity-consistent covariance matrix estimator for the linear regression model. *AstA Advances in Statistical Analysis* 95: 129-146.

Cribari-Neto, F. and Galvao, N.M.S. (2003). A class of improved heteroskedasticity-consistent covariance matrix estimators. *Communications in Statistics - Theory and Methods* 32: 1951-1980.

Cribari-Neto, F. and Lima, M.D.G.A. (2010). Approximate inference in heteroskedastic regressions: A numerical evaluation. *Journal of Applied Statistics* 37: 591-615.

Cribari-Neto, F. and Lima, M.G.A. (2009). Heteroskedasticity-consistent interval estimators. *Journal of Statistical Computation and Simulation* 79: 787-803.

Cribari-Neto, F. and Zarkos, S.G. (2001). Heteroskedasticity-consistent covariance matrix estimation:white's estimator and the bootstrap. *Journal of Statistical*

Computation and Simulation 68: 391-411.

Cribari-Neto, F. and Zarkos, S.G. (2004). Leverage-adjusted heteroskedastic bootstrap methods. *Journal of Statistical Computation and Simulation* 74: 215-232.

Cribari-Neto, F., Ferrari, S.L.P. and Cordeiro, G.M. (2000). Improved heteroscedasticity-consistent covariance matrix estimators. *Biometrika* 87: 907-918.

Cribari-Neto, F., Ferrari, S.L.P. and Oliveira, W.A.S.C. (2005). Numerical evaluation of tests based on diffferent heteroskedasticity-consistent covariance matrix estimators. *Journal of Statistical Computation and Simulation* 75: 611-628.

Cribari-Neto, F., Souza, T.C. and Vasconcellos, K.L.P. (2007). Inference under heteroskedasticity and leveraged data. *Communications in Statistics-Theory and Methods* 36: 1877-1888.

Davidson, R. and MacKinnon, J.G. (1993). *Estimation and Inference in Econometrics,*. New York: Oxford University Press.

Fitrianto, A. and Midi, H. (2010). Diagnostic-robust generalized potentials for identifying high leverage points in mediation analysis. *World Applied Sciences Journal* 11: 979-987.

Fung, W.K. (1993). Unmasking Outliers and Leverage Points: A Confirmation. *Journal of the American Statistical Association* 88: 515-519.

Furno, M. (1996). Small sample behavior of a robust heteroskedasticity consistent covariance matrix estimator. *Journal of Statistical Computation and Simulation* 54: 115-128.

Furno, M. (1997). A Robust heteroskedasticity consistent covariance matrix estimator *A Journal of Theoretical and Applied Statistics* 30: 201-219.

Glejser, H. (1969). A new test for heteroskedasticity *Journal of the American Statistical Association* 64: 316-323.

Godfrey, L.G. (1978). Testing for multiplicative heteroskedasticity. *Journal of Econometrics* 8: 227-236.

Goldfeld, S.M. and Quandt, R.E. (1965). Some tests for homoscedasticity. *Journal of the American Statistical Association* 60: 539-547.

Goldfeld, S.M. and Quandt, R.E. (1972). *Nonlinear Methods in Econometrics*. Amsterdam and London: North-Holland Publishing Company.

Greene, W.H. (1997). *Econometric Analysis,* 3rd Edition. Prentice-Hall Inc, Upper Saddle River, New Jersey 07458.

Gujarati, D. (1995). *Basic Econometrics,* 3th Edition. McGraw-Hill.

Gujarati, D. and Porter, D.C. (2009). *Basic Econometrics,* 5th Edition. McGraw-Hill.

Hadi, A.S. (1992). A new measure of the overall potential influence in linear regression. *Computational Statistics & Data Analysis* 14: 1-27.

Hayes, A.F. and Cai, L. (2007). Using heteroskedasticity-consistent standard error estimators in OLS regression: An introduction and software implementation. *Behavior Research Methods* 39: 709-722.

Hoaglin, D.C. and Welsch, R.E. (1978). The Hat Matrix in Regression and ANOVA. *The American Statistician* 32: 17-22.

Imon, A.H.M.R. (2009). Deletion residuals in the detection of heterogeneity of variances in linear regression. *Journal of Applied Statistics* 36: 347-358.

Imon, A.H.M.R. and Davies, P. (2007). A new graphical display for locating multiple influential observations, high leverage points and outliers in linear regression. *International Journal of Statistical Sciences* 6: 315-330.

Imon, R.A.H.M. and Masoom, A.M. (2005). Simultaneous Identification of multiple outliers and high leverage points in linear regression. *Journal of the Korean Data and Information Science Society* 16: 429-444.

Kim, M.G. (2004). Sources of high leverage in linear regression model. *Journal of Applied Mathematics and Computing* 16: 509-513.

Koenker, R. and Bassett, J.G. (1982). Robust tests for heteroscedasticity based on regression quantiles. *Econometrica* 50: 43-61.

Lee, A.H. (1987). Diagnostic displays for assessing leverage and influence in generalized linear models. *Australian Journal of Statistics* 29: 233243.

Lima, V.M.C., Souza, T,C., Cribari-Neto, F. and Fernandes, G.B. (2009). Heteroskedasticity- robust inference in linear regressions. *Communications in Statis tics-Simulation and Computation* 39: 194-206.

Long, J.S. and Ervin, L.H. (2000). Using Heteroscedasticity consistent standard errors in the linear regression model. *The American Statistician* 54: 217-224.

MacKinnon, J. G. (2013). Thirty years of heteroskedasticity-robust inference. In *Recent advances and future directions in causality, prediction, and specification analysis* (pp. 437-461). Springer New York.

MacKinnon, J.G. and White, H. (1985). Some heteroskedasticity-consistent covariance matrix estimators with improved finite sample propoerties. *Journal of Econometrics* 29: 305-325.

Maddala G.S. (1992). *Introduction to Econometrics,* 2nd Edition. New York: Macmillan Publishing Company.

Midi. H., Norazan, M.R. and Imon, A.H.M.R. (2009). The performance of diagnostic-robust generalized potentials for the identification of multiple high leverage points in linear regression *Journal of Applied Statistics* 36: 507-520.

Ramsey, J.B. (1969). Tests for specification errors in classical linear least-squares regression analysis. *Journal of the Royal Statistical Society. Series B (Methodological)* 31: 350-371.

Rana, Md.S., Midi, H. and Imon A.H.M.R. (2008). A Robust Modification of the Goldfeld-Quandt Test for the Detection of Heteroscedasticity in the Presence of Outliers. *Journal of Mathematics and Statistics* 4: 277-283.

Rana, Md.S., Midi, H. and Imon A.H.M.R. (2012). Robust wild bootstrap for stabilizing the variance of parameter estimates in heteroscedastic regression models in the presence of outliers. *Mathematical Problems in Engineering* 2012.

Rousseeuw, P.J. (1991). A diagnostic plot for regression outliers and leverage points. *Computational Statistics & Data Analysis* 11: 127-129.

West, K.D. (1997). Another heteroskedasticity- and autocorrelation-consistent c-ovariance matrix estimator. *Journal of Econometrics* 76: 171-191.

White, H. (1980). A Heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica* 48: 817-838.

Wooldridge J.M. (2013). *Introductory Econometrics: A Modern Approach,* 5th Edition. South-Western, Cengage Learning.