UNIVERSITI PUTRA MALAYSIA

BLOCK BACKWARD DIFFERENTIATION FORMULAS FOR SOLVING FIRST AND SECOND ORDER FUZZY DIFFERENTIAL EQUATIONS

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By

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BLOCK BACKWARD DIFFERENTIATION FORMULAS FOR SOLVING FIRST AND SECOND ORDER FUZZY DIFFERENTIAL EQUATIONS

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November 2016

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In this thesis, the concerns are mainly in modifying existence method of Block Backward Differentiation Formula (BBDFs) for solving first order fuzzy differential equation, second order non-stiff and stiff fuzzy differential equations (FDEs). This method will solve the Initial Value Problems (IVPs) of FDEs using constant step size. The first part of the thesis discussed the combination of BBDF and Block Simpson into Hybrid method for solving first order FDEs. The subsequent part of the thesis focuses on the modification of BBDF into fuzzy version of BBDF for solving second order non-stiff FDEs and second orders stiff FDEs.

Algorithm was developed to run the FDEs problems in Microsoft Visual C++ environment to obtain exact and approximate solutions. The algorithm of existing BBDF was modified into fuzzy version. The BBDFs method approximates the solution at two points concurrently. Therefore, numerical results show that the proposed methods reduce the execution time when compared to the Backward Differentiation Formula (BDF). In order to compute the error norm, the difference between the approximate solutions and the exact solutions was calculated. The numerical results also show the proposed method produces smaller errors when compared to modified Euler method. The accuracy of the solutions obtained by BBDF and BDF are comparable particularly when the finer step sizes are used. However, in term of execution time, the proposed method BBDF outperformed BDF method. The solutions obtained were illustrated by graphs.

In conclusion, the numerical results clearly demonstrate the efficiency of using BBDF methods proposed in this study for solving fuzzy differential equations. From the results of tests problems, the modified BBDF method reveals that the execution time has been reduced and the numerical result is accurate, which proves its superiority on the existing methods.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk Ijazah Master Sains

FORMULA BLOK PEMBEZAAN KE BELAKANG UNTUK MENYELESAIKAN PERSAMAAN PEMBEZAAN KABUR PERINGKAT KEDUA

Oleh

TIAW KAH FOOK

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Dalam tesis ini, tumpuan utama adalah mengubahsuai kaedah yang sedia ada Formula Blok Pembezaan ke Belakang (FBPB) untuk menyelesaikan persamaan pembezaan kabur (PPK) peringkat pertama, PPK bukan kaku peringkat kedua dan PPK kaku peringkat kedua. Kaedah ini akan digunakan untuk menyelesaikan Masalah Nilai Awal (MNA) PPK dengan saiz langkah yang berterusan. Bahagian pertama tesis ini membincangkan penggabungan FBPB dan Blok Simpson menjadi kaedah hybrid bagi menyelesaikan PPK peringkat pertama. Bahagian berikutnya dalam tesis ini memberi tumpuan kepada pengubahsuaian FBPB ke dalam versi kabur FBPB untuk menyelesaikan PPK bukan kaku peringkat kedua dan PPK stiff peringkat kedua.

Algoritma telah dibangunkan untuk menjalankan masalah PPK dalam persekitaran perisian "Microsoft Visual C ++" untuk mendapatkan penyelesaian tepat dan penyelesaian anggaran. Algoritma FBPB yang sedia ada telah diubahsuai kepada versi kabur. Kaedah versi kabur FBPB menganggar penyelesaian pada dua titik dengan secara serentak. Oleh itu, keputusan berangka menunjukkan bahawa kaedah yang dicadangkan ini dapat mengurangkan masa pelaksanaan apabila ia dibandingkan dengan kaedah Formula Pembezaan ke Belakang (FPB). Dalam usaha untuk mengira ralat norma, perbezaan antara penyelesaian anggaran dan penyelesaian tepat telah dikira. Keputusan berangka ini juga menunjukkan ralat yang lebih kecil jika dibandingkan dengan kaedah Euler yang diubahsuai. Ketepatan penyelesaian FBPB dan FPB adalah setanding terutamanya apabila saiz langkah yang lebih halus digunakan. Walaubagaimanapun, masa pelaksanaan bagi kaedah yang dicadangkan itu dapat mengatasi kaedah FPB. Penyelesaian yang diperolehi telah ditunjukkan melalui graf.

Kesimpulannya, keputusan berangka telah menunjukkan kecekapan penggunaan kaedah FBPB yang dicadangkan dalam kajian ini dalam menyelesaikan persamaan

pembezaan kabur. Daripada keputusan ujian masalah, kaedah FBPB yang telah diubahsuai mendedahkan bahawa masa pelaksanaan telah dikurangkan dan keputusan berangka adalah tepat, maka terbukti bahawa keberkesanan kaedah ini berbanding dengan kaedah yang sedia ada.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirements for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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LIST OF ABBREVIATIONS

FDEs	Fuzzy Differential Equations
ODEs	Ordinary Differential Equations
IVPs	Initial Value Problems
FIVPs	Fuzzy Initial Value Problems
BDF	Backward Differential Formulas
BBDF	Block Backward Differential Formulas
FBBDF	Fuzzy version of Block Backward Differential Formulas
h	Step size
r	Fuzzy numbers with fuzzy bounded r-level interval
$\underline{Y}$	Lower bounded exact solution
$\overline{Y}$	Upper bounded exact solution
$\underline{y}$	Lower bounded approximate solution
$\overline{y}$	Upper bounded approximate solution

CHAPTER 1

INTRODUCTION

1.1 Introduction

In many cases of modeling the real world phenomena, information about the behavior of a dynamical system is uncertain. In order to obtain a more realistic model, these uncertainties have to be taken into account. Fuzzy Differential Equation (FDEs) is a powerful tool for modeling uncertainty and for processing vague or subjective information in mathematical models. Fuzzy model is also adequate for some real-world phenomena.

In recent years, FDEs system can be found in wide varieties of scientific and engineering applications, and they can be used to model problems. For examples,

i. modeling the decay of the biochemical oxygen demand in water by Diniz et al. (2001),

$$[z'(t_i)]^{\propto} \approx \frac{[z(t_i+h)]^{\alpha}-[z(t_i)]^{\alpha}}{h},$$

ii. biology population models by Mengshu et al. (2003),

$$N'(t) = N(t)f_0\big(N(t-\tau)\big) = f(N(t),$$
$$N(t-\tau), t \in R^+,$$
$$N(t) = \varphi(t), \;\; t \in I_0 = [t_0, 0],$$

iii. a fuzzy delay differential equation model for HIV dynamics in medicine by Rosana et al. (2009)

$$\frac{dx(t)}{dt} = \lambda - dx(t) - \beta(t)x(t)v(t)$$
$$\frac{dy(t)}{dt} = \beta(t)x(t)v(t) - \alpha y(t)$$
$$\frac{dv(t)}{dt} = k(t)y(t) - uv(t),$$

iv. genetic programming by Kumaresan et al. (2011).

$$x'(t) = -x(t) + 1, x(0) = x_0.$$

Due to the large potential of fuzzy differential equation involving in these fields, it has become the subject of research projects. In most real life situations, the fuzzy differential equation that models the problem is too complicated to be solved analytically. Therefore, many numerical methods have been developed to obtain numerical approximate solutions. Some of the numerical methods include the use of Taylor series by Abbasbandy et al. (2002), the Runge-Kutta method by Abbasbandy et al. (2004), the predictor-corrector method by Allahviranloo et al. (2007), and the Euler method by Ahmad et al. (2011). However, some of these methods cannot provide a very accurate result and sometimes, the numerical steps are very complicated and difficult to apply. Thereby the ability to obtain accurate numerical approximate solutions plays an important role, especially dealing with stiff FDEs.

Lambert (1992), defined stiffness as follows:

If a numerical method with a finite region of absolute stability, applied to a system with any initial conditions, is forced to use in a certain interval of integration a step size which is excessively small in relation to the smoothness of the exact solution in that interval, then the system is said to be stiff in that interval. There are other characteristics which are exhibited by many examples of stiff problems, but for each there are counter examples, so these characteristics do not make good definitions of stiffness. Nonetheless, definitions based upon these characteristics are in common use by some authors and are good clues as to the presence of stiffness. There is no unique definition of stiffness in the literature. However Lambert refers to these as 'statements' rather than definitions, for the aforementioned reasons. A few of these are:

(a) A linear constant coefficient system is stiff if all of its eigenvalues have negative real part and the stiffness ratio is large.
(b) Stiffness occurs when stability requirements, rather than those of accuracy, constrain the step size.
(c) Stiffness occurs when some components of the solution decay much more rapidly than others.

The most common methods for solving stiff differential equation are based on BDFs. The implementation of BDFs using Newton-like iteration, require repeated solution of linear equation on each step with Jacobian matrix of the differential equation. The matrix operation in the iteration scheme consumed a considerable amount of computational effort. Therefore, appropriate numerical methods are needed to counter these problems. Hence, we are interested in applying the Fuzzy version of Block Backward Differentiation Formulas (FBBDFs) to solve first order fuzzy differential equations, second order differential equations and second order stiff fuzzy differential equations.

1.2 Problem Statement

FDE problems have been studied and solved by different approaches. Most of the existing numerical methods for solving FDEs require a high computation cost. Therefore, it would be more interesting if the numerical solutions can be computed simultaneously. The clear advantage of the method which computes solutions simultaneously is the low computation cost. This again will lead to a quicker execution time. Although it is possible to integrate a second order FDEs $y" = f(x, y, y')$ by reducing it to first order systems, the numerical methods can be derived via integrating $y" = f(x, y, y')$ directly without using the first derivatives. These approaches should provide significant increase in efficiency and decrease computation work.

1.3 Objectives of Study

The objectives of the research are:

(i) to develop hybrid method based on BBDF, Ibrahim (2006) and block Simpson, Adegboye et al. (2014) to solve first order FDEs.

(ii) to modify BBDF, Ibrahim (2008) into fuzzy version in order to solve second order non-stiff and stiff FDEs directly.

(iii) to develop algorithms in C programming environment for solving FDEs.

(iv) to compare the numerical results obtained.

1.4 An Overview of Thesis

A brief description of the organization of the thesis is as follows: Chapter 1 is a brief introduction to Fuzzy Differential Equation (FDEs) and the scope of the study. Chapter 2 begins with literature review, which covers discussion on methods used to solve FDEs proposed by several researchers. For Chapters 3, 4 and 5, each chapter represents a separate study that has its own introduction, including objective, methods, results, discussion and conclusion.

In Chapter 3, we will propose a new fuzzy version hybrid method by combining block backward differentiation formulas (BBDFs) that have been proposed by Ibrahim et al. (2006) and block Simpson formula that is proposed by Adegboye et al. (2014). The proposed hybrid method is used to solve first order fuzzy differential equation. Numerical results obtained by the proposed hybrid method will be compared with the numerical results obtained by Euler method and BBDF. The accuracy and efficiency of the methods will be discussed in details. Graphs will be plotted to compare the approximate solutions with the exact solutions.

For Chapter 4, this study extends the block backward differentiation formulas (BBDFs) that have been proposed by Ibrahim et al. (2006) and the BBDFs is modified into fuzzy version BBDF (FBBDFs) to solve second order fuzzy differential equations. Numerical results obtained by the proposed method will be compared with the numerical results obtained by backward differential formula (BDF) and modified Euler methods in term of accuracy and execution time. Graphs will be presented and compared.
In Chapter 5, the numerical method developed in Chapter 4 will be used to solve second order stiff fuzzy differential equations, the approximate solution obtained is then compared with the exact solution. One of the examples in engineering application is also shown in this chapter.

Finally, Chapter 6 summarizes the conclusions of the research and recommendations for further study will be suggested.

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