



**UNIVERSITI PUTRA MALAYSIA**

***TWO FLUID SCENARIO FOR DARK ENERGY MODELS IN A BIANCHI  
TYPE I UNIVERSE***

**SYED NASRULLAH ALI QAZI**

**IPM 2015 20**



# **TWO FLUID SCENARIO FOR DARK ENERGY MODELS IN A BIANCHI TYPE I UNIVERSE**

By

**SYED NASRULLAH ALI QAZI**

Thesis Submitted to the School of Graduate Studies, Universiti Putra  
Malaysia, in Fulfilment of the Requirements for the Degree of Master of  
Science

October 2014

All material contained within the thesis, including without limitation text, logos, icons, photographs and all other artwork, is copyright material of Universiti Putra Malaysia unless otherwise stated. Use may be made of any material contained within the thesis for non-commercial purposes from the copyright holder. Commercial use of material may only be made with the express, prior, written permission of Universiti Putra Malaysia.

Copyright © Universiti Putra Malaysia



© COPYRIGHT UPM

## TWO FLUID SCENARIO FOR DARK ENERGY MODELS IN A BIANCHI TYPE I UNIVERSE

By

SYED NASRULLAH ALI QAZI

October 2014

**Chair:** Associate Professor Hishamuddin B. Zainuddin, Ph.D.

**Faculty:** Institute for Mathematical Research

The current observations reveal the fact that the universe is expanding at an accelerating rate. The observations indicate that the driving force behind the accelerating expansion is the mysterious component known as dark energy. The only known information about dark energy is that it constitutes 73% of the total mass energy density of universe, its pressure is negative which acts as anti-gravitational force and it dominates at late times. However its nature still remains mysterious and it poses a challenge to the theoretical and observational cosmologists. Besides dark energy another mystery component in the universe that occupy large portion of universe after dark energy is dark matter. Since the nature of these dark components are relatively unknown to us, we could assume some sort of interaction between these components. The aim of this research is to study the nature of the dark energy by considering its physical quantity known as equation of state parameter  $\omega_D = \frac{p_D}{\rho_D}$  that specifies the model for dark energy. The general theory of relativity has the important role in explaining the dynamics of the universe as gravity governs the largest scale structure of the universe. Considering the fact that at present times that the universe to be in the form of perfect fluid, the equation of state describes the type of matter energy present in universe. The measurements from cosmic microwave background reveals the anisotropies in the temperature and almost spatially flat universe. Therefore in this research, I study the dark energy models in the framework of the anisotropic Bianchi I spacetime. The cosmological models studied here are mainly concerned with the recent epoch of universe. The behavior of the  $\omega_D$  is considered for two scenarios. First is the non-interacting scenario where the dark energy and dark matter are minimally coupled to each other. Second is the interacting scenario where there exists an interaction between the dark components and this is connected by an interaction term  $Q$ . I investigate the behavior of the  $\omega_D$  parameter by considering

the function of universe scale factor of two types. First is the scale factor in the form of power function law and it is found that  $\omega_D$  parameter for both non-interacting and interacting cases varies in the phantom region  $\omega_D < -1$ . Second is using the solutions of the Einstein field equation where I derive the general form of the  $\omega_D$  parameter for Bianchi I spacetime. Then I use the scale factor in terms of hyperbolic function with general  $\omega_D$  parameter. It is shown that in non-interacting case, depending on the value of the anisotropy parameter  $K$ , the dark energy EoS parameter is varying from phantom  $\omega_D < -1$  to quintessence  $\omega_D > -1$  whereas in interacting case EoS parameter vary in quintessence region. However, eventually all of these models tend to reach  $\omega_D = -1$ . Generally it is found that the behavior of the EoS parameter from all these models is that the dark energy model behaves like scalar field models such as quintessence, phantom and quintom.



## SENARIO DUA-BENDALIR BAGI MODEL TENAGA GELAP DALAM ALAM SEMESTA JENIS BIANCHI-I

Oleh

SYED NASRULLAH ALI QAZI

Oktober 2014

**Pengerusi: Profesor Madya Hishamuddin Zainuddin, Ph.D.**

**Fakulti: Institut Penyelidikan Matematik**

Cerapan terkini menunjukkan kenyataan bahawa alam semesta mengembang dengan kadar memecut. Pencerapan mendapati bahawa daya di sebalik pengembangan memecut ini adalah suatu komponen bermisteri yang dipanggil sebagai tenaga gelap. Maklumat yang diketahui mengenai tenaga gelap hanyalah tenaga ini memenuhi 73% daripada jumlah ketumpaan jirimtenaga alam semesta, tekanannya yang negatif berperanan sebagai daya anti-graviti dan tenaga ini kini mendominasi alam. Walau bagaimanapun sifat tenaga ini masih tidak diketahui dan kekal menjadi cabaran bagi komuniti teoris dan pencerap. Selain tenaga gelap, satu lagi komponen bermisteri yang diketahui memenuhi sebahagian besar alam ini adalah jirim gelap. Oleh sebab, sifat komponen-komponen gelap ini tidak diketahui, kita boleh mengandaikan adanya sejenis saling tindakan antara komponen-komponen ini. Tujuan penyelidikan ini adalah untuk mengkaji sifat tenaga gelap dengan pertimbangan kuantiti fizikal yang dikenali sebagai parameter persamaan keadaan  $\omega_D = \frac{p_D}{\rho_D}$  yang menentukan model tenaga gelap. Teori kerelatifan umum mempunyai peranan penting dalam menjelaskan dinamik alam semesta kerana graviti menguasai struktur skala besar alam ini. Dengan pertimbangan pada masa kini bahawa alam semesta berada dalam bentuk bendalir sempurna, persamaan keadaan akan menjelaskan jenis jirim-tenaga yang akan wujud dalam alam semesta. Pengukuran dari latar belakang mikrogelombang kosmik menunjukkan anisotropi dalam suhu dan alam semesta yang hampir datar. Dengan itu, kajian ini menyelidiki model tenaga gelap dalam kerangka ruang-masa anisotropi Bianchi-I. Model kosmologi yang dikaji di sini adalah lebih tertumpu pada zaman semasa. Sifat  $\omega_D$  dipertimbang dalam dua senario. Pertama adalah senario tak bersaling tindak yang mana tenaga gelap dan jirim gelap hanya terganding antara satu sama lain secara minimal. Kedua adalah senario bersaling tindak yang mana wujud saling tindakan antara komponen-komponen gelap ini dan dihubungkan melalui sebutan saling tindakan  $Q$ . Pertama adalah faktor skala dalam bentuk fungsi kuasa dan didapati bahawa parameter  $\omega_D$  bagi kedua-dua kes

tiada saling tindakan dan bersaling tindak, parameter berubah dalam rantau fantom  $\omega_D < -1$ . Kedua, menggunakan penyelesaian persamaan medan Einstein, bentuk umum parameter  $\omega_D$  diterbitkan untuk ruangmasa Bianchi I dan seterusnya faktor skala dalam bentuk fungsi hiperbolik. Dalam kes tanpa saling tindakan, ditunjukkan bahawa, bergantung pada parameter anisotropi  $K$ , parameter persamaan keadaan tenaga gelap berubah dari fantom  $\omega_D < -1$  ke kuintesens  $\omega_D > -1$ . Manakala untuk kes bersaling tindak, parameter persamaan keadaan hanya berubah dalam rantau kuintesens. Walau bagaimanapun kesemua model ini sampai ke had  $\omega_D = -1$ . Secara umumnya didapati bahawa perlakuan parameter persamaan keadaan bagi semua model tenaga gelap menyamai model medan skalar kuintesens, fantom dan kuintom.



## ACKNOWLEDGEMENTS

First of all I would like to thank to Almighty Allah the Most Gracious and Most Merciful for giving me lot of patience and determination to do my research during my study and making my dream possible to study in the field that I am really interested in. Next I would like to express my deepest and sincere gratitude to my supervisor Associate Professor Hishamuddin Zainuddin and post-doctoral fellow Dr Hassan Amirhashchi for their invaluable help and support. It has been honor for me to have Prof Hishamuddin Zainuddin as my supervisor as his knowledge and expertise in theoretical physics is totally immense. I am extremely grateful to him for his guidance and invaluable support in my research and as well for giving me a chance of doing research in cosmology. Next I am really indebted to the Post doctoral fellow Dr Hassan Amirhashchi who really acted as my co-supervisor during my research years. Dr Hassan is one of the finest theoretical cosmologist I have known as his research expertise in the field of theoretical cosmology is high. During his stay in the INSPEM as a postdoctoral fellow from 2012-2013, he gave me an opportunity to work on the problem and collaborate with him on the study of the dark energy models. During these years Dr.Hassan has been like a mentor to me as his guidance and his valuable help and support has allowed to finish my thesis on time. In fact I have learned a lot from Dr Hassan in the field of cosmology and doing research. For this I will always be grateful to him and acknowledge him for playing the important role behind the motivation of my study. I also wish to thank the other member of my supervisor committee Dr.Mahmudur Rahman for also supporting me and giving invaluable advice.

Next I would like to thank my colleagues, friends and others etc for being supportive and being helpful in my research as having discussion and exchanging ideas with them on the field of physics have brought influences in my research. Next I would like to express my sincere gratitude to the UPM and the INSPEM for letting me study in this wonderful university. I would like to express my thanks to the UPM lecturers especially from my department for teaching and training me into the field of mathematics and physics.

Finally I would like to express my deepest gratitude and sincere acknowledgement to my beloved family members especially my mother and father. I will always be grateful to them for providing their encouragement and moral support in my decision to study in the field. Lastly I will always be grateful to my brothers Ahmed and Fahd for providing me moral support and financially helping me during my Master study. Without my family members I wouldn't had the opportunity to study in the UPM and therefore I dedicate my thesis to them.



## Approval Sheet 1

I certify that a Thesis Examination Committee has met on (**insert the date of viva voce**) to conduct the final examination of (**insert the student's name**) on his (or her) thesis entitled "**Title of thesis**" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the (**insert the name of relevant degree**).

Members of the Thesis Examination Committee were as follows:

**Rakimov Isamiddin, Ph.D.**

Professor  
Faculty of Science  
Universiti Putra Malaysia  
(Chairperson)

**Anvarjon Ahmedov, Ph.D.**

Associate Professor  
Faculty of Science  
Universiti Putra Malaysia  
(Internal Examiner)

**Zaidan Abdul Wahab, Ph.D.**

Associate Professor  
Faculty of Science  
Universiti Putra Malaysia  
(Internal Examiner)

**Name of External Examiner, Ph.D.**

Title (e.g. Professor/Associate Professor/Ir) – Omit if not relevant  
Name of Department and/or Faculty  
Name of Organisation (University/Institute)  
Country  
(External Examiner)

---

**ZULKARNAIN ZAINAL, Ph.D.**

Professor and Deputy Dean  
School of Graduate Studies  
Universiti Putra Malaysia

Date:

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Master of Science.

The members of the Supervisory Committee were as follows:

**Hishamuddin B. Zainuddin, Ph.D.**

Associate Professor  
Institute of Mathematical Research  
Universiti Putra Malaysia  
(Chairperson)

**M. Mahmudur Rahman, Ph.D.**

Senior Lecturer  
Faculty of Science  
Universiti Putra Malaysia  
(Member)



---

**BUJANG BIN KIM HUAT, Ph.D.**

Professor and Dean  
School of Graduate Studies  
Universiti Putra Malaysia

Date:

## Declaration by graduate student

I hereby confirm that:

- this thesis is my original work;
- quotations, illustrations and citations have been duly referenced;
- this thesis has not been submitted previously or concurrently for any other degree at any other institutions;
- intellectual property from the thesis and copyright of thesis are fully-owned by Universiti Putra Malaysia, as according to the Universiti Putra Malaysia (Research) Rules 2012;
- written permission must be obtained from supervisor and the office of Deputy Vice-Chancellor (Research and Innovation) before thesis is published (in the form of written, printed or in electronic form) including books, journals, modules, proceedings, popular writings, seminar papers, manuscripts, posters, reports, lecture notes, learning modules or any other materials as stated in the Universiti Putra Malaysia (Research) Rules 2012;
- there is no plagiarism or data falsification/fabrication in the thesis, and scholarly integrity is upheld as according to the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) and the Universiti Putra Malaysia (Research) Rules 2012. The thesis has undergone plagiarism detection software.

Signature: \_\_\_\_\_ Date: \_\_\_\_\_  
Name and Matric No.: \_\_\_\_\_

## Declaration by Members of Supervisory Committee

This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) are adhered to.

Signature: \_\_\_\_\_  
Name of  
Chairman of  
Supervisory  
Committee: \_\_\_\_\_

Signature: \_\_\_\_\_  
Name of  
Member of  
Supervisory  
Committee: \_\_\_\_\_



# TABLE OF CONTENTS

	Page
<b>ABSTRACT</b>	i
<b>ABSTRAK</b>	iii
<b>ACKNOWLEDGMENTS</b>	v
<b>APPROVAL</b>	vi
<b>DECLARATION</b>	viii
<b>LIST OF TABLES</b>	xii
<b>LIST OF FIGURES</b>	xiii
<b>LIST OF ABBREVIATIONS</b>	xiv
<b>CHAPTER</b>	
<b>1 INTRODUCTION</b>	1
1.1 Objectives of the Research	2
<b>2 LITERATURE REVIEW</b>	3
2.1 Observational overview on Dark Energy	3
2.2 Models of Dark Energy	6
2.2.1 Cosmological constant	6
2.2.2 Dynamic models	7
2.2.3 Interacting models	9
<b>3 THEORY</b>	11
3.1 Basics of General Relativity	11
3.1.1 Spacetime metric tensor	11
3.1.2 The curvature tensor and scalar	12
3.1.3 Energy momentum tensor	13
3.1.4 Einstein field equations	14
3.2 Basics of cosmology	15
3.2.1 The Bianchi models	15
3.2.2 Bianchi-I cosmological model	17
3.2.3 Cosmological parameters	18
3.2.4 Energy conditions	20
<b>4 TWO FLUID SCENARIO FOR DARK ENERGY MODELS FOR THE FUNCTION OF SCALE FACTOR IN THE FORM OF POWER LAW</b>	22
4.1 Introduction	22
4.2 The Metric and Field Equations	23
4.3 Non-interacting two fluid model	29
4.4 Interacting two fluid model	35
4.5 Discussion and Conclusion	41

<b>5</b>	<b>TWO FLUID SCENARIO FOR DARK ENERGY MODELS FOR THE FUNCTION OF SCALE FACTOR IN THE FORM OF HYPERBOLIC</b>	43
5.1	Introduction	43
5.2	The Metric and Field Equations	44
5.3	Non-interacting two fluid model	48
5.4	Interacting two fluid model	54
5.5	Discussion and Conclusion	59
<b>6</b>	<b>DISCUSSION AND CONCLUSION</b>	61
6.1	Models for $\omega_D < -1$	61
6.2	Models for $\omega_D < -1$ to $\omega_D > -1$	61
6.3	Models for $\omega_D > -1$	62
6.4	Note on interacting models	62
6.5	Conclusion	62
6.6	Future problems	62
	<b>REFERENCES/BIBLIOGRAPHY</b>	63
	<b>APPENDICES</b>	69
	<b>BIODATA OF STUDENT</b>	93
	<b>LIST OF PUBLICATIONS</b>	94

# LIST OF TABLES

Table	Page
3.1 The classification of the Bianchi models	17



© COPYRIGHT UPM

Figure	Page
4.1 The mean anisotropy parameter $A_m$ versus $z$ for $\alpha = 2$ , $\beta = 2.1$ and $\gamma = 2.2$	28
4.2 The deceleration parameter $q$ versus $z$ for $\alpha = 2$ , $\beta = 2.1$ and $\gamma = 2.2$	28
4.3 The energy density of dark fluid $\rho_D$ versus $z$ for $\alpha = 2$ , $\beta = 2.1$ and $\gamma = 2.2$ , $\rho_0 = 3.0$	31
4.4 The pressure of dark fluid $p_D$ versus $z$ for $\alpha = 2$ , $\beta = 2.1$ and $\gamma = 2.2$ , $\rho_0 = 3.0$	31
4.5 The EoS parameter of dark energy $\omega_D$ versus $z$ for $\alpha = 2$ , $\beta = 2.1$ and $\gamma = 2.2$ , $\rho_0 = 3.0$	32
4.6 The plot of $\Omega_D$ , $\Omega_m$ and $\Omega_{tot}$ versus $z$ for $\alpha = 2$ , $\beta = 2.1$ and $\gamma = 2.2$ , $\rho_0 = 3.0$	33
4.7 The plot of speed of sound $v_s$ versus $z$	34
4.8 The plot of energy conditions versus $z$	35
4.9 The energy density of dark fluid $\rho_D$ versus $z$ for $\alpha = 2$ , $\beta = 2.1$ and $\gamma = 2.2$ , $\rho_0 = 3.0$ , $\sigma = 0.01, 0.02, 0.03$	37
4.10 The pressure of dark fluid $p_D$ versus $z$ for $\alpha = 2$ , $\beta = 2.1$ and $\gamma = 2.2$ , $\rho_0 = 3.0$ , $\sigma = 0.01, 0.02, 0.03$	37
4.11 The EoS parameter of dark energy $\omega_D$ versus $z$ for $\alpha = 2$ , $\beta = 2.1$ and $\gamma = 2.2$ , $\rho_0 = 3.0$ , $\sigma = 0.01, 0.02, 0.03$	38
4.12 The plot of $\Omega_D$ , $\Omega_m$ and $\Omega_{tot}$ versus $z$ for $\alpha = 2$ , $\beta = 2.1$ and $\gamma = 2.2$ , $\rho_0 = 3.0$	39
4.13 The plot of speed of sound $v_s$ versus $z$	40
4.14 The plot of energy conditions versus $z$	41
5.1 The mean anisotropy parameter $A_m$ versus $z$ for $L = 0.02$ , $L = 0.08$ and $L = 0.14$	49
5.2 The deceleration parameter $q$ versus $z$	49
5.3 The EoS parameter $\omega_D$ versus $z$ for $K = 0.01, 0.04, 0.07$ corresponding with $L = 0.02, 0.08, 0.14$ and $\Omega_{0m} = 0.24$ .	51
5.4 The plot of the DM, DE, and total energy densities ( $\Omega_m, \Omega_D, \Omega$ ) versus $z$ for $K = 0.001$ . The dots show the present value of $\Omega_m$ and $\Omega_D$ .	52
5.5 The plot of speed of sound $v_s$ versus $z$	53
5.6 The plot of the weak $\rho_D \geq 0$ , dominate $\rho_D + p_D \geq 0$ , and strong $\rho_D + 3p_D \geq 0$ energy conditions versus $z$ .	54
5.7 The plot of the dark energy EoS parameter $\omega_D$ versus $z$ . Here, we fix the parameter $K = 0.01$ and vary $\sigma$ as $0.01, 0.02, 0.03$ .	56
5.8 The plot of the DM, DE, and total energy densities ( $\Omega_m, \Omega_D, \Omega$ ) versus $z$ for $K = 0.001$ . The dots show the present value of $\Omega_m$ and $\Omega_D$ .	57
5.9 The plot of speed of sound $v_s$ versus $z$	58
5.10 The plot of the weak $\rho_D \geq 0$ , dominate $\rho_D + p_D \geq 0$ , and strong $\rho_D + 3p_D \geq 0$ energy conditions versus $z$ .	59



## LIST OF ABBREVIATIONS

EoS	Equation of state parameter
SNIa	Type Ia supernova
CMB	Cosmic microwave background
BAO	Baryon acoustic oscillation
WMAP	Wilkinson Microwave Anisotropy Probe
WEC	Weak energy conditions
DEC	Dominant energy conditions
SEC	Strong energy conditions
$\Lambda$	Cosmological constant
$\omega_D$	Equation of state parameter
$\Omega_D$	Dark energy density
$\Omega_m$	Dark matter density
$Q$	Interaction term
$\sigma$	constant coupling
$\sigma^2$	shear scalar

## INTRODUCTION

Before the end of twentieth century, we believed that our universe comprised of mostly matter and its gravitational influence is causing universe's expansion rate to slow down. However, this perception was changed in 1998 when the observation from the Supernovae Type Ia data from two independent teams, High-redshift Supernovae Search Team (Riess et al., 1998) and Supernovae Cosmology Team Project (Perlmutter et al., 1999) showed the universe is expanding at an accelerating rate which is caused by unknown energy. Since this kind of energy does not have interaction with the ordinary matter (energy) its nature still remains enigma for us and for this reason cosmologists call it 'dark energy'. Furthermore, numerous observation such as Cosmic Microwave Background, Baryon Acoustic Oscillation, and etc. show that the dark energy occupies 73%, dark matter 23% and matter only 4% in the total energy density of universe. These observations indicate that our universe is mostly dominated by dark components. Thus this opens a new perspective about our understanding of our universe and also gives some fundamental unsolved problem in cosmology about the nature of dark energy and dark matter and the possibility of detecting them.

From 1998 many observational as well as theoretical attempts (Zlatev et al., 1999; Caldwell, 2002; Alam et al., 2004; Feng et al., 2005; Astier et al., 2006; Copeland et al., 2006; Kowalski et al., 2008; Zhao et al., 2012) have been made in order to investigate the nature and properties of dark energy. Until now, there are so many candidates available to explain the model of dark energy. The first and simplest candidate for dark energy is the cosmological constant  $\Lambda$  which is constant vacuum energy in empty space and acts as a repulsive force opposing the gravitational force with having constant negative pressure. However it suffers from many problems among them are the fine-tuning and cosmic coincidence (Weinberg, 1989; Perlmutter et al., 1999; Carroll, 2001). The other models for dark energy are the dynamical models (Ratra and Peebles, 1988; Caldwell et al., 1998; Caldwell, 2002; Feng et al., 2005; Copeland et al., 2006; Singh, 2008; Tiwari and Singh, 2012) that could resolve to address the issues posed by cosmological constant. Such dynamical models include scalar field models (quintessence, phantom, quintom, k-essence), varying cosmological term models, and chaplygin gas. So far the dark energy models have been studied mainly in the framework of homogeneous and isotropic universe described by FRW (Friedmann-Robertson-Walker) metric. However the FRW model is not suited to describe at certain epoch in universe such as the moments after the Big Bang or satisfactorily explain the anisotropic radiation background in the universe observed at present times. Therefore, it is possible to consider to study models of universe with different background such as anisotropic Bianchi models, inhomogeneous models which are less symmetric than FRW and alternate to standard cosmology. There are also theoretical argument to explain the phase from anisotropic to isotropic phase (Misner, 1968). This is further supported from the observations of the cosmic microwave background radiation which reveals anisotropy in background radiation (Spergel et al., 2003). Since theoretical and experimental supports the universe to have anisotropic background phase, this motivate us to study the dark energy models

in universe with anisotropic background. The Bianchi models are such models that describe the universe having anisotropic and homogeneous property and also generalization of FRW models. The simplest model is the Bianchi I spacetime which is generalization of flat spacetime, is suitable to provide the description of the universe from the early anisotropic phase to isotropic.

Another possible way to understand the nature of dark energy is to allow the possibility of its interaction with the rest of the matter energy content in universe. In fact the detection of dark energy and dark matter by their gravitational effects, implies that there could be some sort of interaction between them. The study of the interaction models could possibly alleviate the 'Coincidence problem. In this research the evolution of dark energy is studied within the framework of homogeneous Bianchi I model. The behavior of the evolution of the dark energy parameter is investigated in two circumstances i) absence of interaction between dark energy and barotropic fluid ii) presence of interaction between dark energy and barotropic fluid. Therefore, the objectives set in this research are:

### **1.1 Objectives of the Research**

1. To study the dark energy models in Bianchi I universe by considering the behavior of scale factor as power law.
2. To study the dark energy models in Bianchi I universe by considering the behavior of scale factor as hyperbolic function that is applied in the exact solutions of the metric.
3. To interpret the solutions of these models and discuss their physical behavior with respect to cosmological evolution.

I start off with Chapter 2 summarizing the on the observational evidence and status of dark energy as well as reviewing the various models of dark energy. In Chapter 3, I review the some basics in cosmology and general relativity which would be relevant for this research. In Chapters 4 and 5 I solve for the dark energy models in non-interacting and interacting case for different scale factors respectively. Chapter 6 summarize the results for this research and suggest future problems.

- Alam, U., Sahni, V. and Starobinsky, A. 2004. The case for dynamical dark energy revisited. *Journal of Cosmology and Particle Physics* 2004.
- Amendola, L., Campos, G. C. and Rosenfeld, R. 2007. Consequences of dark matter-dark energy interaction on cosmological parameters derived from type Ia supernova data. *Physical Review D* 75 (8).
- Amirhashchi, H. 2013. Phantom instability of viscous dark energy in anisotropic space-time. *Astrophysics and Space Science* 345 (2): 439–447.
- Amirhashchi, H., Pradhan, A. and Zainuddin, H. 2011. An Interacting and Non-interacting Two-Fluid Dark Energy Models in FRW Universe with Time Dependent Deceleration Parameter. *International Journal of Theoretical Physics* 50 (11): 3529–3543.
- Amirhashchi, H., Pradhan, A. and Zainuddin, H. 2013. Interacting two-fluid viscous dark energy models in a non-flat universe. *Research in Astronomy and Astrophysics* 13 (2): 129138.
- Astier, P., Guy, J., Regnault, N., Pain, R., Aubourg, E., Balam, D., Basa, S., Carlberg, R., Fabbro, S., Fouchez, D., Hook, I., Howell, D., Lafoux, H., Neill, J., Palanque-Delabrouille, N., Perrett, K., Pritchet, C., Rich, J., Sullivan, M., Taillet, R., Aldering, G., Antilogus, P., Arsenijevic, V., Balland, C., Baumont, S., Bronder, J., Courtois, H., Ellis, R. S., Filio, M., Gonalves, A., Goobar, A., Guide, D., Hardin, D., Lusser, V., Lidman, C., McMahon, R., Mouchet, M., Mourao, A., Perlmutter, S., Ripoche, P., Tao, C. and Walton, N. 2006. The Supernova Legacy Survey: measurement of  $\Omega_m$ ,  $\Omega_\Lambda$  and  $\omega$  from the first year data set. *Astronomy and Astrophysics* 447 (1): 31–48.
- Bennett, C. L., Larson, D., Weiland, J. L., Jarosik, N., Hinshaw, G., Odegard, N., Smith, K. M., Hill, R. S., Gold, B., Halpern, M., Komatsu, E., Nolte, M. R., Page, L., Spergel, D. N., Wollack, E., Dunkley, J., Kogut, A., Limon, M., Meyer, S. S., Tucker, G. S. and Wright, E. L. 2013. Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results. *The Astrophysical Journal* 208 (2).
- Berman, M. S. 1983. A special law of variation for Hubbles parameter. *Il Nuovo Cimento B Series* 11 74 (2): 182–186.
- Bernardis, P. D., Ade, P. A. R., Bock, J. J., Bond, J. R., Borrill, J., Boscaleri, A., Coble, K., Crill, B. P., Gasperis, G. D., Farese, P. C., Ferreira, P. G., Ganga, K., Giacometti, M., Hivon, E., Hristov, V. V., Iacoangeli, A., Jaffel, A. H., Lange, A. E., Martinis, L., Masi, S., Mason, P. V., Mauskopf, P. D., Melchiorri, A., Miglio, L., Montroy, T., Netterfield, C. B., Pascale, E., Piacentini, F., Pogosyan, D., Prunet, S., Rao, S., Romeo, G., Ruh, J. E., Scaramuzzi, F., Sforna, D. and Vittorio, N. 2000. A flat Universe from high-resolution maps of the cosmic microwave background radiation. *Nature* 404: 955–959.

- Bertolami, O., Pedro, F. G. and Delliou, M. L. 2009. The Abell cluster A586 and the detection of violation of the equivalence principle. *General Relativity and Gravitation* 41 (12): 2839–2846.
- Caldwell, R. 2002. A phantom menace? Cosmological consequences of a dark energy component with super-negative equation of state. *Physical Letters B* 545 (1-2): 23–29.
- Caldwell, R. R., Dave, R. and Steinhardt, P. J. 1998. Cosmological Imprint of an Energy Component with General Equation of State. *Physics Review Letters* 80 (8): 15821585.
- Caldwell, R. R., Kamionkowski, M. and Weinberg, N. N. 2003. Phantom Energy: Dark Energy with  $\omega < -1$  Causes a Cosmic Doomsday. *General Relativity and Gravitation* 91 (7).
- Carroll, S. 2004. Gravitation. In *Spacetime and Geometry - An Introduction to General Relativity*, 174–176. Pearson Education, San Francisco: Addison Wesley, chapter 4.
- Carroll, S., Hoffman, M. and Trodden, M. 2003. Can the dark energy equation-of-state parameter  $\omega$  be less than -1? *Physical Review D* 68 (2).
- Carroll, S. M. 2001. The cosmological constant. *Living Reviews in Relativity* 4 (1).
- Chen, S. and Jing, J. 2009. A dark energy model interacting with dark matter and unparticle. *Classical and Quantum Gravity* 26 (15).
- Chimento, L. P., Jakub, A. S. and Pavón, D. 2003. Dark energy, dissipation, and the coincidence problem. *Physical Review D* 67 (8).
- Copeland, E., Sami, M. and Tsujikawa, S. 2006. Dynamics of dark energy. *International Journal of Modern Physics D* 15 (11).
- Cotsakis, S. 2001. Current Trends in Mathematical Cosmology. *arxiv* arXiv:gr-qc/0107090.
- Dalal, N., Abazajian, K., Jenkins, E. and Manohar, A. V. 2001. Testing the Cosmic Coincidence Problem and the Nature of Dark Energy. *Physical Review Letters* 87 (14).
- Delliou, M. L., Bertolami, O. and Pedro, F. G. 2007. Dark Energ-Dark Matter Interaction from the Abell Cluster A586 and violation of the Equivalence Principle. *AIP Conference Proceedings* 957 (421).
- Einstein, A. 1917. Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie. *Preussische Akademie der Wissenschaften, Sitzungsberichte* 142–152.
- Eisenstein, D. J., Zehavi, I., Hogg, D. W., Scoccamarro, R., Blanton, M. R., Nichol, R. C., Scranton, R., Seo, H. J., Tegmark, M., Zheng, Z., Anderson, S. F., Annis, J., Bahcall, N., Brinkmann, J., Burles, S., Castander, F. J., Connolly, A., Csabai, I., Doi, M., Fukugita, M., Frieman, J. A., Glazebrook, K., Gunn, J. E., Hendry,

- J. S., Hennessy, G., Ivezi, Z., Kent, S., Knapp, G. R., Lin, H., Loh, Y. S., Lupton, R. H., Margon, B., McKay, T. A., Meiksin, A., Munn, J. A., Pope, A., Richmond, M. W., Schlege, D., Schneider, D. P., Shimasaku, K., Stoughton, C., Strauss, M. A., SubbaRao, M., Szalay, A. S., Szapudi, I., Tucker, D. L., Yanny, B. and York, D. G. 2005. Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies. *The Astrophysical Journal* 633 (2): 560574.
- Ellis, G. F. R. and Elst, H. V. 1998. Cosmological models. In *Proceedings of the NATO Advanced Study Institute on Theoretical and Observational Cosmology* (ed. M. Lachi), 1–116. NATO science series, Boston, USA: Boston: Kluwer Academic.
- Feng, B., Wang, X. and Zhang, X. 2005. Dark energy constraints from the cosmic age and supernova. *Physics Letters B* 607 (1-2): 35–41.
- Feng, C., Wanga, B., Abdalla, E. and Su, R. 2008. Observational constraints on the dark energy and dark matter mutual coupling. *Physics Letters B* 665 (2-3): 111–119.
- Grøon, O. and Hervik, S. 2007. *Einsteins General Theory of Relativity: with Modern Applications in Cosmology*. New York: Springer.
- Guo, Z. K., Ohta, N. and Tsujikawa, S. 2007. Probing the coupling between dark components of the universe. *Physical Review D* 76 (2).
- Hartle, J. B. 2003. The Source of Curvature. In *Gravity An Introduction to Einstein's General Relativity*, 1st edn., 12–36. San Fransico: Addison Wesley, chapter 21.
- He, J. H. and Wang, B. 2008. Eects of the interaction between dark energy and dark matter on cosmological parameters. *Journal of Cosmology and Astroparticle Physics* 2008 (6).
- He, J. H. and Wang, B. 2010. The interaction between dark energy and dark matter. *Journal of Physics: Conference Series* 222 (1).
- Islam, J. N. 2004. Introduction to general relativity. In *An Introduction to Mathematical Cosmology*, 2nd edn., 12–36. Cambridge University Press, chapter 2.
- Komatsu, E., Dunkley, J., Nolta, M. R., Bennett, C. L., Gold, B., Hinshaw, G., Jarosik, N., Larson, D., Limon, M., Page, L., Spergel, D. N., Halpern, M., Hill, R. S., Kogut, A., Meyer, S. S., Tucker, G. S., Weiland, J. L., Wollack, E. and Wright, E. L. 2009. Five-year wilkinson microwave anisotropy probe observations: Cosmological interperations. *The Astrophysical Journal* 180 (2): 330–376.
- Kowalski, M., Rubin, D., Aldering, G., Agostinho, R., Amadon, A., Amanullah, R., Balland, C., Barbary, K., Blanc, G., Challis, P., Conley, A., Connolly, N., Covarrubias, R., Dawson, K., Deustua, S., Ellis, R., Fabbro, S., Fadeyev, V., Fan, X., Farris, B., Folatelli, G., Frye, B., Garavini, G., Gates, E. L., Germany, L., Goldhaber, G., Goldman, B., Goobar, A., Groom, D., Haissinski, J., Hardin, D., Hook, I., Kent, S., Kim, A., Knop, R., Lidman, C., Linder, E., Mendez, J., Meyers, J., Miller, G., Moniez, M., Mouro, A., Newberg, H., Nobili, S., Nugent,

- P., Pain, R., Perdereau, O., Perlmutter, S., Phillips, M., Prasad, V., Quimby, R., Regnault, N., Rich, J., Rubenstein, E., Ruiz-Lapuente, P., Santos, F., Schaefer, B., Schommer, R., Smith, R. C., Soderberg, A., Spadafora, A., Strolger, L., Strovink, M., Suntzeff, N., Suzuki, N., Thomas, R., Walton, N., Wang, L., Wood-Vasey, W. and J.L., Y. 2008. Improved Cosmological Constraints from New, Old, and Combined Supernova Data Sets. *The Astrophysical Journal* 686 (2): 749778.
- Krauss, L. M. and Turner, M. S. 1995. The cosmological constant is back. *General Relativity and Gravitation* 27 (11): 1137–1144.
- Kumar, S. and Singh, C. P. 2007. Anisotropic Bianchi type-I models with constant deceleration parameter in general relativity. *Astrophysics and Space Science* 312 (1-2): 57–62.
- Larson, D., Dunkley, J., Hinshaw, G., Komatsu, E., Nolta, M. R., Bennett, C. L., Gold, B., Halpern, M., Hill, R. S., Jarosik, N., Kogut, A., Limon, M., Meyer, S. S., Odegard, N., Page, L., Smith, K. M., Spergel, D. N. and Tucker, G. S., Weiland, J. L., Wollack, E. and Wright, E. L. 2011. Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Power Spectra and WMAP derived parameters. *The Astrophysical Journal* 192 (2).
- MacTavish, C. J., Ade, P. A. R., Bock, J. J., Bond, J. R., Borrill, J., Boscaleri, A., Cabella, P., Contaldi, C. R., Crill, B. P., de Bernardis, P., Gasperis, G. D., Oliveira-Costa, A. D., Troia, G. D., Stefano, G. D., Hivon, E., Jaffe, A. H., Jones, W. C., Kisner, T. S., Lange, A. E., Lewis, A. M., Masi, S., Mauskopf, P. D., Melchiorri, A., Montroy, T. E., Natoli, P., Netterfield, C. B., Pascale, E., Piacentini, F., Pogosyan, D., Polenta, G., Prunet, S., Ricciardi, S., Romeo, G., Ruhl, J. E., Santini, P. and Tegmark, M. 2006. Cosmological Parameters from the 20003 Flight Boomerang. *The Astrophysical Journal* 647 (2): 799812.
- Misner, C. W. 1968. The Isotropy of the Universe. *The Astrophysical Journal* 151 (2): 431–458.
- Pavón, D. and Wang, B. 2009. Le Châtelier-Braun principle in cosmological physics. *General Relativity and Gravitation* 41 (1): 1–5.
- Pavón, D. and Zimdahl, W. 2005. Holographic dark energy and cosmic coincidence. *Physics Letters B* 628 (3-4): 206–210.
- Perlmutter, S., Aldering, G., Goldhaber, G., Knop, R. A., Nugent, P., Castro, P. G., Deustua, S., Fabbro, S., Goobar, A., Groom, D. E., Hook, I. M., Kim, A., Kim, M., Lee, J., Nunes, N., Pain, R., Pennypacker, R., Quimby, R., Lidman, C., Ellis, R., Irwin, M., McMahon, R., Ruiz-Lapuente, P., Walton, N., Schaefer, B., Boyle, B., Filippenko, A., Matheson, T., Fruchter, A., Panagia, N., Newberg, H. and Couch, W. 1999. Measurements of  $\Omega$  and  $\Lambda$  from 42 High-Redshift Supernovae. *The Astrophysical Journal* 517 (2): 565–586.
- Petrosian, V., Salpeter, E. and Szekeres, P. 1967. Quasi-Stellar Objects in Universes with Non-Zero Cosmological Constant. *The Astrophysical Journal* 147: 1222–1226.

- Pradhan, A., Amirhashchi, H. and Saha, B. 2011. An interacting and non-interacting two-fluid scenario for dark energy in FRW universe with constant deceleration parameter. *Astrophysics and Space Science* 333 (1): 343–350.
- Ratra, B. and Peebles, P. J. E. 1988. Cosmological consequences of a rolling homogeneous scalar field. *Physical Review D* 37 (12): 34063427.
- Riess, A., Filippenko, A., Challis, P., Clocchiatti, A., Diercks, A. Garnavich, P., Gilliland, R., Hogan, C., Jha, S., Kirshner, R., Leibundgut, B. M., Phillips, M. Reiss, D., Schmidt, B., Schommer, R. A., Smith, C., Spyromilio, J., S. C., Suntzeff, N. and Tonry, J. 1998. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal* 116 (3): 1009–1038.
- Riess, A. G., Strolger, L. G., Tonry, J., Casertano, S., Ferguson, H. C., Mobasher, B., Challis, P., Filippenko, A. V., Jha, S., Li, W., Chornock, R., Kirshner, R. P., Leibundgut, B., Dickinson, M., Livio, M., Giavalisco, M., Steidel, C. C., Bentz, T. and Tsvetanov, Z. 2004. Type Ia Supernova Discoveries at  $z > 1$  from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution. *The Astrophysical Journal* 607 (2): 665687.
- Saha, B. 2011. Bianchi type I universe with viscous fluid. *Modern Physics Letters A* 20 (28).
- Saha, B., Amirhashchi, H. and Pradhan, A. 2012. Two-fluid scenario for dark energy models in an FRW universe-revisited. *Astrophysics and Space Science* 342 (1): 257–267.
- Setare, M. R. 2007. Interacting holographic phantom. *The European Physical Journal C* 50 (4): 991–998.
- Sherwin, B. D., Dunkley, J., Das, S., Appel, J. W., Bond, J. R., Carvalho, C. S., Devlin, M. J., Dunner, R., Hileman, T. E., Fowler, J. W., Hajian, A., Halpern, M., Hasseleld, M., Hincks, A. D., Hlozek, R., Hughes, J. P., Irwin, K. D., Klein, J., Kosowsky, A., Marriage, T. A., Marsden, D., Moodley, K., Menanteau, F., Niemack, M. D., Nolta, M. R., Page, L. A., Parker, L., Reese, E. D., Schmitt, B. L., Sehgal, N., Sievers, J., Spergel, D. N., Staggs, S. T., Swetz, D. S., Switzer, E. R., Thornton, R., Visnjic, K. and Wollack, E. 2011. Evidence for dark energy from the cosmic microwave background alone using the Atacama Cosmology Telescope lensing measurements. *Physical Review Letters* 107 (2).
- Sheykhi, A. 2011. Thermodynamical description of interacting entropy-corrected new agegraphic dark energy. *Modern Physics Letters A* 26 (25).
- Singh, J. 2008. A cosmological model with both deceleration and acceleration. *Astrophysics and Space Science* 318 (1-2): 103–107.
- Singh, J. P. and Tiwari, R. K. 2008. Perfect fluid Bianchi Type-I cosmological models with time varying  $G$  and  $\Lambda$ . *Pramana* 70 (4): 565–574.



- Singh, T. and Chaubey, R. 2012. An interacting two-fluid scenario for dark energy in a Bianchi type-I cosmological model. *Research in Astronomy and Astrophysics* 12 (5): 473484.
- Spergel, D. N., Verde, L., Peiris, H. V., Komatsu, E., Nolta, M. R., Bennett, C. L., Halpern, M., Hinshaw, G., Jarosik, N., Kogut, A., Limon, M., Meyer, S. S., Page, L., Tucker, G. S., Weiland, J. L., Wollack, E. and Wright, E. L. 2003. First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters. *The Astrophysical Journal* 148 (1): 175194.
- Tiwari, R. K. and Singh, D. 2012. Anisotropic Bianchi Type-I Model with a Varying  $\Lambda$  Term. *Chinese Physics Letters* 29 (3).
- Wang, B., Gong, Y. and Abdalla, E. 2005. Transition of the dark energy equation of state in an interacting holographic dark energy model. *Physics Letters B* 624 (3-4): 141146.
- Wang, B., Zang, J., Lin, C. Y., Abdalla, E. and Micheletti, S. 2007. Interacting dark energy and dark matter: Observational constraints from cosmological parameters. *Nuclear Physics B* 778 (1-2): 6984.
- Weinberg, S. 1989. The cosmological constant problem. *Review of Modern Physics* 61 (1): 1–23.
- Zhang, X. 2005. Statefinder diagnostic for coupled quintessence. *Physics Letters B* 611 (1-2): 1–7.
- Zhao, G., Crittenden, R., Pogosian, L. and Zhang, X. 2012. Examining the Evidence for Dynamical Dark Energy. *Physics Review Letters* 109 (17).
- Zhao, G. B., Xia, J. Q., Li, H., Tao, C., Virey, J. M., Zhu, Z. H. and Zhang, X. 2007. Probing for dynamics of dark energy and curvature of universe with latest cosmological observations. *Physics Letters B* 648 (1): 813.
- Zimdahl, W. 2012. Models of Interacting Dark Energy. *arXiv* .
- Zlatev, I., Wang, L. and Steinhardt, P. 1999. Quintessence, Cosmic Coincidence, and the Cosmological Constant. *Physical Review Letters* 82 (5): 896–899.