UNIVERSITI PUTRA MALAYSIA

MAGNETOHYDRODYNAMICS BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER A PERMEABLE STRETCHING/SHRINKING SHEET

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By

SITI SUZILLIANA PUTRI BT MOHAMED ISA

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Doctor of Philosophy

August 2016

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DEDICATIONS

To my parents, husband, siblings and daughter

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the Degree of Doctor of Philosophy

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August 2016

Chairman : Associate Professor Norihan Md. Arifin, PhD
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A theoretical study that describes boundary layer flow and heat transfer, which is induced by a moving plate in a quiescent ambient fluid has been presented herein. In this study, five problems are discussed in details. First problem related to the fluid flow and heat transfer in the boundary layers on a nonlinearly stretching sheet with a variable sheet temperature and suction, in the presence of magnetic field and non-uniform heat source. The effects of magnetic parameter, suction parameter, the temperature parameter, the space dependent heat source parameter and the temperature dependent heat source parameter have been studied. Magnetohydrodynamics (MHD) boundary layer flow and heat transfer of a viscous fluid over an exponentially permeable stretching sheet is analysed in the second problem, where the system is suppressed by thermal radiation. Velocity, thermal as well as mass slips are considered at the boundary. The boundary layer flow and heat transfer of a viscous fluid on an exponentially shrinking sheet is described in the third problem. The shrinking sheet is permeable and the system is suppressed by an exponential variation of magnetic field. The impacts of the magnetic parameter, the suction parameter and the mixed convection parameter are considered in the third problem. Fourth problem contains steady MHD mixed convection boundary layer flow of a Casson fluid over an exponentially permeable shrinking sheet. The results exhibit that the Casson fluid parameter, mixed convection parameter, magnetic parameter and suction parameter would significantly affect the number of multiple solutions obtained from numerical calculations. The final problem is about the unsteady boundary layer flow of a viscous fluid past a permeable curved stretching/shrinking surface in the presence of a uniform magnetic field. The effects of magnetic parameter, dimensionless curvature, suction parameter, unsteadiness parameter and mixed convection parameter are calculated numerically. For all the tested problems, the governing nonlinear partial differential equations are converted into ordinary differential equations by a similarity transformation. The converted equations are then solved numerically using the shooting method in Maple programming software. The results showed that the values of skin friction coefficient, local Nusselt number, local Sherwood number and the profiles of velocity,

temperature and concentration are changed by the governing parameters on the system. Additionally, the existences of multiple solutions are contributed by the applied numerical method (shooting) and the involvement of certain parameters in the system.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk Ijazah Doktor Falsafah

ALIRAN LAPISAN SEMPADAN MAGNETOHIDRODINAMIK DAN PEMINDAHAN HABA KE ATAS KEPINGAN TELAP MEREGANG/MENGECUT

Oleh

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Kajian teori yang menerangkan aliran lapisan sempadan dan pemindahan haba, yang telah diaruhkan oleh plat bergerak di dalam lingkungan bendalir yang tenang telah dibentangkan di sini. Dalam kajian ini, lima persoalan telah dibincangkan dengan lebih lanjut. Persoalan pertama berkaitan dengan aliran bendalir dan pemindahan haba dalam lapisan sempadan terhadap kepingan meregang secara tidak linear dengan suhu kepingan yang berubah-ubah dan sedutan, dengan kehadiran medan magnet dan ketidakseragaman sumber haba. Kesan parameter magnet, parameter sedutan, parameter suhu, parameter ruang bersandar sumber haba dan parameter suhu bersandar sumber haba telah dikaji. Aliran lapisan sempadan Magnetohidrodinamik (MHD) dan pemindahan haba di dalam bendalir yang likat ke atas kepingan telap meregang secara eksponen telah dianalisis di dalam persoalan kedua, di mana sistem telah dikenakan sinaran terma. Gelinciran halaju, terma serta jisim telah dipertimbangkan di sempadan. Aliran lapisan sempadan dan pemindahan haba di dalam cecair likat ke atas kepingan mengecut secara eksponen telah diterangkan dalam persoalan ketiga. Kepingan mengecut bersifat telap dan sistem dikenakan medan magnet yang berubah secara eksponen. Kesan parameter magnetik, parameter sedutan dan parameter olakan campuran telah dipertimbangkan dalam persoalan ketiga. Persoalan keempat mengandungi kestabilan olakan bercampur dalam aliran lapisan sempadan magnetohidrodinamik (MHD) di dalam bendalir Casson ke atas kepingan telap meregang secara eksponen. Keputusan menunjukkan bahawa parameter bendalir Casson, parameter olakan campuran, parameter magnetik dan parameter sedutan berpengaruh besar terhadap bilangan penyelesaian berganda yng diperoleh daripada pengiraan berangka. Persoalan terakhir berkenaan dengan ketidakstabilan aliran lapisan sempadan dalam bendalir likat merentasi permukaan telap melengkung yang meregang/mengecut dengan kehadiran medan magnet yang seragam. Kesan parameter magnetik, lengkungan berdimensi, parameter sedutan, parameter ketidakstabilan dan parameter regangan/kecutan telah dihitung secara berangka. Untuk semua persoalan yang diuji, persamaan pembezaan separa tak linear dijelmakan kepada persamaan pembezaan biasa menggunakan penjelmaan keserupaan. Persamaan yang telah

dijelmakan seterusnya diselesaikan secara berangka dengan menggunakan kaedah tembakan dalam perisian pengaturcaraan Maple. Keputusan menunjukkan bahawa nilai-nilai pekali geseran kulit, nombor Nusselt setempat, nombor Sherwood setempat dan profil-profil halaju, suhu dan kepekatan berubah disebabkan parameter-parameter yang mengawal sistem. Selain itu, kewujudan penyelesaian berganda telah disumbangkan oleh aplikasi kaedah berangka (tembakan) dan penglibatan parameter-parameter tertentu di dalam sistem.

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I certify that a Thesis Examination Committee has met on 5 August 2016 to conduct the final examination of Siti Suzilliana Putri bt Mohamed Isa on her thesis entitled "Magnetohydrodynamics Boundary Layer Flow and Heat Transfer Over a Permeable Stretching/Shrinking Sheet " in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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LIST OF ABBREVIATIONS

a	Dimensional constants related with velocity of curved stretching/shrinking sheet
A^*	Parameters of the space dependant internal heat generation/absorption
B	Magnetic field
B_0	Constant characteristic magnetic field
B^*	Parameters of the temperature dependant internal heat generation/absorption
c_p	Specific heat at constant pressure
C	Concentration of the fluid
C_f	Skin friction coefficient
C_{fr}	Reduced skin friction coefficient
C_w	Concentration of the fluid near to the plate
C_0	Reference concentration of the fluid due to an exponential stretching/shrinking sheet
C_∞	Concentration of the fluid in the free stream
D	Coefficient of mass diffusivity
e_{ij}	Component of the deformation rate
f	Non-dimensional stream function
F	Mass slip factor
F_1	Initial value of mass slip factor
g	Acceleration due to gravity
G	Dimensional constants related with velocity of nonlinear stretching/shrinking sheet
Gr	Grashof number

h	Index of power-law variation of nonlinear stretching/shrinking temperature
H	Dimensional constants related with temperature of nonlinear stretching/shrinking sheet
J	Thermal slip factor
J_1	Initial value of thermal slip factor
k	Thermal conductivity
k^*	Absorption coefficient
K	Dimensionless curvature
L	Characteristic length of the sheet
L_*	Slip length
m^*	Mass slip parameter
m_w	Mass transfer rate at the surface flux at the wall
M	Hartman number (magnetic field parameter)
n	Index of power-law velocity of nonlinear stretching/shrinking sheet
N	Velocity slip factor
N_1	Initial value of velocity slip factor
Nu_x	Local Nusselt number
p_y	Yield stress of fluid
P_*	Pressure
$\Pr$	Prandtl number
q'''	Non-uniform heat source
q_r	Radiative heat flux
q_w	Heat transfer rate at the surface flux at the wall

r	Radial coordinate in curved stretching/shrinking surface
R	Radiation parameter
R_0	Characteristic radius curvature
R_*	Radius of a circle
$\mathrm{Re}_x, \mathrm{Re}_s$	Local Reynolds number in the *x*- and *s*- directions, respectively
s	Arc length coordinate in curved stretching/shrinking surface
S	Suction parameter
S_a	Singularity point of suction parameter
S_c	Critical point of suction parameter
Sc	Schmidt number
Sh_x	Local Sherwood number
t	Time
t^*	Thermal slip parameter
T	Fluid temperature
T_0	Reference temperature of exponential stretching/shrinking sheet
T_w	Stretching/shrinking sheet temperature
T_∞	Ambient fluid temperature
u	Velocity components in the *x*- direction
u_w	Velocity of the stretching/shrinking surface
U	Velocity components in the *s*- direction
U_0	Reference velocity of exponential stretching/shrinking sheet
v	Velocity components in the *y*- direction
v^*	Velocity slip parameter

v_w	Velocity of the mass transfer/suction
V	Velocity components in the r- direction
x	Cartesian coordinates along the flat surface of the sheet
y	Cartesian coordinates normal to the flat surface of the sheet

Greek Symbols

α	Thermal diffusivity
β	Unsteadiness parameter
β^*	Thermal expansion coefficient
γ	Casson parameter
δ	Accelerating/decelerating flow
ε	Dimensionless constant stretching/shrinking
η	Similarity variable
θ	Dimensionless temperature
λ	Mixed convection parameter
μ	Viscosity of the Newtonian fluid
μ_B	Plastic dynamic viscosity of the non- Newtonian fluid
υ	Kinematic viscosity
π	Product of the component of the deformation rate with itself
π_c	Critical value of the product based on the non-Newtonian model
ρ	Fluid density
σ	Electrical conductivity
σ^*	Stefan-Boltzman constant

τ_{ij}	Rheological equation of state for an isotropic and incompressible flow for Casson fluid
τ_w	Shear rate at the surface
ψ	Stream function
ω	Nonlinear stretching parameter

Subscripts

a	Indication at singularity point
c	Indication at critical point
w	Condition at the surface of stretching/shrinking sheet
0	Reference value
1	Value of slip factor at initial condition
∞	Free stream condition

Superscripts

$'$	Differentiation with respect to η

CHAPTER 1

INTRODUCTION

1.1 History of Fluid Dynamics

Fluid dynamics is a branch of fluid mechanics, whereas aerodynamics and hydrodynamics are sub-branches of fluid dynamics. Fluid dynamics deals with fluids (liquids and gases), and how they are influenced by forces. Aerodynamics, on the other hand, deals with the study of air and other gases that are in motion. Hydrodynamics deals with the study of liquids in motion. The study of fluid dynamics was begun by a scientist known as Euler in 1977. In his book "General Principles of Motion of Fluids," he described most of the equations necessary for ideal in viscid fluids (Johnson, 1998). The history of fluid dynamics is well reported by Rouse and Ince (1957) and Tokaty (1971). Meanwhile, Anderson (1997) also presented the history of both fluid dynamics and aerodynamics.

The law of conservation of energy, mass and momentum govern the field of fluid dynamics. In a closed system, the total amount of mass, energy and linear momentum based on the law remains constant. Additionally mass and energy cannot be created nor destroyed rather can only change forms. Conclusively, this law contributes a lot of basic assumptions involving the study of fluid.

1.2 Heat Transfer and Convection

1.2.1 Heat Transfer

Heat transfer can be described as the process of exchange of thermal energy between physical systems, through the dissipation of heat. The process of transferring heat is induced by pressure and temperature difference that occurs within the involved physical systems. Moreover, the mechanism of heat transfer is started from an area of high temperature to that of low temperature, which can be described by the second law of thermodynamics. In the case an object or fluid has a different temperature than the surroundings, heat transfer will occur. This mechanism will occur until thermal equilibrium is attained in the object and its surroundings. The process of transferring heat between the surrounding and object when their temperature differences in proximity cannot be stopped but rather can be slowed down. Generally, heat transfer can further be subdivided into convection, conduction and radiation. Therefore, this thesis will concentrate on the first type of heat transfer, also known as convective heat transfer.

1.2.2 Convection

Convection describes the heat transfer from one place to another through the mass motion of fluids, for instance, air and water. This type of heat transfer occurs when the heated fluid moves away from the source of heat and carries away with it the energy acquired. Therefore, convection describes the main way by which heat is transferred to gases and fluids. The ideal gas law describes that convection on a hot surface occurs when heated air (temperature increases) is expanded (volume increases), becomes less dense (more buoyant), and then rises.

Circulation in a liquid is an illustration of the principles of convection. One of the best ways to describe the mechanism of circulation in a liquid is when water is heated in a cooking pot. The hot water at the bottom once heated, becomes less dense and then rises making the cold water at the top which is denser to flow to the bottom as illustrated in Figure 1.1. Additionally, Figure 1.2 describes convection in the air induced by a heater. These two figures illustrate a circulation pattern due to the ascending of heated fluid and descending of cooled fluid.

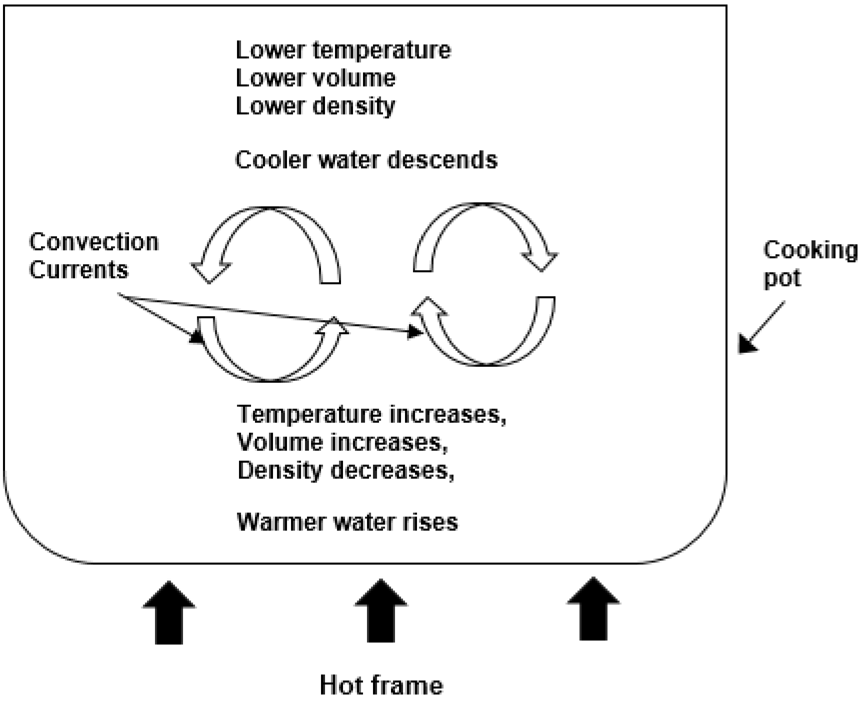

Figure 1.1 : Convection circulation in a water heated in cooking pot in accordance with ideal gas law

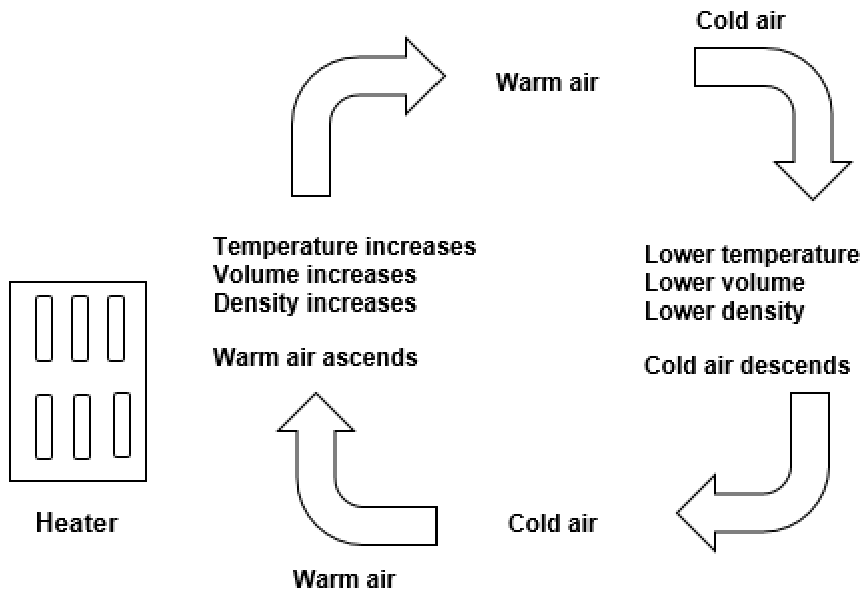

Figure 1.2 : Demonstration of convection due to a heater in accordance with ideal gas law

1.2.3 Types of Convection

Convection can further be subdivided into various forms that include natural convection, forced convection and mixed convention. Natural convection occurs when the fluid has density differences. This density difference results from temperature variations in a fluid. During heating, a density variation at boundary layer causes the less dense heated liquid to rise and then replaced by the denser cooler liquid. Then, this cooler liquid heats up, expands and rises, therefore, resulting in a natural convention phenomenon. Forced convention, on the other hand, describes fluid flow that is induced by external forces that may be caused by a suction device, pump or a fan. Refrigeration, central heating and air conditioning describe the best example of an application in forced convention (Rathore & Kapuno, 2011).

Mixed convention occurs when forced and natural convections occur simultaneously. This type of convention describes the interaction of buoyant forces and pressure during the heat transfer process. Nature of fluid, orientation, geometry, temperature and flow describes the extent to which the form of convention influences the process of heat transfer (Rathore & Kapuno, 2011).

In general, mixed convection can further be subdivided into three cases. The first case arises when the forced convention is supported by natural convention. In this case, buoyant forces move in the same direction to facilitate heat transfer. The second case occurs when the forced convection and natural convection are in opposite direction, therefore, reducing the heat transferred. The final case also known as transverse flow occurs when the vectors of buoyant forces and the forced motion are orthogonal. As a result, the result of fluid mixing in the third case leads to the increment of heat transfer (Rathore & Kapuno, 2011).

1.3 Boundary Layer

In 1905, the concept of fluid viscosity was introduced by Prandtl (Andersson, 1997). Before he published the reports, the viscosity effects were neglected in the ideal flow solutions, therefore, the equations regarding viscosity became complicated. In addition, before the concept of boundary layer was introduced, the Navier-Stokes equations were noted to give correct solutions for flows with small Reynolds number. The Reynolds number in the Navier-Stokes equations is defined as the ratio of inertial forces to viscous forces. In contrast, the Navier-Stokes equations gave insignificant solutions for flows with high Reynolds number. The equations for flows with high Reynolds number are highly non-linear, second order and elliptic in space. As a result, solutions of the boundary layer equations pose great mathematical difficulties.

Therefore, the concept introduced by Prantl (Andersson, 1997) stated that viscosity has large impact at the solid boundary, and this effect is insignificant in areas further away from the solid boundary. Consequently, the flow past a solid boundary can be divided into two regions. The first region is thin and very close to the solid boundary, which is termed as boundary layer. In the boundary layer, fluid has great significant effects on the flow. The second region is one that is away from the solid boundary, but bordering the boundary layer. The fluid viscosity has very low effect in second region. Referring to the concept presented by Prantl, there are various terms that can be neglected in the Navier-Stokes equations through the assumption of a thin boundary layer. The transformation of elliptic equations to parabolic ones made them easier to solve.

1.4 Types of Fluid

Substances that have the ability to flow freely, include both liquids and gases are known as fluids. The Newton's law of viscosity is the basic principles that govern the fluid flow. Therefore, Newton's law of viscosity holds that there is a direct proportion between the shear stress with the fluids viscosity and shear rate. In the Newton's law of viscosity, shear stress is a measure of friction force from a fluid that is exerted on a body in the path of that fluid. Besides, the shear rate in the Newton's law of viscosity is defined as the rate of change of the velocity fluid at which adjacent layers of fluid move with respect to each other (Papaioannou and Stefanadis, 2005).

Therefore, fluids can be categorized as either non-Newtonian or Newtonian based on the above principles. Newtonian fluid refers to the fluid that obeys the Newton's law of viscosity. Besides, any fluid that flows unabated regardless of an applied force or the rate at which is stirred or mixed is referred as a Newtonian fluid. A good example of this fluid is water, honey, thin motor oil, organic solvents and air.
On the other hand, non-Newtonian fluid is a fluid with different characteristics in any way from those of Newtonian fluids. These fluids demonstrate either shear thickening

or shear thinning. Shear thinning happens when the viscosity of the fluid reduces as the shear rate enhances. On the contrary, shear thickening produces when the viscosity of the fluid increases due to the decrement of shear rate. A good example of these fluids is magma, saliva, lava, soap solutions, mucus, ketchup and shampoo. For a proper insight, ketchup and toothpaste flow more easily when the tube is pressed harder.

Casson fluid is a good example of the non-Newtonian fluid. This kind of fluid is assumed to have an infinite viscosity when a shear rate is zero, and a zero viscosity when a shear rate has an infinity value. A model of blood flow through narrow arteries is an example of demonstrating the Casson fluid behaviours.

Rheogram is a plotted relationship shear stress τ versus the shear rate γ, as illustrated in Figure 1.3. The rheogram of a Newtonian fluid is a straight line that passes through an origin and has a slope given by the value of dynamic viscosity of the fluid. The viscosity of a Newtonian fluid is independent of the shear rate. Rheograms of non-Newtonians fluid does not pass through the origin and or does not result in a linear relationship between shear stress τ and rate of shear γ. The viscosity of a non-Newtonian fluid is not constant, but rather dependent on the shear rate.

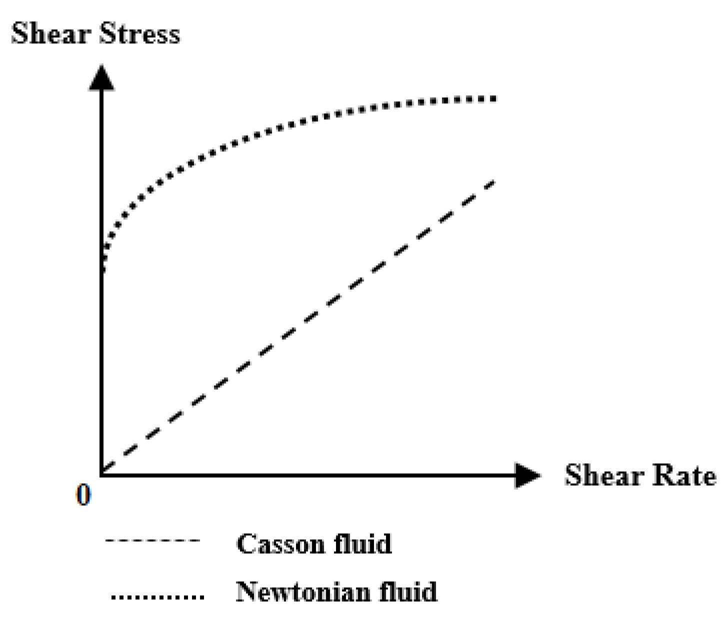

Figure 1.3 : Rheogram of a Newtonian and Non-Newtonian fluid

1.5 Types of Flow

1.5.1 Steady and Unsteady Flows

The condition of fluid properties at any point of a fluid flow can either be categorized to be steady or unsteady. In a steady flow, the fluid properties at each point in the flow do not change with time and they remain constant. This means that the variation of time does not influence the properties of the fluid. Steady flows can be represented mathematically by $\partial P/\partial t = 0,$ where $P = P(x, y, z)$ which refers to fluid properties such as velocity, pressure and density. On the other hand, unsteady flow fluid properties that include velocity, pressure and density are usually time dependent. Therefore, this signifies that fluid properties at any point in the flow vary with time.

1.5.2 Compressible and Incompressible Flows

For starters, it is vital to note that gases are usually compressible. A flow in which fluid density varies especially when subjected to high-pressure differences defines a compressible flow. Large pressure differences in unsteady conditions may make a fluid to completely compressible. It is significant to point out that in a compressible fluid the application of a force on one end of a system does not lead to an immediate flow throughout the system. Instead, the density of the fluid rises near where the force was applied, in reaction to the force. Consequently, the fluid compresses at the point near the applied force. Specifically the neighbouring particles are affected as the compressed fluid expands which therefore makes the particles to compress. Subsequently, the neighbouring particles create a motion wave pulses throughout the system.

On the other hand, the flow of a fluid whose density remains constant to the changing pressure refers to an incompressible fluid flow. However, where conditions are steady and provided that pressure changes are small, it is assumed that the density of the fluid is constant to simplify the analysis of the flow as incompressible. An example of incompressible fluids used commonly is liquids.

1.5.3 Magnetohydrodynamics (MHD)

Magnetohydrodynamics is one of the latest Fluid Mechanics branches. It is mainly concerned with the study of electrically conducting fluids and how they are influenced by magnetic fluids. Based on Faraday’s laws of electromagnetism, when a conductor is passed through a magnetic flux, a current gets induced in the conductor. This current is in a direction that is mutually perpendicular to both the direction of the motion of the conductor and magnetic field. On the other hand, when a conductor carrying an electric current is placed in a magnetic flux, a conductor experiences a significant force that is in a direction that is mutually perpendicular to both the direction of the current and the magnetic field. Based on these it is, therefore, true to state that electromagnetic forces result within an electrically conducting fluid when in the influence of a magnetic field. Hydrodynamic forces within the fluid combine with the electromagnetic forces resulting to what is termed as magnetohydromagnetic (MHD) flow.

The model of MHD flow can be described by considering that the equations of motion account for the effects of electromagnetic forces and other forces such as inertial and hydrodynamic forces. The equations of motion are a combination of the Maxwell's equations of electromagnetism and Navier-Stokes equations of fluid dynamics. Therefore, they need to be solved simultaneously. Electrofluid mechanical energy conversion is linked to the interaction of the magnetic fluids with the electrically conducting fluids. The impacts of this interaction can clearly be observed in plasmas, two-phase mixtures, gases and liquids.

The latter presented applications have diverse technological applications ranging from heating and flow control in metals processing, two-phase mixtures resulting in power generation and the magnetic confinement of high-temperature plasmas. Magnetogasdynamics, magnetofluidmechanics, and the widely used magnetohydrodynamics can be used to describe the extensive effects of electromagnetism in the electrically conducted fluids.

1.5.4 Stretching or Shrinking Sheet Flow

Stretching sheet flow defined as the flow of fluid is induced when the elastic sheet in the incompressible fluid being extended by an application of a stress. This sheet has an elasticity behaviour, means by an ability of a sheet to resist a distorting stress and to return to its original size and shape when the stress is removed. The movement of the stretched or shrieked sheet has velocity varying with the distance from a fixed point. In spite of that, shrinking sheet has an opposite nature with stretching one; the sheet is compressed and influences the fluid flow and the rate of transferring heat.

1.6 Factors Applied in the Mathematical Formulation of the Problem

In this thesis, the effect of suction, heat source (heat generation/absorption), thermal radiation, and partial slip at boundary conditions are included in mathematical formulation of the problems.

1.6.1 Suction Effects

Suction is one of the factor that influences the boundary layer control. Reduction of the pull on bodies in an external flow or reduction of the losses of energy in channels is one of the purpose of adding the effect of suction. In 1904, Prandtl suggested suction as one of the methods in the impediment of boundary layer separation (Andersson, 1997). Suction implementation requires the surface to have holes which can be expounded to refer as perforations, slots and porous sections. The holes are vital for the sucking the portion of the boundary layer that is closest to the wall and which is travelling to the lowest possible velocity.

Injection or suction of a liquid or fluid through the bounding surface, for instance, in the mass transfer cooling has a significant variation in the flow field which consequently influences the heat transfer rate from the plate. Generally suction alleviates the heat transfer coefficients and skin frictions (Al-Sanea, 2004).

Practically, to increase the efficiency of diffusers that have a greater compression ratio of the working fluid (with large convergence angles), suction is applied to delay early boundary layer separation. Additionally, the increase in the lift and decrease drag of aerofoils operating at great incidence angles occur when the boundary layer suction through slots is exerted located close the trailing edge. Practically it has been demonstrated that suction through slots is less effective compared to suction in a porous wall. For instance, for aerofoil, a similar increase of lift force can be attained by sucking a smaller amount of fluid through pores than slots.

1.6.2 Internal Heat Generation/Absorption Effects

The latter two variables (coefficient of space and temperature dependent heat source/sink) are additionally included in the heat generation/absorption formulae. The definition of heat generation is when the coefficient of space and temperature dependent heat source/sink is greater than zero. Move over, the internal heat is absorbed when these two variables are less than zero. The heat source/sink formulation is part of the energy equation which is managed together with both momentum and mass equations. Therefore, a mathematical formulation of the three equations is presented of the heat and flow problems.

The study of heat generation is vital in several physical problems which include fluids that are undergoing endothermic and exothermic chemical reaction. Alteration of the temperature distribution may result due to possible heat generation. Subsequently, the changes of temperature distribution affect the rate of particle deposition in electric chips, nuclear reactors and semiconductor wafers. The process of melting and impediment of freezing increase due to the effect of internal heat generation, which occur in material processing, cryogenic, nuclear and geologic. However, the mechanism of heat absorption is opposite compared to heat generation. Dealing with problems such as fluids undergoing exothermic or endothermic chemical reaction can be tackled by the study of the heat generation and absorption in the moving fluids as it is vital. (Dinesh et al., 2014).

1.6.3 Thermal Radiation Effects

Radiation is a system for transmission of heat that does not need any contact between heated object and the heat source. Infrared radiation defines as the thermal radiation of heat through an empty space. Electromagnetic radiation from an object caused by temperature simply defines thermal radiation. Thermal radiation increases in both power and frequency with the increasing temperature. The conversion of thermal energy to electromagnetic radiation by movement of protons and electrons leads to thermal radiation generation. Thermal radiation examples are a radiant bulb emitting light. Additionally other examples include sun electromagnetic radiation, common electric heater and household radiator.

Moreover, the effect of thermal radiation plays a vital role in the heat transfer control in the polymer manufacturing. The quality of the final product as discussed earlier in the paper greatly affects the heat transfer; therefore any heat controlling features is of great importance in such processes. The thermal radiation additionally have great influence on the flow and transmission of heat processes which is vital to the design of all advanced power convection systems that operate at a greater temperature. The emission from a hot wall surfaces as well as from a fluid flow determine the rate of thermal radiation in such systems.

Particularly the influences of thermal radiation are more pronounced in the cases of the great differences between ambient temperature and surfaces. Therefore in the control of mass and heat transfer, thermal radiation is one of the major factors. Moreover in the stretching problem, thermal radiation has a crucial role in the enhancement of the thermal diffusivity of the cooling liquid in the stretching sheet problem. The production of components that have desired features can be boosted by the knowledge on thermal radiation.

1.6.4 No Slips and Partial Slips Boundary Conditions

When the velocity component of the fluid parallel to the sheet $u(x,y)$ gets equal to the velocity of the sheet $u_w(x)$, the flow fields is assumed to obey the no-slip condition at the boundary. The velocity of the fluid at the wall is assumed to be similar to the velocity of the moving surface and it changes continuously within the fluid based on the non-slip boundary condition. This condition is additionally used in the modelling of the viscous flows and contributes significantly to the diffusion coefficient close to the sheet.

However in certain situation the no slip assumption does not apply and therefore needs to be replaced with a partial slip boundary condition. The latter function is represented by $u(x,y)=u_w(x)+L_*(\partial u/\partial y)$ at $y=0$, which relates the fluid velocity u to the shear stress $\partial u/\partial y$ at the boundary, $u_w(x)>0$ is the velocity of the stretched sheet proportional to the distance of the origin $x=0$, whereas the velocity of the shrieked surface refer to $u_w(x)<0$. Here, velocity slip is defined as $L_*(\partial u/\partial y)$, L_* is the slip length, and y denotes the coordinate perpendicular to the surface. The length L_* is zero if there is no slip. Based on the fact that partial no slip boundary condition is more consistent to the physical features within the practical flow situations approves the selection of partial slip instead of no slip. The partial slip between the moving surface and the fluid happens in circumstances where the fluid is particulate. The examples of fluid particulates are suspensions, foams, emulsions and finally polymer solutions. The technology such as internal cavities and polishing heart valves are applications of the boundary slip in the flow field (Mahantesh et al., 2013). In the situation where the fluid is not sticking to the solid boundary, the velocity slip arises.

1.7 Industrial Applications of the Flow and Heat Transfer Induced by a Stretching/Shrinking Sheet

The theoretical study on boundary layer flow and heat transfer, driven by a stretching/shrinking sheet have numerous investigations, for the reason that this field has industrial applications. Some of the aforementioned industrial applications of stretching sheet flow are extrusion of polymer sheets from a die, drawing of plastic films, wire drawing, polyester thin wall heat shrink tubing, and in glass as well as paper production.

It is worth noting that the quality of the final product in industrial applications depend largely on the heat transfer rate at the stretching/shrinking surface. Therefore, in order to achieve the desired properties of the material being manufactured, proper cooling fluid should be chosen and the flow of the cooling fluid due to the stretching/shrinking sheet must be controlled. As a result, this calls for extra attention to be drawn for both flow and heat transfer characteristics of the cooling fluid medium in the manufacturing processes involving stretching/shrinking sheet (Van De Ven, 2003).

1.8 Aim and Research Objectives

The aim**s** of this thesis are to model, analyse and to obtain the numerical solutions of the following five problems:

i. The MHD flow and heat transfer in the boundary layers on a nonlinearly permeable stretching sheet with a variable wall temperature, in the presence of non-uniform heat source. Under this, the effects of the magnetic parameter *M*, suction parameter *S*, the temperature parameter *h*, the space dependent heat source A^* and the temperature dependent heat source B^* will be studied.
ii. The magnetohydrodynamic (MHD) boundary layer flow and heat transfer of a viscous fluid over an exponentially permeable stretching sheet, where the system is suppressed by thermal radiation. Under this, velocity, thermal, as well as mass slips will be considered at the boundary. The governing parameters involved are velocity slip parameter v^*, thermal slip parameter t^*, mass slip parameter m^*, magnetic parameter *M*, suction parameter *S*, radiation parameter *R* and Schmidt number *Sc*.
iii. MHD flow and heat transfer of a viscous fluid on an exponentially shrinking sheet, where the shrinking sheet is permeable. Under this objective, features of the flow and heat transfer will be obtained for various values of the magnetic parameter *M*, suction parameter *S* and the mixed convection parameter λ.
iv. The steady magneto hydrodynamic mixed convection boundary layer flow of a Casson fluid over an exponentially permeable shrinking sheet. In this

problem, we discuss the additional parameter of Casson γ together with the effect of the governing parameters as in Problem 3.

v. The MHD flow of a viscous fluid past a permeable curved stretching/shrinking surface. In this problem, we analyse the effects of magnetic parameter M, dimensionless curvature K, suction parameter S, unsteadiness parameter β and stretching/shrinking parameter ε.

1.9 Scope of the Thesis

Motivated by similar studies in the aforementioned problems, the present study emphasizes on steady two-dimensional boundary layer flow (x, y) of a viscous incompressible electrically conducting fluid over an isothermal stretching/shrinking surface in the presence of an externally applied magnetic field $(0, B(x))$. The variation of the velocity of stretched/shrieked surface is exponential or nonlinear. The characteristics of the flow and heat transfer are analyzed for viscous and Casson fluid. The scope also covered the problem of the unsteady two-dimensional boundary layer flow of a viscous and incompressible fluid over a curved stretching/shrinking surface in the presence of a uniform magnetic field. Using a similarity transformation, the governing equations of continuity, momentum, energy and specie diffusion have been converted into ordinary differential equations. Then, shooting method is used to solve the ordinary differential equations in numerical method.

1.10 Thesis Outline

The background of the field of dynamics of fluid is presented as an introductory of the first chapter. Subsequently, the concepts of heat transfer and convection are defined, with the related phenomenon. The explanation of various types of convection, boundary layer concept, types of fluid and the patterns of fluid flow are also given. The external factors that exerted in the system of mathematical formulation are introduced in this chapter, together with their definition and applications in engineering field. The related factors are the effects of suction, heat generation/absorption, thermal radiation and when the system is bounded by partial slips. Subsequently, the industrial applications of the fluid flow and heat transfer driven by stretching/shrinking sheet are stated. Finally, the objectives and the scope of the present thesis are given.

Chapter 2 reviews the pioneering studies of the flat or curved sheet which is stretched or shrieked, and this condition of sheet influences the flow and heat transfer characteristics of an incompressible magnetohydrodynamics fluid. Linear, non-linear and exponential forms of velocity are taken into account for the stretching or shrinking sheet. The systems of the pioneering studies in the flat sheet are subjected to the following external factors: mass suction applied at the sheet, partial slips at the edge of the fluid layer, the additional heat generated or absorbed in the system, the flow is opposed or assisted by the forces of buoyancy, and radiation of thermal. In addition, the background studies of the curved surface to search the characteristics of fluid flow and transmission heat restricted to the following aspects: unsteady state, sheet curvature and suction.

In Chapter 3, we define the basic mathematical model and discuss its limitations, which are extended in Chapters 4 to 8. This chapter is divided into two sections. First section described how similarity variables are used to transform the related governed equations, to the set of ordinary differential equations (ODEs). These ODEs are solved numerically by applying shooting method.

Chapter 4 to 8 report the problem solved in the thesis (Problems 1 to 5). All the chapters begin with the introduction of the related problem, method of solution extended from Chapter 3, results and discussion and ended by conclusion. Results are illustrated in the form of tables and graphs, and the comparisons with the previous researchers are also tabulated. The results are restricted to the illustrations of velocity, temperature, skin friction coefficient and local Nusselt number. In addition, the representation of concentration is added in Chapter 5, since concentration equation is considered together with continuity, momentum and energy equations in Problem 2. The effects of related factors in the system of all the problems are listed in conclusion section.

The conclusions of all the tested problems (Problems 1 to 5) are summarized in the last chapter, and the potential further works are also recommended. The future recommendations are the guidelines for future researcher to obtain new results, by using similar method or by developing advanced computational techniques.

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