UNIVERSITI PUTRA MALAYSIA

BORNOLOGICAL STRUCTURES ON SOME ALGEBRAIC SYSTEMS

ANWAR NOORULDEEN IMRAN

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BORNOLOGICAL STRUCTURES ON SOME ALGEBRAIC SYSTEMS

By

ANWAR NOORULDEEN IMRAN

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

January 2018

DEDICATIONS

To my husband and my children
and
my great supervisor

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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ANWAR NOORULDEEN IMRAN

January 2018

Chairman : I.S. Rakhimov, PhD
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This work concerns the notion of determining the boundedness of some algebraic structures such as groups and rings.

Firstly, a new structure bornological semigroup is considered to determine the boundedness of algebraic structure semigroups. Then, some properties are investigated. Some of these properties are shared with bornological groups, but some properties are not. Further properties of bornological groups are studied to give sufficient condition of bornology to bornologize every group. In particular, we show that a left (right) translation in bornological groups is a bornological isomorphism and therefore the bornological groups structures are homogeneous.

Next, bornological group actions (BGA) are constructed to prove some basic results which hold true just for bornological actions. In particular, we show that a bornological group action can be deduced from its boundedness at the identity and a bornological group acts on a bornological set by a bornological isomorphism. The effect of bornological action is to partition bornological sets into orbital bornological sets. Furthermore, the morphisms between G-bornological sets to be bounded maps are introduced. This motivated us to construct the category of G-bornological sets.

For this purpose, we construct chorology theory for bornological groups based on bounded cochains and study some of its basic properties. We show that the cohomology theory of bounded cochains and the cohomology theory of homogenous cochains are isomorphic.

Furthermore, the equivalent classes of bornological group in terms of a semi-bounded sets and s-bounded maps are presented to restrict the condition of boundedness for bornological group.

Lastly, the concept of bornological semi rings is introduced to determine the boundedness of rings and semi rings, and the fundamental constructions in the class of bornological semi rings are discussed. The general results in this chapter concerning projective limits and inductive limits as well as an isomorphism theorem are established.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

STRUKTUR BORNOLOGI TERHADAP BEBERAPA SISTEM ALJABRAS

Oleh

ANWAR NOORULDEEN IMRAN

Januari 2018

Pengerusi : I.S. Rakhimov, PhD
Fakulti : Sains

Kerja ini adalah berhubung dengan tanggapan untuk menentukan keterbatasan beberapa struktur aljabras (kumpulan, gelang).

Pertama sekali, suatu struktur semikumpulan bornologi yang baru dipertimbangkan untuk menentukan keterbatasan semikumpulan struktur algebra. Kemudian, beberapa sifat disiasat. Sesetengah sifat ini dikongsi dengan kumpulan-kumpulan bornologi, tetapi sesetengah sifat tidak. Sifat-sifat tambahan kumpulan-kumpulan bornologi dikaji untuk memberi keadaan bornologi yang mencukupi untuk membornologikan setiap kumpulan. Khususnya, kami menunjukkan bahawa suatu terjemahan kiri (kanan) dalam kumpulan-kumpulan bornologi adalah isomorfisme dan justeru itu struktur-struktur kumpulan-kumpulan bornologi tersebut adalah homogen.

Seterusnya, tindakan kumpulan bornologi (BGA) dibina untuk membuktikan beberapa hasil asas yang bersifat benar hanya untuk tindakan bornologi. Khususnya, kami menunjukkan bahawa tindakan sesuatu kumpulan bornologi boleh disimpulkan daripada keterbatasannya di identiti dan kumpulan bornologi bertindak ke atas sesuatu set bornologi melalui isomorfisme bornologi. Kesan tindakan bornologi adalah untuk memetakkan set-set bornologi menjadi set-set bornologi orbitan. Tambahan lagi, morfisme-morfisme antara set-set bornologi-G untuk menjadi peta-peta terbatas diperkenalkan. Ini memberi motivasi kepada kami untuk membina kategori set G-set bornologi.

Untuk tujuan ini, kami membina teori korologi untuk kumpulan-kumpulan bornologi berdasarkan pada korantai terbatas dan mengkaji beberapa sifat-sifat asasnya. Kami menunjukkan bahawa teori kohomologi korantai terbatas dan teori kohomologi korantai homogen adalah isomorfisme.

Tambahan pula, kelas-kelas kumpulan bornologi setara dari segi set-set semi-terbatas dan peta-peta terbatas-s dibentangkan bagi menghadkan keadaan keterbatasan untuk kumpulan-kumpulan bornologi.

Akhir sekali, konsep separuh gelang bornologi diperkenalkan untuk menentukan keterbatasan (gelang, separuh gelang), dan pembinaan asas dalam kelas separuh gelang bornologi dibincangkan. Hasil umum dalam bab ini yang berkenaan dengan had-had unjuran dan had-had induktif serta teorem isomorfisme ditubuhkan.

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Finally, I would also like to thank my mother for her prayers and her never-ending unconditional support. I wish to send special thanks to my husband, Amear, for his support and encouragement.

I certify that a Thesis Examination Committee has met on 8 January 2018 to conduct the final examination of Anwar Nooruldeen Imran on her thesis entitled "Bornological Structures on Some Algebraic Systems" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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Declaration by graduate student

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Name of Member of Supervisory Comittee : SHARIFAH KARTINI SAID HUSAIN

TABLE OF CONTENTS

LIST OF ABBREVIATIONS

β	A Bornology
(X, β)	A Bornological set
(G, β)	A Bornological group
(S, β)	A Bornological semigroup
(R, β)	A Bornological ring
(L, β)	A Bornological semi ring
Born	Category of bornological sets
BornSG	Category of bornological semigroups
BornSR	Category of bornological semi rings
$\mathbb{N}$	Set of all natural numbers
$\mathbb{Z}$	Set of all integer numbers
$\mathbb{R}$	Set of all real numbers
$\mathbb{Q}$	Set of all rational numbers
$\mathbb{K}$	A field
B	Bounded set
A	Semi bounded set
$\lvert x\rvert$	Absolute value of a real number x
$\lVert v\rVert$	Norm of a vector v
T	Operator
$\varnothing$	Empty set
G_x	Orbit of an element under a bornological group action
$(V, \lVert \cdot \rVert)$	Normed space
$\mathbf{G-Born}$	The category of G-bornological sets
$\mathcal{P}(X)$	Power set of X

CHAPTER 1

INTRODUCTION

1.1 Basic Concepts

In this section some definitions related to group actions and cohomology groups are presented.

Definition 1.1 *An action of a group G on a set X is a map $\sigma : G \times X \longrightarrow X$ with*

i. $\sigma(e, x) = x$*, where e is the identity element of G and $x \in X$*

ii. $\sigma(g, \sigma(h, x)) = \sigma(gh, x)$*, for any $g, h \in G$ and $x \in X$.*

We shortly write gx for $\sigma(g, x)$, and call X a G-set.

Let G be a group and X be a set. Suppose that H is a subgroup of G and A is a nonempty subset of X, respectively. We put, $\sigma(H, A) = HA = \{ha : h \in H, a \in A\}$. We say that A is H-invariant if $HA \subset A$. For more detail in group action we refer the readers to Malik et al. (1997), Dummit and Foote (2004) and Atiyah (1994).

Now, let G be a group and $(A, +)$ be any abelian group, such that G acts on A, then A is called a G-module. It is the same group action with extra condition.

$\sigma(g, (a_1 + a_2)) = \sigma(g, a_1) + \sigma(g, a_2)$, for any $g \in G$ and $a_1, a_2 \in A$.

Let us define the set

$$C^n(G, A) = \{f : G^n \longrightarrow A\}.$$

To prove group structure on this set we need to choose binary operation which is defined as follows

$$(f_1 \oplus f_2)(g_1, ..., g_n) = f_1(g_1, ..., g_n) + f_2(g_1, ..., g_n)$$

for every $(g_1, ..., g_n) \in G^n$ and $f_1, f_2 \in C^n(G, A)$. Thus, $(C^n(G, A), \oplus)$ is a group. Indeed, first of all we have to prove this set is closed under $\oplus$ operation, i.e., for every $(g_1, ..., g_n) \in G^n$ and $f_1, f_2 \in C^n(G, A)$, such that

$$f_1(g_1, ..., g_n) \in A$$

and

$$f_2(g_1, ..., g_n) \in A,$$

then

$$(f_1 \oplus f_2)(g_1, ..., g_n) = f_1(g_1, ..., g_n) + f_2(g_1, ..., g_n) \in A.$$

We can relate these groups to each other by specific homomorphism which depends on n degree argument, such that

$$d_n : C^{n-1}(G, A) \longrightarrow C^n(G, A),$$

Definition 1.2 *Let C be a sequence of abelian group homomorphisms:*

$$0 \longrightarrow C^0(G, A) \xrightarrow{d_1} C^1(G, A) \xrightarrow{d_2} ...C^{n-1}(G, A) \xrightarrow{d_n} C^n(G, A) \longrightarrow ...$$

i. The sequence C is called a cochain complex if the composition of any two successive maps is zero: $d_{n+1} \circ d_n = 0$ for all n.

ii. If C is a cochain complex, its n^{th} cohomology group is the quotient group $kerd_{n+1}/imaged_n$ and is denoted by $H^n(C)$

Bounded cohomology of groups was first defined by Johnson (1972) in the context of Banach algebras. As an independent and very active research field, however, bounded cohomology started to develop in 1982, by Gromov (1982), where the definition of bounded cohomology was extended to deal also with topological spaces. There are many different equivalent definitions of group cohomology. The question that has been pondered is, what is the right generalization of group cohomology to a cohomology theory for bornological groups $H_{bd}(G, M)$?

Certainly, we would want a cohomology theory for bornological groups to satisfy cohomology groups properties, but one can hope for more, since there is bornological data on G and M.

1.2 Literature Review

Historically, the idea of a bounded subset of a topological vector space was introduced by von Neumann (1935), it played an important role in functional analysis that motivated the concept of more general and abstract classes of bounded sets, the so called bornology, see (Mackey, 1943). That means, it is applied to solve the questions of boundedness for any space or set X in general way not just by usual definition of bounded set, but we take a collection β of subset of X such that satisfy three conditions, β covers X, and β stable under hereditary also finite union. Basically, a bornological space is a type of space which possesses the minimum amount of structure needed to address questions of boundedness of sets and functions. In addition, since bornology have shown to be a very useful tool in various aspects of functional analysis, they have been considered by several researchers in different contexts see (Akkar (1970), Hogbe-Nlend (1977), Waelbroeck (2006)). Specifically, while Lechicki et al. (2004) examined bornology in relation to topology and Bernardes Jr (1994) examined bornology in relation to topological algebra and Voigt (2008) in cyclic homology.

There has been a good deal of researches done on this subject in recent years as a glance at the bibliography will show (see, Beer and Levi (2009), Meyer (1999), Mathai and Stevenson (2007), Lechicki et al. (2004), Vroegrijk (2009), Rump (2011), Meyer (2004), Valdivia (1971), Waelbroeck (1986)).

Interestingly, researchers have begun determining and solving the quotient of boundedness for group of objectives rather than sets of elements and introduce the concept of bornological groups. Accordingly, the crux of this study is the observation that most of the known bornological structures were considered to satisfy the common property of being compatible with abelian group structure. The theory of bornological groups has been studied from different perspectives by (Garrett (1997), Bambozzi (2015), Abramenko and Brown (2008), Funakosi (1976), Van Daele and Wang (2010)).

Pombo Jr (2012) study fundamental construction of bornological groups. Bambozzi (2014) discussed that bornological abelian groups form a quasi-abelian category which it is additive categories with kernels and cokernels. In Pombo Jr (2014) certain abelian bornological groups of continuous mappings from topological spaces into abelian topological groups are shown to be isomorphic to bornological projective limits of abelian bornological groups of continuous mappings.

Up to now, we have to keep in mind that in the setting of modules over a commutative ring a rich line of investigation was introduced by Pombo, in the last two decades. He extensively considered the notion of a linear bornology on module and its relation to the notion of a linear topology on a module, see (Pombo Jr, 1993a), (Pombo Jr, 1996b). After that, he consider the notion of a module bornology (a bornology on a module compatible with its module structure), see (Pombo Jr, 1996a), bornological modules over bornological rings in (Pombo Jr, 1998) and topologies and bornologies on module in (Pombo Jr, 1993b), such that compatible topologies and bornologies in the context of modules.

However, Barros and Bernardes (2014) completed the works by presenting a study of tenser products of bornological modules by means of an elementary approach, over bornological commutative rings such that this work in the setting of modules over a commutative ring.

If we compare the literature on bornological groups with the literature on topological groups, we find that there are still much work to be done on bornological groups. Therefore, we study some further properties of bornological group in Chapter 3.

In the present study, we determine the boundedness for algebraic structures (groups, semigroups, rings, semi rings). Consequently, we develop the cohomology theory for bornological groups, the case when a bornological group acts on abelian bornological group. Accordingly, in order to achieve this goal, we will use the efficient language of bornological group. Moreover, there is no strong literature on bornological groups on which the results of the present study is based, we need to start from search with theory of bornological groups and we develop it as far we need for our scope. Consequently, the bornological group actions will be explored and eventually tilted to suit the context of the present study. In this way we can construct cohomology theory for bornological group. Hence, a theory of cohomology bornological group is thus used to study the complex of homogeneous cochains. When, we define the complex of homogeneous cochains by setting $C^n_{hbd}(G, M)$ to be the set of G-equivariant maps.

1.3 Problem Statement

This study addresses the following problems regarding bornological structure, which are summarized as follows:

on Bornological groups: The idea of a bornological group is to determine the boundedness for groups which is a set with two structure group and bornology such that the product and inverse maps are bounded. Thus, the main problem is to bornologize groups. Thus, it is natural to ask these questions.

1. What are the ways to bornologize the group?

   Of course, this informal question can be given different formal interpretations; for example, we could look for bornologies which would make G into a bornological group. To exclude this trivial solution, we should look for non discrete bornologizations of G. Since not every group is bornological group. Therefore, the following question can be asked.

2. What conditions should added on bornology to bornologize every group?

   An effective way to deal with this problem is to study further properties of bornological group with the following questions in focus.

3. Is it every left (right) translation in bornological groups are bornological isomorphism?

4. Is it possible to determine the boundedness for anther algebraic structure, for example (semigroups)?

5. How can introduce the fundamental construction of bornological semigroups. In particular, projective limits and inductive limits?

   Since bornological abelian groups form a quasi-abelian category which it is additive categories with kernels and cokernels were totally discussed by Bambozzi (2014). For bornological semigroup we have the following question

6. What are the similar and different between the category of bornological group and the category of bornological semigroups?

   As we mention earlier the case of bornological groups is that the product and inverse maps are bounded. So, the following question is raised.

7. How can we restrict the condition of boundedness for bornological groups?

   Our main goal is to require less restrictive condition on the group operations neither of the operation is required to be bounded.

   on Bornological group actions:

   It is known that every left (right) translation is an example of group action and in the present study, it is proven that every left (right) translation is bounded in bornological groups. So, there are group actions which are bounded. Hence, the following questions are proposed

8. Is it possible to construct bornological actions and prove some basic facts which hold true only for actions of bornological groups on bornological set?

9. Can we construct the category of G-bornological set?

10. Is it possible to develop the cohomology of bornological group.

11. Is it possible to define a homogenous cochain in such away to be isomorphic

for bounded cochain? Accordingly, the study is employed in this work for constructing: the bornological semi ring structure.
On bornological rings: The notion of bornological ring is a set R with two structure ring and bornology.

12. Is it possible to study new structure bornological semi ring and construct the category for this new structure?

13. Is the product of two bornological semi rings again bornological semi ring?

1.4 Objectives

The objectives of this research are to solve the problem of boundedness for algebraic structure (group, semigroup, ring, semi ring) by introducing new structures bornological semigroups, bornological semi rings and investigate further properties of bornological group. Moreover, define and study new classes of bornological groups in order to restrict the condition of bornological group and find solutions to the problems in Section $1-3$. More specifically,

1. To determine the boundednees for another algebraic structures (semigroups, semi rings), by study the concepts of a bornological semigroups and a bornological semi rings.

2. To furnishes sufficient conditions for a bornology to determine the boundedness for every group by investigating further properties of bornological groups.

3. To construct bornological group actions and the category of G-bornological sets.

4. To introduce cohomology theory for bornological group and prove the isomorphic between homogenous cochains and bounded cochains

5. To restrict the condition of bornological group and study new classes of bornological groups.

1.5 Outlines of Study

In Chapter 2, we provide a brief review on bornological structures, which are used for our work. We recall the notion of bornology on a set: a bornology on a set X is a collection of subsets $\beta \subset \mathcal{P}(X)$ which is an ideal of the boolean algebra $\mathcal{P}(X)$. We study algebraic bornological structures: bornological groups, bornological rings which we define as algebraic structures over bornological sets.

In Chapter 3, we solve the question of boundedness for algebraic structure group when we investigate further properties of bornological groups. As well as, we construct the category of bornological semigroups and discuss the concept of the product, coproduct and fiber product. More specifically, we study fundamental construction of bornological semi groups.

In Chapter 4, we define and study bornological group actions (BGA). We extend all terminology concerning on G-sets to G-bornological sets. Then, we prove some basic facts which hold true for bornological group actions. In particular, we show that a bornological group action can be deduced from its boundedness at the identity. The orbital bornological set is also investigated and the quotient X/G succeed to the quotient bornology, which is called the quotient bornological set of the action. Furthermore, we introduce morphisms between G-bornological sets to be bounded maps compatible with action of G. This motivated us to construct the category of G-bornological sets.

In Chapter 5, we construct cohomology theory for bornological groups based on bounded cochain and study some of its basic properties. We show that for bornological groups and abelian bornological group the cohomology groups of bounded group cochains and of group cochain that are equivariant are isomorphic.

In Chapter 6, we study the possibility to introduce semi-bounded set with respect to bornological sets and we call the new bounded maps such as s-bounded map, s^*-bounded and s^{**}-bounded map. We prove that every bounded map is s-bounded map, s^*-bounded and s^{**}-bounded map, but the converse is not true, we provide the counterexamples for every case. Additionally, we introduce semi-bounded set with respect to a bornological ideals. Finally, new classes of bornological groups are discussed to restrict the condition of bornological groups with respect to these new bounded maps.

In Chapter 7, we generalize the theory of algebraic semi rings from the algebraic setting to the framework of bornological sets. More specifically, the concept of a new structure bornological semi ring (BSR) is introduced and some constructions in the class of bornological semi rings are discussed. In particular, the existence of arbitrary projective limits and arbitrary inductive limits of bornological semi rings is ensured. Additionally, the description of the category of bornological semi rings is presented. We also discuss the concept of product, coproduct and fibre product in the category of bornological semi rings. In the context under consideration, general results concerning projective limits and inductive limits as well as an isomorphism theorem are established.

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