UNIVERSITI PUTRA MALAYSIA

CANONICAL GROUP QUANTISATION ON ONE-DIMENSIONAL COMPLEX PROJECTIVE SPACE

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CANONICAL GROUP QUANTISATION ON ONE-DIMENSIONAL COMPLEX PROJECTIVE SPACE

By

AHMAD HAZAZI AHAMAD SUMADI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

November 2015
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DEDICATIONS

Dedicated in Humble Gratitude to my parents; Ahamad Sumadi Hj. Marzuki and Hasnah Hj. Omar, parent-in law; Abdul Rahman Jaffar and Hasnah Hasan, and especially to my beloved other half and son; Rahmah and Luqman ‘Atif, who inspires me to seek knowledge.

“Ramai orang datang bertamu
Di bawah pohon rimbun tertutup
Bersungguh-sungguh menuntut ilmu
Moga jadi pedoman hidup”

“Buah pedada batang keladi
Kembang berseri bunga senduduk
Marilah menurut resminya padi
Semakin berisi semakin menunduk”

“Pergi ke pasar membeli kangkung
Kangkung dimasak bersama tenggiri
Janganlah diturut resminya jagung
Semakin berisi semakin meninggi”

-Malay Pantun
Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

CANONICAL GROUP QUANTISATION ON ONE-DIMENSIONAL COMPLEX PROJECTIVE SPACE

By

AHMAD HAZAZI AHAMAD SUMADI

November 2015

Chairman: Associate Professor Hishamuddin Zainuddin, PhD
Faculty: Science

In this thesis we study the idea of quantisation approach to study the mathematical formalism of quantum theory with the intent to relate it with the idea of geometry of quantum states, particularly, Isham’s group-theoretic quantisation technique to quantise compact manifold. The core of the discussions is based upon the Isham’s quantisation programme and the compact classical phase space $S^2$ and $CP^1$. In Chapter 2, we review some of the literature that give some motivations to our investigation and also of those closely related to our present work.

In Chapter 3, we emphasize on reviewing several mathematical ingredients needed and also the idea of Isham’s group-theoretic quantisation method and discussed some insights to further the investigation in the subsequent chapter.

Chapter 4 consists of the author’s original contributions to the thesis. In this chapter, by using the aforementioned technique proposed in Chapter 3, we quantise the systems on one-dimensional complex projective space which is topologically homeomorphic to two-dimensional sphere. These two topological spaces are regarded as the underlying compact phase spaces for which there is no longer a cotangent bundle structure. These spaces have natural symplectic structure that allows one to use them for quantisation. The crucial part is to identify canonical group that acts on the phase space. The first phase is completed by finding all the algebras related to the groups.

With the canonical groups $SO(3)$ and $SU(2)$ found, we complete the quantisation process by finding representations of the canonical groups for $CP^1$. It is also discussed that Isham’s group-theoretic quantisation can be used for quantising complex projective spaces in general and study the complex projective space from group theoretical aspects for infinite-dimensional Hilbert space. Finally, Chapter 5 is a con-
clusion, in this chapter we summarise all our work and suggest some idea for future research.
Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Sarjana Sains

PENGQUANTUMAN KUMPULAN BERKANUN KE ATAS RUANG UNJURAN KOMPLEKS BERMATRA SATU

Oleh

AHMAD HAZAZI AHAMAD SUMADI

November 2015

Pengerusi: Profesor Madya Hishamuddin Zainuddin, PhD
Fakulti: Sains

Dalam tesis ini kami mengkaji idea pengquantuman bagi menyelidiki formulasi matematik bagi teori quantum dengan bermatlamat untuk mengaitkannya dengan idea geometri keadaan quantum, terutamanya teknik pengquantuman berteori-kumpulan Isham untuk mengquantumkan manifold padat. Teras perbincangan adalah berdasarkan program pengquantuman Isham pada ruang fasa klasik padat $S^2$ dan $CP^1$. Dalam Bab 2, kami melakukan tinjauan susastera yang berkait rapat dan memberikan motivasi kepada kajian kami.

Dalam Bab 3, kami memberi ulasan kepada beberapa topik matematik yang diperlukan bagi memahami keseluruhan kerja-kerja dan program Isham ini serta kami juga mengulas secara terperinci kaedah pengquantuman berteori-kumpulan Isham dan membincangkan beberapa pandangan untuk melanjutkan siasatan dalam bab berikutnya.


Setelah kumpulan-kumpulan berkanun $SO(3)$ dan $SU(2)$ dijumpai, kami melengkapkan proses pengquantuman dengan mencari semua perwakilan tak setara bagi
kumpulan berkanun. Kami juga membincangkan bahawa pengquantuman berteori-
kumpulan Isham ini boleh digunakan untuk mengquantumkan ruang unjuran kom-
pleks secara umum dan mengkaji ruang unjuran kompleks dari sifat teori kumpulan
bagi ruang Hilbert bermatra ketakterhingga. Akhir sekali Bab 5 adalah kesimpu-
lan dengan kami merumuskan semua kerja-kerja kami dan mencadangkan beberapa
pandangan untuk kajian akan datang.
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In The Name of Allah Most Merciful, Most Compassionate. Praises be to Allah alone, and may Allah bless Prophet Muhammad (pbuh), his family and companions and grant them mercy.

First and foremost, I would like to pay tribute to my teachers and mentors, a long line of illustrious people includes my generous supervisor Assoc. Prof. Dr. Hishamuddin Zainuddin who exposed me to the works of Chris J. Isham and the area of Quantization, Quantum Foundations and Quantum Information Theory. Also, for his valuable guidances on doing theoretical and mathematical physics research, and for teaching me how to realise ideas into concrete work. His insights and passion in both quantum theory and mathematical physics become an inspiration for young physicist like me.

Many thanks to Dr. Nurisya Mohd Shah for her enlightening discussions on measure theory and group representation theory, and to Dr. Mohammad Alinor Abdul Kadir for allowing me to borrow many of his mathematics books from his personal library and to whom I was introduced to mathematical logic and algebraic topology. Also, to my friend, Umair, for valuable consultations on LaTeX programming; to my mathematician friend, Taufik, for interesting discussions on pure mathematics, and to my fellow researchers in UPM Theoretical and Computational Physics Group for stimulating discussions every week!

I also would like to thank my Committee Examiners, Assoc. Prof. Dr. Zuriati Ahmad Zukarnain, Assoc. Prof. Dr. Jesni Shamsul Shaari and Dr. Md Mahmudur Rahman for their constructive remarks on previous version of this thesis.

Thanks also to the people who directly and indirectly involved in the process of preparing this thesis especially INSPEM staff, also many thanks are due to Universiti Putra Malaysia for providing me with a Graduate Research Fellowship scheme and also to INSPEM for some financial support.

Last but not least, I would like to thank my family members and especially my heart-felt appreciation to my beloved other half, Rahmah Abdul Rahman, for her love, patience and understanding. Without her life itself would be bereft of joy and happiness.
I certify that a Thesis Examination Committee has met on 26 November 2015 to conduct the final examination of Ahmad Hazazi Ahamad Sumadi on his thesis entitled “Canonical Group Quantisation on One-Dimensional Complex Projective Space” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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<td>Canonical commutation relation</td>
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<td>C-R</td>
<td>Cauchy- Riemann relation</td>
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<td>$T_xM$</td>
<td>Tangent space of manifold $M$ at point $x$</td>
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<td>$\mathcal{C}$</td>
<td>Classical phase space $\mathcal{Q}$</td>
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<td>$T^*\mathcal{Q}$</td>
<td>Set of cotangent bundle of configurations space $\mathcal{Q}$</td>
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<td>Holomorphic tangent bundle</td>
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<td>$N(\mathcal{C})$</td>
<td>Complex line bundle</td>
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<td>$CP^n$</td>
<td>Set of $n$-dimensional complex projective space</td>
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<td>$\mathbb{R}^n$</td>
<td>Set of $n$-dimensional real number</td>
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<td>$\mathbb{R}^+$</td>
<td>Set of positive real line</td>
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<td>$\mathbb{C}^n$</td>
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<td>$\mathbb{C}$</td>
<td>Set of complex line</td>
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<td>$\Omega^2(M)$</td>
<td>The space of differential 2-forms on $M$</td>
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<td>$\mathbb{C}^*$</td>
<td>Complex plane</td>
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<td>$\mathcal{P}\mathcal{H}$</td>
<td>Projective Hilbert space</td>
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<td>$K$</td>
<td>Kähler manifold</td>
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<td>$\mathbb{Z}$</td>
<td>Set of integers</td>
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<td>$\text{Pic}(\mathcal{C})$</td>
<td>Picard group</td>
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<td>$p_1$</td>
<td>Projection mapping of the bundle</td>
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<td>$T(M)$</td>
<td>Set of tangent bundle $M$</td>
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<td>Set of frame bundle $M$</td>
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<td>$GL(n, \mathbb{C})$</td>
<td>General linear group with $n \times n$ complex matrix</td>
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<td>General linear group with $n \times n$ real matrix</td>
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<td>Semi-direct product group of a dual vector space with $SU(2)$</td>
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<td>$\mathfrak{su}(2)$</td>
<td>Set of Lie algebra of $SU(2)$</td>
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<td>Diff($M$)</td>
<td>A diffeomorphisms group of symplectic transformation of the symplectic manifold $M$</td>
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<td>FLT</td>
<td>Fractional-Linear Transformation</td>
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<td>$\mathcal{L}(\mathcal{G})$</td>
<td>Lie algebra of canonical group $\mathcal{G}$</td>
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<td>Endomorphism of $TM$</td>
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<td>Left hand side</td>
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<td>Right hand side</td>
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<td>$\bar{\hbar}$</td>
<td>Planck’s constant</td>
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CHAPTER 1

INTRODUCTION

1.1 Preamble

Historically, in twentieth century physics, the two major theories that had been a subject of discourse among physicists are Einstein’s theory of relativity and quantum theory. The former caused a major reformulation of the concepts of space and time or space-time (Einstein and Lawson, 1920); the latter is the theory that revolutionised physics through the discovery of the “discreteness” energy of the hypothetical black body radiation by a German-born physicist Max Planck (Jammer, 1966). It has been used widely to understand the nature of microscopic world especially in Planck’s scale. There are many applications of quantum theory in various fields of physics such as solid state physics, nuclear physics, atomic and molecular physics, condensed matter physics, etc.

Recently, the twenty-first century physics has brought us to open a new horizon of research in which there are group of computer scientists, mathematicians and physicists working together to discover a new ideas in relatively fresh research area viz. quantum information theory. The advent of this field has found a renewal of interest into basic quantum theory, asking new kinds of questions and making more development on the theory, and at the same time also reawakening interest in the foundational issues of quantum theory itself. For example, attempts are currently being made to understand quantum entanglement from the information-theoretic point of view (Bengtsson and Życzkowski, 2007). In this thesis we use quantisation approach in order to study the mathematical formalism of quantum theory with the intent to relate it with the idea of geometry of quantum states.

In general, the word “quantisation” often means the discretisation of particle’s energy from the ground state energy of atom in microscopic world, but in our spectrum of discussion it is different and can be understood as an appropriate procedure to construct the quantum analogue of a given classical system with a specific phase space $\mathcal{S}$. Despite of being a century old, an effort of investigating the precise mathematical formalisms are of interest for both physicists and mathematicians ever since its birth. Indeed, there are different ways to quantise a classical theory such as Feynman-path integral quantisation, Weyl-Wigner quantisation, C*-algebra quantisation, Moyal quantisation, stochastic quantisation, quantisation by *-product, geometric quantisation, Schwinger’s quantisation etc (Ali and Englis, 2005; Shaharir, 2005; Feynman and Hibbs, 1965). From all the quantisation programmes mentioned, they differ in the fundamental structures assumed on the phase space. There is no one unique quantisation prescription that converts a classical theory to the quantum one, producing a “well-defined” quantum formalism.
In our research, we are using a particular quantisation programme initiated by Chris J. Isham called the group-theoretic quantisation\(^1\) (Isham, 1984). Isham’s first attempt was to apply this programme to quantise gravity based on 3-metrics (Isham and Kakas, 1984a,b). The programme has been generalised by others for different cases (Jung, 2012; Benevides and Reyes, 2010; Bouketir, 2000; Zainuddin, 1990, 1989). Group-theoretic quantisation, mathematically, shares the same mathematical language with geometric quantisation, but it emphasises the group-theoretical aspects. The idea is to focus on the construction of a canonical group describing the symmetries of the phase space of the system under study. The canonical groups play a pivotal role corresponding to a global analogue of the canonical commutation relations (CCR) in Dirac canonical quantisation given by

\[
[q_i, p_j] = i\hbar\delta_{ij}; \quad [q_i, q^j] = 0; \quad [p_i, p_j] = 0
\]

where the \(\hbar\) is a Planck’s constant, \(q^i\) are position variables of the configuration space of the system studied and their corresponding conjugate momenta \(p_j\). Note that in general, many physicists claimed that the starting point would seem to be the imposition of this CCR. For the other schemes, they usually have the CCR built in as an outcome at a later stage of the procedure. However, the CCR may be inappropriate as a basis for quantising classical systems on non-linear configuration spaces. For this reason one has to look for another guiding procedure to serve as a basis for quantisation. A natural ingredient would be the consideration of symmetries of the system to be quantised. Thus, one of the advantage of this programme is described by the nature of geometrical notions that allows one to understand the topological and global aspects of quantum theory in a group-theoretical context.

1.2 Organisation

In Chapter 2, we will review in general some literature that are related to our work and see how this quantisation scheme is used in a particular system and its generalisation.

In Chapter 3 we present the discussions on theory and methodology that we will use in our research. Here we will further the discussions by reviewing the preliminaries of the mathematical ingredients that are used to understand this quantisation scheme such as symplectic manifolds (the underlying mathematical structure of classical mechanics), complex and Kähler manifolds, and fibre bundle theory. Note that these geometrical tools are used in geometric quantisation programme (Brian, 2013; Woodhouse, 1997; Weinstein and Bates, 1997). In addition, group-

---

\(^1\) Jung (2012); Benevides and Reyes (2010) and some authors named this scheme as “Canonical Group Quantisation”, and we adopted this term as our main title of this thesis.
theoretical features based on Lie group actions on manifold and representation theory are extensively used in this quantisation programme and hence, we will briefly introduce an overview of, more or less, the fundamentals axioms of quantum theory known among physicists and followed by introducing the mathematical framework of Isham’s group-theoretic quantisation programme with the detailed elaborations. The discussions on comparison between Isham’s group-theoretic and geometric quantisation are further discussed.

In Chapter 4 is the author’s contribution where this technique is applied to quantise a simple system on a compact phase spaces. The phase space chosen here is one-dimensional complex projective space $CP^1$ that is topologically homeomorphic to the two-sphere $S^2$. The case considered is slightly different compared to those of Isham and others, since the phase space $\mathcal{S}$ is no longer a cotangent bundle. Albeit, to this particular case we proceed to the next step by finding the appropriate canonical groups and its relevant unitary irreducible representations.

The final chapter is to summarise all the author’s research findings and proposed some possible generalisations for future work. We suggest that, in terms of geometry of quantum states, one can study the idea of describing multiple qubit or qudit states that arise geometrically from this quantisation framework and hence to understand the idea of quantum entanglement (Bengtsson and Zyczkowski, 2007).

### 1.3 Problem Statements and Objectives

Based on Isham’s approach, the quantisation procedure utilise geometry of phase spaces in the form of cotangent bundle $T^*Q$. From there, one has to find its appropriate canonical groups and followed by the unitary irreducible representation of the groups. This were done by others in several cases, for instance this programme is applied on a system of a particle on a two-torus $T^2$ with a background field (Zainuddin, 1989), system on a homogeneous space $SU(2)/U(1)$ (Benevides and Reyes, 2010) and system of a particle on $\mathbb{R}^+$ (positive real line) with boundary conditions (Jung, 2012) etc. Furthermore, in comparison, the case of quantisation on compact phase space is well-known in geometric quantisation school (Woodhouse, 1997; Hurt, 1983; Sniatycki, 1980; Woodhouse and Simms, 1976).

Notwithstanding, motivated from the geometric quantisation school, the question arises whether it is possible or not that Isham’s method be applied to the case of non-cotangent bundle structure. From here we proposed our premise of argument that we want to generalise Isham’s method for the case of compact manifold as an underlying phase space.

Furthermore, complex projective spaces has been used recently in studying the ge-
ometrical feature of quantum states in the area of geometric quantum information. In this study we also want to understand the structure of this space, from the group-theoretical technique, realised as infinite-dimensional Hilbert space.

Therefore, the objectives and motivations for this thesis are as follows;

- To quantise a classical system described by simplest compact manifold. In this study they are the two-dimensional sphere $S^2$ and one-dimensional complex projective space $CP^1$.
- To find the appropriate canonical groups of both topological spaces $S^2$ and $CP^1$.
- To find inequivalent quantisations through inequivalent representations of the canonical group for $CP^1$.

These three objectives form the bases of our premise of arguments in Chapter 4 later on.
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