UNIVERSITI PUTRA MALAYSIA

MIXED CONVECTION BOUNDARY LAYER FLOW EMBEDDED IN POROUS MEDIUM WITH NANOFLUIDS

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By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Master of Science

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To My Beloved Wife, Lecturers and Friends

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Master of Science

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June 2013

Chair: Associate Professor Norihan Md. Arifin, PhD

Faculty: Institute for Mathematical Research

Convective flow and heat transfer in a saturated porous medium has been the subject of many investigations during the last decades and has been extensively studied. This fact has been motivated due to the importance of this process which occurs in many engineering and natural systems. An analysis of the steady mixed convection boundary layer flow past a vertical surface embedded in a porous medium saturated by a nanofluid is performed in this thesis.

The objective of this thesis is to investigate the effect of internal heat generation, effect by using stratified porous medium and effect of suction and injection for present problem. The effect of internal heat generation is important in several applications including reactor safety analysis, metal waste form development for spent nuclear fuel, fire and combustion studies, and storage of radioactive materials.

The similarity equations are solved numerically for three types of metallic or nonmetallic nanoparticles such as copper (Cu), alumina (Al_2O_3) and titania (TiO_2), in a water-based fluid to investigate the effect of the solid volume fraction or nanoparticles volume fraction parameter φ of the nanofluid on the flow and heat transfer characteristics with suction or injection.

Shooting method from MAPLE is used to solve the problem. This well-known technique is an iterative algorithm which attempts to identify appropriate initial conditions for a related initial value problem (IVP) that provides the solution to the original boundary value problem (BVP). The surface velocity, the local Nusselt number, the velocity profiles and temperature profiles are presented and discussed. Based on the results, it is noticed that these parameters can be used to control the convection boundary layer flow. The suction or injection parameters have the effects to increase the thermal boundary layer thickness, thus reduce the heat transfer at the interface.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

ALIRAN OLAKAN LAPISAN SEMPADAN MENERUSI MEDIUM BERLIANG DENGAN CAMPURAN BENDALIR NANO

Oleh

MOHD HAFIZI BIN MAT YASIN

Jun 2013

Pengerusi: Profesor Madya Norihan Md. Arifin, PhD

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Aliran olakan dan pemindahan haba menerusi medium tepu yang berliang telah menjadi satu subjek penyiasatan yang terkenal sejak kebelakangan ini. Fakta ini telah dimotivasi oleh kepentingan proses itu yang banyak digunakan di dalam bidang kejuruteraan dan sistem alam semulajadi. Satu analisis tentang olakan campuran aliran lapisan sempadan bagi permukaan menegak yang dilarutkan di dalam bendalir nano telah dibincangkan di dalam tesis ini.

Objektif utama tesis ini adalah untuk mengkaji tentang kesan penjanaan haba dalaman, kesan penggunaan lapisan medium berliang, kesan sedutan dan semburan terhadap masalah yang telah dinyatakan. Kesan penjanaan haba dalaman adalah penting dalam beberapa aplikasi termasuk keselamatan analisis reaktor, pembangunan bagi sisa bahan api nuklear yang berbentuk logam, kajian pembakaran

dan penyimpanan bahan radioaktif. Persamaan keserupaan telah diselesaikan secara berangka untuk tiga jenis zarah nano logam atau bukan logam seperti tembaga (Cu), alumina (Al_2O_3) dan titania (TiO_2), di dalam cecair yang berasaskan air untuk mengkaji kesan pecahan isipadu pepejal atau parameter pecahan isipadu zarah nano, φ bagi bendalir nano terhadap ciri-ciri aliran dan pemindahan haba dengan sedutan atau semburan.

Kaedah tembakan daripada pengaturcaraan MAPLE telah digunakan untuk menyelesaikan masalah tersebut. Teknik ini terkenal sebagai lelaran algoritma yang cuba untuk mengenal pasti keadaan masalah awal yang menyediakan penyelesaian kepada masalah nilai sempadan asal. Halaju permukaan, nombor Nusselt, profil halaju dan profil suhu telah dibentangkan dan dibincangkan. Daripada keputusan yang telah diperolehi, didapati bahawa parameter-parameter tersebut boleh digunakan untuk mengawal olakan aliran lapisan sempadan. Parameter sedutan atau semburan boleh meningkatkan ketebalan lapisan sempadan terma, seterusnya mengurangkan pemindahan haba di permukaan.

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Last but not least; I thank my wife for the unconditional love and encouragement throughout the entire period of my study.

I certify that a Thesis Examination Committee has met on 18th June 2013 to conduct the final examination of Mohd Hafizi Bin Mat Yasin on his thesis entitled "Mixed Convection Boundary Layer Flow Embedded In Porous Medium With Nanofluids" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the degree of Master of Science.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

MOHD HAFIZI BIN MAT YASIN

Date: 31 August 2013

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LIST OF ABBREVIATIONS

a	positive constant
b	positive constant
c	positive constant
C_f	skin friction coefficient
C_p	specific heat at constant temperature
e	exponential function
f	non-dimensional stream function
f_0	constant mass transfer parameter
g	acceleration due to gravity
Gr	Grashoff number
k	thermal conductivity
m	power law parameter
Pe	Peclet number
Pr	Prandtl number
Ra	Rayleigh number
Re	Reynolds number
S	Stratification parameter
t	time
T	fluid temperature within the thermal boundary layer
T_0	ambient temperature at the leading edge
T_w	surface temperature
T_∞	free-stream temperature
u	velocity component in the x direction

U_∞	free-stream fluid velocity
v	velocity component in the y direction
v_w	dimensionless suction or injection velocity
x	non-dimensional Cartesian coordinate along the surface
y	non-dimensional Cartesian coordinate normal to the surface

Greek symbols

α	thermal diffusivity
β	coefficient of thermal expansion
λ	constant mixed convection parameter
δ	velocity boundary layer thickness
η	similarity variable
θ	non-dimensional temperature
ν	kinematic viscosity
μ	dynamic viscosity
φ	nanoparticles volume fraction
ρ	fluid density
τ	shear stress
ψ	stream function

Subscripts

c	critical value
f	fluid
nf	nanofluid
s	solid
∞	free-stream condition

Superscripts

x'	differentiation with respect to x

CHAPTER 1

INTRODUCTION

1.1 Introduction

Heat is the thermal energy that flow when a temperature difference exist across a medium (Rudramoorthy & Mayilsamy (2006)). So, we can say that, “heat is the energy transfer between two objects from higher temperature to lower temperature because of the temperature difference”. However, heat cannot transfer from lower temperature to medium that has higher temperature.

Temperature is a property that measures the level of heat in a medium (Rudramoorthy & Mayilsamy (2006)). Temperature can be defined as a property that measures the level of heat in medium or property that measures of the average molecular kinetic energy of a system. The temperature of a substance is determined by the speed of its moving molecules whether it is a solid, liquid or gas (Blanc et al. (1971)). The heat comes out from the kinetic energy when the molecules of a substance collide with each other. The speed of the molecules will increase because of the friction and will lead the kinetic energy in each molecule to increase. Therefore, the greater of the number of collisions among molecules, the kinetic energy of the molecules or the amount of heat will increase”. Temperature also will control the direction of internal energy flow between two systems. Two systems are said to be in a state of thermal equilibrium which is there is no exchange of heat when that two system are in the same temperature. Therefore, we can conclude that heat exists only when the transfer occurs between two systems.

Heat transfer occurs through three phenomenon i.e. radiation, conduction and convection. Radiation and conduction depend on the temperature difference only but convection depends on the temperature difference and mass transport on the fluid. In convection and conduction, heat transfer only can take place through a material (Rudramoorthy & Mayilsamy (2006)).

1.2 Research Background

Convection is a vertical circulation of the heat transfer between two medium that happen when the medium have different temperature through to the motion or movement of the fluid. Based on physical science, fluid can be defined as substances that flow and has no definite shape i.e. liquids and gases. Convection cannot occur in solids since significant diffusion or current flows cannot take place in solids. It may happen in fluids at all scales.

Convection of heat transfer refer to the sum of both by heat is transported by advective and diffusive. Advective can be defined as a larger-scale motion of currents in the fluid and diffusive as a Brownian motion of individual particles. However, some researcher just defined convection based on advective phenomena only like transport equation.

Convection heat transfers take place in two ways or can be divided by two type of convection i.e. natural convection (free convection) and forced convection while mixed convection is consisting both of this two types.

1.2.1 Natural Convection

Natural or free convection is a type or mechanism of heat transport where the fluid motion does not involved with any external or outside factor but only based on density differences in the fluid that occur when the temperature gradient existed. Rohsenow et al. (1998) defined natural convection as a motion that results from the interaction of gravity with density differences within a fluid. The differences may result from gradients in temperature, concentration or composition. Natural convection can take place between a fluid and a solid surface when they exist at different temperatures and also contact in each other (Rudramoorthy & Mayilsamy (2006)).

In natural convection, fluid that surrounding by heat source will receive or gain the heat. Therefore, that particular fluid will becomes less density and will rises which is it will change place between the cooler fluid that have high density. The cooler fluid at the bottom then heated by heat source and will rise to the top. This process is called as the convection circulation.

This topic becomes very popular to the researcher since its presence in natural environment and most importantly in engineering applications. As an easy example in natural environment is the rain phenomenon. The heat from the sunlight will warm the sea and formed the convection cells that rise and form the cloud. There a lot of applications in the engineering, like cooling of molten metal, solar ponds and nuclear power station.

1.2.2 Forced Convection

Nield and Bejan (1992) defined forced convection as a fluid flow that caused or forced by outside factor or external agent which is not related to the heating effect. Besides that, Rudramoorthy and Mayilsamy (2006) stated that forced convection is a type or mechanism of heat transport where the fluid motion involved by any external factor likes pump or fan. Forced convection usually used to increase the rate of heat transport because it will produce result more quickly compare to the natural convection. For example, a convection oven has a fan which rapidly circulates hot air and force heat into food faster compared to what will naturally happen without any external factors.

Another type of mechanism for heat transfer that discussed in this thesis which is mixed convection heat transfer. Mixed convection defined as a combine effect of natural convection and forced convection. Since it caused by two effects, internal and external factor that make mixed convection is the best type of convection compare to the natural convection and force convection, respectively.

1.2.3 Convection in Porous Media

A porous medium is a material that containing pores or interconnected voids. The pores will allow a fluid to through it. They are typically filled with a fluid weather liquid or gas. Single phase flow mean that medium just have saturated by a single fluid and two phase flow is saturated by a liquid and gases. The easy examples for porous media are wood, sand, sponge and human lung.

A porous media is most often characterized by its porosity. By Cutnell & Johnson (2001), the porosity of the porous medium is defined as the fraction of the total volume of the medium that is occupied by void space. The concept of porous media is used in many areas of applied science and engineering, geosciences, biology and material science.

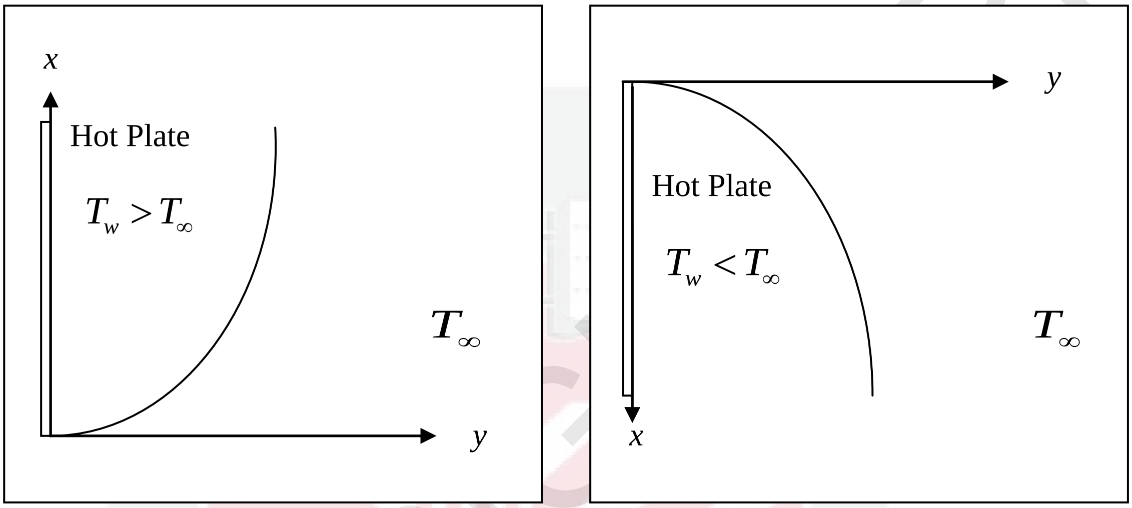
1.2.4 Nanofluids

Nanofluid is fluid that consists of nanometres-sized particles, known as nanoparticles. In term of nanotechnology, a particle is described as a miniature object that act as a whole unit in terms of its transport and properties. Particles are categorized according to the size. Das et al. (2008) defined that fine particles have a range between 100 and 2500 nanometers for their diameter, while ultrafine particles have a diameter between 1 and 100 nanometers. Same just like the ultrafine particles, nanoparticles have a diameter between 1 and 100 nanometers. From Wang & Mujumdar (2008), research about the suspending micro sized solid particles in fluids to enhance the thermal conductivity of base fluids already came out since 19^{th} century when Maxwell's theoretical work (Maxwell (1881)) was published. That is because; thermal conductivity of solid is typically higher comparing to the liquids. However, they cannot use the conventional fluids that have millimeter or micrometer sized particles since it can clog the tiny vessels of the equipment in the miniaturized technology. Hence, this method has not yet been commercialized.

On the other hand, nanofluids are created by suspending nanoparticles with average size below 100 nm in traditional heat transfer fluids such as water and oil. Fluids such as water, oil and ethylene glycol are poor heat transfer fluids, since the thermal conductivity of these fluids play important role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface (Ahmad & Pop (2010)). Choi (1995) in his experiment showed that the thermal conductivity of the fluid increases approximately two times when he added the small amount that less than 1% by volume of nanoparticles to conventional heat transfer liquid such as water and oil. Therefore, the effective thermal conductivity of nanofluids is expected to enhance heat transfer compared to the conventional heat transfer liquids.

1.3 Mathematical Formulation

In discussion on the convective heat transfer in flowing fluid, we need to consider three basic equations i.e. continuity equation, momentum equation and energy equation. These three equations are derived from the laws of conservation of mass, conservation of momentum and conservation of energy (Nield and Bejan (1992)).



a) Assisting Flow b) Opposing Flow

Figure 1.1: Velocity boundary layer on a vertical plate (Ahmad and Pop (2010))

Consider the steady mixed convection boundary layer flow past a vertical semi-infinite plate as a Figure 1.1. It is assumed that the ambient temperature that far flow from the plate is T_∞ and the temperature of the plate is T_w. In this case, we consider $T_w > T_\infty$ corresponds to a heated plate which is assisting flow and $T_w < T_\infty$ corresponds to a cooled plate which is opposing flow. To apply the Boussinesq approximation, convecting fluid and the porous medium are assumed in local thermodynamic equilibrium, the physical properties of the fluid except the density are constant and the viscous dissipation is neglected. From Gray and Giorgini (1975), Boussinesq approximation is density that assumed constant except when it directly causes buoyant forces. Its mean that continuity equation has its incompressible form and that density is considered variable only in the gravitational term of the momentum equation. Consider the present problem case, continuity equation, momentum equation and energy equation can be defined as,

Continuity equation,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1.1}$$

Momentum equation,

$$\frac{\mu_{nf}}{\mu_f}u=\frac{\mu_{nf}}{\mu_f}U_\infty+\frac{gK\left[\varphi\rho_s\beta_s+(1-\varphi)\rho_f\beta_f\right]}{\mu_f}(T-T_\infty) \tag{1.2}$$

Energy equation,

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}. \tag{1.3}$$

Here μ_{nf} and μ_f is the dynamic viscosity of the nanofluid and base fluid, respectively, g is the acceleration due to gravity, φ is the nanoparticles volume fraction, ρ_f and ρ_s are the densities of the fluid and of the solid fractions, respectively, β_f and β_s are the coefficients of thermal expansion of the fluid and of the solid, respectively, T is the temperature of the nanofluid, v is the mass transfer velocity and α_{nf} is the thermal diffusivity of the nanofluid.

Viscosity of the nanofluid, μ_{nf} was given by Brinkman (1952) and Oztop and Abu-Nada (2008) as

$$\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}}. \tag{1.4}$$

Thermal diffusivity of the nanofluid, α_{nf} was given by Oztop and Abu-Nada (2008) and Ahmad and Pop (2010) as

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \tag{1.5}$$

while the heat capacitance of the nanofluid, $(\rho C_p)_{nf}$ was given by Oztop and Abu-Nada (2008)

$$(\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s. \tag{1.6}$$

The effective thermal conductivity of the nanofluid is approximated by the Maxwell-Garnetts model (Oztop and Abu-Nada (2008))

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\varphi(k_f - k_s)}{(k_s + 2k_f) + \varphi(k_f - k_s)}. \tag{1.7}$$

The surface temperature is assumed to be a power-law function of the location. Hence, the boundary conditions at $y = 0,$ can be defined as

$$v(x,0) = 0, \tag{1.8}$$

$$T = T_w. \tag{1.9}$$

Meanwhile far from the surface, the boundary conditions are written as

$$u(x,\infty) = U_\infty, \tag{1.10}$$

$$T(x,\infty) = T_\infty. \tag{1.11}$$

1.4 Problem Statements

Nanofluids have a significant impact on the technology applications since it is potentially useful in many applications in heat transfer like power nuclear technology, hybrid power engines, electronic components and industrial heat exchange. Nanofluid enhanced thermal conductivity and the convective heat transfer coefficient compared to the base fluid, because of its novel properties. The main problem discussed in this thesis is about the comparison of nanofluids models with the existing Navier-Stokes or Newtonian mathematical models in describing the nature of boundary layer and mixed convection flows. Besides that, the effect of the uniform suction or injection on the heat transfer characteristics and the effects of nano parameter on the wall shear stress (skin friction) and wall heat flux (heat transfer from the surface) as well as fluid velocity and temperature profiles will also be studied.

1.5 Objectives and Scope

The objective of the thesis is to study numerically for the following problems.

a) Mixed convection boundary layer flow on a vertical surface in a porous medium saturated by a nanofluid with suction and injection.
b) Mixed convection boundary layer flow embedded in a thermally stratified porous medium saturated by a nanofluid.
c) Mixed convection boundary layer flow on a vertical surface in a porous medium saturated by a nanofluids with internal heat generation.

Meanwhile the scope of this study is limited to mixed convection boundary layer flow on a vertical surface in a porous medium. The nanofluids model that we used in this thesis is proposed by Tiwari and Das (2007) only, where the flows consider in local thermodynamic equilibrium and viscous dissipation is neglected.

1.6 Outline of the Thesis

Basically this thesis is divided into seven chapters. Chapter 1 consists of the general introduction about convection boundary layer, research background, problem statements and objectives of the thesis. Chapter 2 consists of the literature review and concentrate on the major part of this thesis, nanofluid. These thesis based on Ahmad and Pop (2010)'s problem where we extend their problem study. Therefore, the general formulation for Ahmad and Pop (2010) was shown in section 1.3. Meanwhile, Chapter 4 until 6 discusses the three main problems that have been mentioned in section 1.4.

In Chapter 4, we studied the mixed convection boundary layer flow on vertical surface in a porous medium saturated by a nanofluid with effect of suction and injection. Chapter 5 discusses on the mixed convection boundary layer flow embedded in a thermally stratified porous medium saturated by a nanofluid. In Chapter 6, we investigate the mixed convection boundary layer flow on a vertical surface in a porous medium saturated by nanofluids with internal heat generation.

Lastly, Chapter 7 contains the summary of the study as well as possible further research that can be carried out.

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