UNIVERSITI PUTRA MALAYSIA

SIMPLE MOTION EVASION DIFFERENTIAL GAME OF MULTIPLE PURSUERS AND SINGLE EVADER WITH INTEGRAL CONSTRAINTS ON CONTROL FUNCTION

YUSRA BINTI SALLEH

SIMPLE MOTION EVASION DIFFERENTIAL GAME OF MULTIPLE PURSUERS AND SINGLE EVADER WITH INTEGRAL CONSTRAINTS ON CONTROL FUNCTION

By

YUSRA BINTI SALLEH

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

December 2013

Dedication to all my family members:

My beloved husband, Ahmad

My dear son, Uwais

My Parents, Ayah & Ummi

All my siblings:

Along & Kak Sha & Fateh

Angah

Atam

Ateh

Achik

Aniq

Uda

Ya

Widad

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

SIMPLE MOTION EVASION DIFFERENTIAL GAME OF MULTIPLE PURSUERS AND SINGLE EVADER WITH INTEGRAL CONSTRAINTS ON CONTROL FUNCTION

By

YUSRA BINTI SALLEH

December 2013

Chair: Gafurjan Ibragimov, PhD
Faculty: Science

The term "Differential games" is applied to a group of problems in applied mathematics that share certain characteristics related to the modelling of conflict. Differential games are games in which the position of the players develops continuously in time. In a basic differential game, there are two actors (a pursuer and an evader) with conflicting goal. The pursuer wishes is to catch the evader, while the evader's mission is to prevent this capture.

The main steps in studying evasion games are to:

1) construct a strategy for the evader,
2) show admissibility of this strategy,
3) show that evasion is possible.

For the main result, we consider evasion differential game of multiple pursuers and single evader with integral constraints in the plane $\mathbb{R}^2$. The game is described by simple equations. Different from constraints on control functions of other works, here, each component of the control functions of players are subjected to integral constraint. We say that evasion is possible if the state of the evader does not coincide with that of any pursuer. To construct a strategy of the evader we use controls of the pursuers with time lag. We obtained a sufficient condition of evasion from many pursuers. At the end of this thesis we provide an illustrative example.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

PERMAINAN PEMBEZAAN PENGELAKAN GERAKAN MUDAH BAGI PENGEJAR BERGANDA DAN PENGELAK TUNGGAL DENGAN KEKANGAN KAMIRAN TERHADAP FUNGSI KAWALAN

Oleh

YUSRA BINTI SALLEH

Disember 2013

Pengerusi: Gafurjan Ibragimov, PhD
Fakulti: Sains

Istilah "Permainan Pembezaan" digunakan kepada sekumpulan masalah dalam matematik gunaan yang berkongsi ciri-ciri tertentu yang berkaitan dengan model konflik. Permainan pembezaan adalah permainan di mana kedudukan pemain membangun secara berterusan dengan masa. Dalam permainan pembezaan asas terdapat dua pemain (pengejar dan pengelak) dengan matlamat yang berbeza. Hasrat pengejar adalah untuk menangkap pengelak, manakala misi pengelak adalah untuk mengelakkan diri dari penangkapan ini.

Langkah-langkah utama dalam mengkaji permainan pengelakan adalah untuk:

1) membina strategi untuk pengelak,
2) menunjukkan kebolehterimaan strategi ini,
3) menunjukkan bahawa pengelakan mungkin berlaku.

Untuk keputusan utama, kami mempertimbangkan permainan pembezaan pengelakan pengejar berganda dan pengelak tunggal dengan kekangan kamiran dalam satu satah $\mathbb{R}^2$. Permainan digambarkan oleh persamaan mudah. Berbeza dengan halangan pada fungsi kawalan kerja-kerja lain, di sini setiap komponen fungsi kawalan pemain adalah tertakluk kepada kekangan kamiran. Kami mengatakan bahawa pengelakan boleh dilakukan jika keadaan pengelak tidak bersamaan dengan setiap pengejar. Untuk membina satu strategi pengelak, kami menggunakan kawalan daripada pengejar dengan jarak. Kami memperolehi syarat yang mencukupi untuk

pengelakan daripada banyak pengejar. Pada akhir tesis ini, kami menyediakan satu contoh ilustrasi.

ACKNOWLEDGEMENTS

Bismillahirrohmanirrohim,

Assalamu'alaikum w.b.t, Alhamdulillah, thanks to Allah s.w.t, whom with His willing and blessing for giving me the opportunity to complete this Master Research Thesis which is title Simple Motion Evasion Differential Game of Multiple Pursuers And Single Evader With Integral Constraints On Control Function. This thesis was prepared for School Graduated Studies, University Putra Malaysia (UPM), basically for postgraduate student to complete the master program that leads to the degree of Master Science of Applied Mathematics.

First of all, I would like to express my sincere appreciation to my soul mate which is my husband and also my beloved parents, for their cooperation, encouragement, constructive suggestion and full of support for the thesis completion, from the beginning till the end. They always are by my side whenever I need their support.

Thanks belong to my supervisor, Prof. Madya Dr. Gafurjan Ibragimov, the one who always treat me like his own daughter. I really appreciate his lectures, advices and guidance in the compilation and preparation this research thesis. Furthermore, he inspired me greatly to work in this research. His concern, patience, and constructive comments are very meaningful to me.

Foremost, I would like to give special thanks to my co-supervisor, Prof. Madya Dr. Zanariah for her valuable time to read and give suggestion for the improvement of this research. Besides, deepest thanks to the School of Graduated Studies (SGS) of UPM for the Graduate Research Fellowship (GRF) and Ministry of Education Malaysia for the MyMaster that supported me throughout my studies. Last but not least, I would like to thanks all my friends, colleagues and department staff during my studies at UPM.

I certify that a Thesis Examination Committee has met on 24 December 2013 to conduct the final examination of Yusra binti Salleh on her thesis entitled "Simple Motion Evasion Differential Game of Multiple Pursuers and Single Evader with Integral Constraints on Control Function" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

Members of the Thesis Examination Committee were as follows:

Siti Hasana binti Sapar, PhD
Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Mansor bin Monsi, PhD
Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Adem Kilicman, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Kuchkarrov Atamurat, PhD
Professor
National University of Uzbekistan
Uzbekistan
(External Examiner)

NORITAH OMAR, PhD
Associate Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 10 March 2014

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

Gafurjan Ibragimov, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Zanariah Abd Majid, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

____
BUJANG BIN KIM HUAT, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

DECLARATION

Declaration by graduate student

I hereby confirm that:

- this thesis is my original work;
- quotations, illustrations and citations have been duly referenced;
- this thesis has not been submitted previously or concurrently for any other degree at any other institutions;
- intellectual property from the thesis and copyright of thesis are fully-owned by Universiti Putra Malaysia, as according to the Universiti Putra Malaysia (Research) Rules 2012;
- written permission must be obtained from supervisor and the office of Deputy Vice-Chancellor (Research and Innovation) before thesis is published (in the form of written, printed or in electronic form) including books, journals, modules, proceedings, popular writings, seminar papers, manuscripts, posters, reports, lecture notes, learning modules or any other materials as stated in the Universiti Putra Malaysia (Research) Rules 2012;
- there is no plagiarism or data falsification/fabrication in the thesis, and scholarly integrity is upheld as according to the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) and the Universiti Putra Malaysia (Research) Rules 2012. The thesis has undergone plagiarism detection software.

Signature: ______________________ Date: ______________

Name and Matric No.: ________________________________

Declaration by Members of Supervisory Committee

This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) are adhered to.

Signature: ____________________
Name of Chairman of Supervisory Committee: ____________________

Signature: ____________________
Name of Member of Supervisory Committee: ____________________

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF ABBREVIATIONS

In this page, some of the abbreviations invoked in this thesis are listed.

P	Pursuer
E	Evader
$\dot{x}$	First derivative of x with respect to t
$\dot{y}$	First derivative of y with respect to t
$\mathbb{R}^n$	n-dimensional Euclidean space
x_{ij}	j coordinate of the ith pursuers
y_j	j coordinate of the evader
$\langle \ \rangle$	Inner product
$(\ )$	Vector
$\| \ \|$	Norm for infinite dimensional space
$\lVert \ \rVert$	Norm for Hilbert space

CHAPTER 1

INTRODUCTION

1.1 Differential Games

Differential Games are a special kind of problems for dynamic systems, particularly in moving objects. The concept of the differential games concentrates such conception as conflict, control, optimization, current information and equilibrium. Principally, there are applicable to politics, economics, sports and other spheres. In addition, differential games are an attractive mathematical task and can be developed more advanced in country. The Theory of Differential Games was intensively developed during 1960-80's.

In game theory, differential games are a group of problems related to the modeling and analysis of conflict in the context of a dynamical system. The problem usually consists of two actors with conflicting goals - one which wishes to maximize some given quantity and one wishing to minimize it. We will follow Isaacs (1965) and call them E (for Evader) and P (for Pursuer) respectively. The Pursuer wish is to catch the evader, while the evader's mission is to prevent this capture. In order to have a concrete technical example, we imagine one airplane pursuing another airplane. The objective of the first airplane is to overtake the second one; meanwhile the aim of the second airplane is to prevent from pursuit plane. Each pilot chooses a control, bearing in mind his purpose and having information regarding the situation available. The information consists of two parts. The first part is the complete knowledge of the performance capabilities of both planes. The second part of the information concerns the present and the past behavior of the airplanes, but nothing is known about their future behavior. We must give a mathematical idealization that retains the essential features of the technological problem. For example, the motions of objects P and E are described by differential equations (Pontryagin L.S (1981)).

$$P: \ddot{x} - \alpha \dot{x} = u, \quad |u| \leq \rho, \tag{1.1}$$

$$E: \ddot{y} - \beta \dot{y} = v, \quad |v| \leq \sigma, \tag{1.2}$$

The first part of the information mentioned above is that for the objects P and E, the equations (1.1) and (1.2) are always assumed to be known. In considering the second part is, to construct the control parameter u, for the example, P uses $x(t), y(t), v(t)$ at present time t. Indeed, the dynamics of the pursuer and the evader are modeled by systems of differential equations.

These "games" are modelled mathematically by first defining state variables that represent the position and perhaps velocity of the participants, determining (differential) equations of motion for the rivals and then describing sets in the state space called target sets. The study of these games has implication for real-life air combat and for artificial intelligence. Differential games also are related closely with optimal control problems. Specifically, in an optimal control problem there is single control *u(t)* and a single criterion to be optimized. Next, differential game theory generalizes this to two controls *u(t), v(t)* and two criteria, one for each player. Hence, each player attempts to control the state of the system so as to achieve the goal respectively, and the system responds to the inputs of both players.

Besides, duration of the game is unfixed and numbers of pursuers are countably many. Therefore, our research is in the intersection of the following differential games:

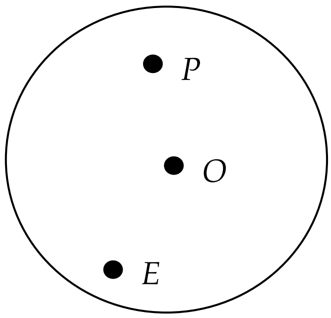
- evasion differential games
- linear differential games
- games with integral constraints
- multi-pursuer differential games.

Because we shall study an evasion differential game, which is described by linear differential equations. Moreover, control functions are subjected to integral constraints. In addition, number of pursuers several (multi-pursuer).

1.2 Lion And Man Problem

The classical Lion and Man problem is a game posed as to determine a strategy for a pursuer (lion) to capture the evader (man) in a given environment (J.E Littlewood (1986)). By capture, we mean that the man and the lion come at the same position after a finite time. The aim of the lion is to capture the man for any trajectory. The man wins the game if it can avoid capture indefinitely. Both the Lion and Man have identical motion capabilities. Capture strategies are important in surveillance where we would like to detect and capture equally agile intruders. For example, we consider the following:

Let: P = Pursuer (Lion);
E = Evader (Man)



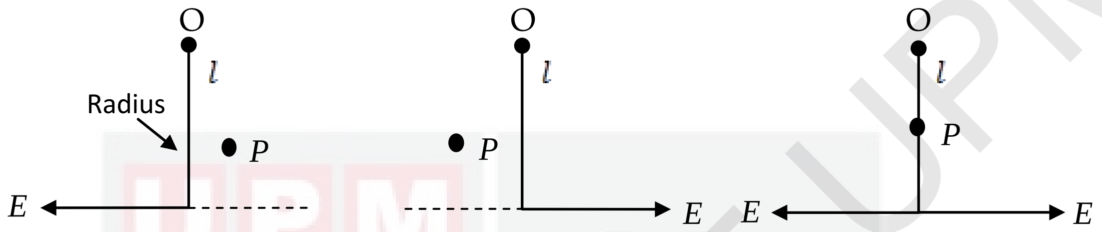


Figure 1.1: Movement of Lion and Man

From Figure 1.1, we consider a Lion, P and a Man, E are confined to a circular arena of radius 1 and move with maximum speeds bounded by 1. P and E have perfect information about each other's position, but they have contrary objectives; P wants to decrease his distance from E to value 0 in finite time, while E wants to avoid being captured by P in this way. If the position of the pursuer coincides with that of the evader then pursuit is said to be completed.

<u>Theorem 1.1</u> (J.E Littlewood (1986))
Evasion is possible in the Lion and Man game.

<u>Proof</u>

i) Construction a strategy for the evader.
Without any loss of generality we assume that E is inside the circle. We change directions of the point E at times $t_1, t_2, \ldots, t_n, \ldots$, where t_i is the time at which distance of the point E from the circumference equals $\frac{r}{i+1}$, where r is distance of E from the circumference of $t_0 = 0$.

In Figure 1.2, position of the evader at t_i is denoted by E_i. At each time t_i, we pass straight line l_i through the points O and E_i (see Figure 1.1). Three cases are possible. The pursuer P is on the right of l_i, or on the left of l_i, or on the line l_i. The evader uses the following strategy. (Figure 1.1) If the pursuer is on the right of l_i, then E moves to the left perpendicularly to l_i. If P is on the left from l_i, then E moves to the right perpendicularly to l_i. Finally, if P is on l_i, then E moves either to the left or to the right of l_i. We take the left direction for definiteness in the last case. Without restriction of generality we assume that P is always on the right or on l_i, i.e., P is not on the left of l_i. Then E moves in the left direction at each time t_i.

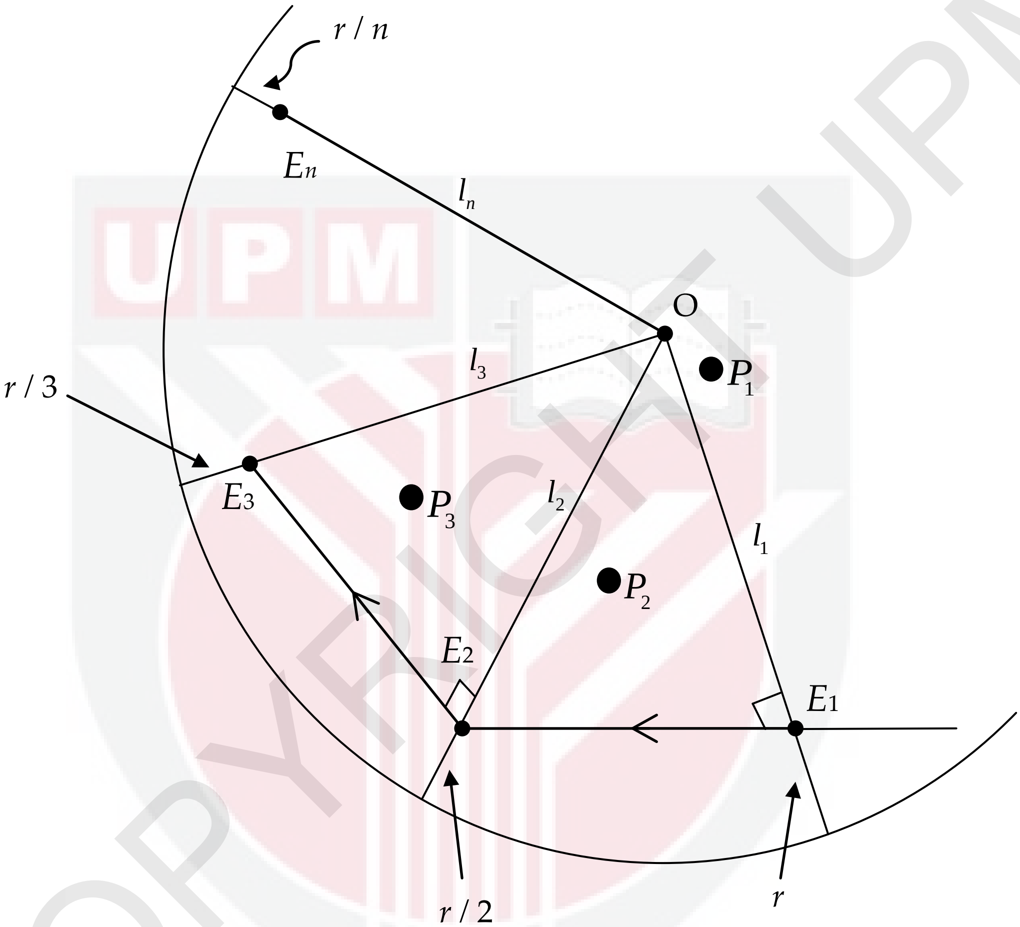


Figure 1.2: The trajectory of the Evader

ii) On each section evasion is possible.

Let $E_i = E(t_i)$. We show that on each section E_iE_{i+1} evasion is possible. Note that $P(t_i)$ is not on the left of l_i (see Figure 1.3).

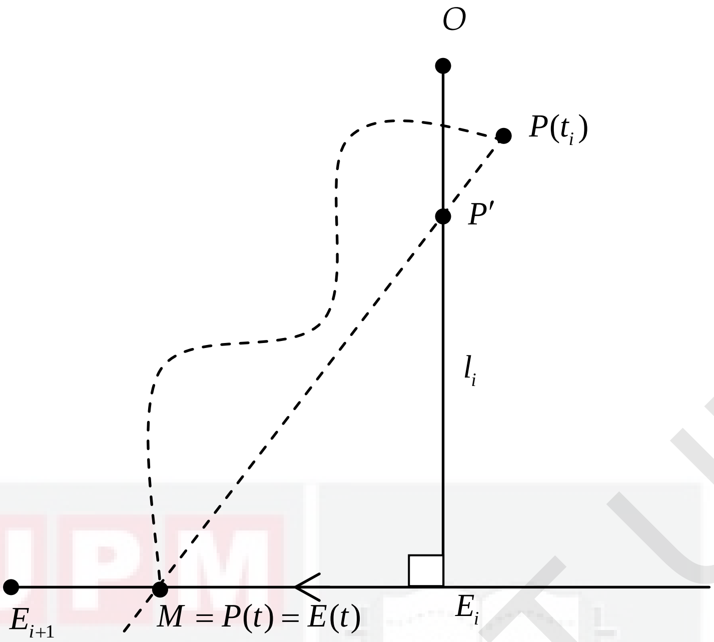


Figure 1.3: Evasion on section E_iE_{i+1}

We assume the contrary. Let

$$P(t) = E(t) = M$$

at some time t. Denote by P' intersection point of the interval $P(t_i)M$ with the line l_i.

Then (see Figure 1.3)

$$\begin{aligned} t = \frac{E_iM}{1} = \frac{\operatorname{arc}\left(P(t_i)M\right)}{1} &\geq \frac{P(t_i)M}{1} \\ &\geq \frac{P'M}{1} \\ &> \frac{E_iM}{1} = t, \end{aligned}$$

where $\operatorname{arc}\left(P(t_i)M\right)$ is length of the curve PM, and 1's in denominators are speeds of players and curve PM is the trajectory of the pursuer on the time interval $[0,t]$.

Then, it is a contradiction.

Therefore on each section E_iE_{i+1} the evader is not captured by the pursuer.

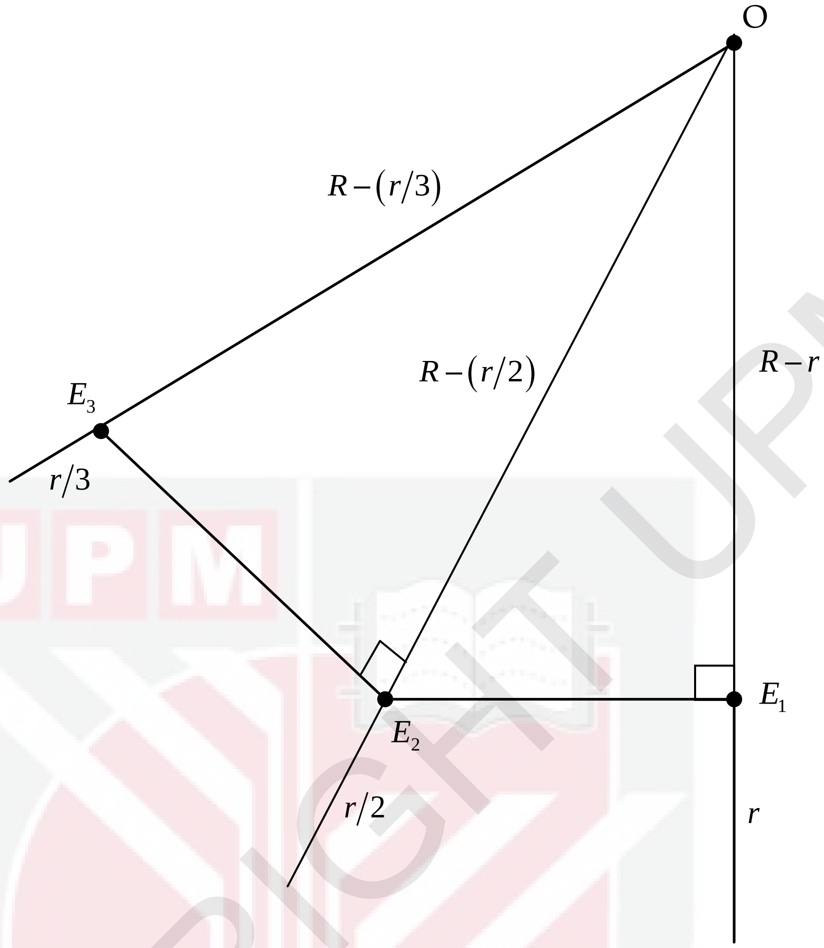


Figure 1.4: Figure for Estimation of total time

iii) Estimation of the total time.

We have shown that on each section E_iE_{i+1} evasion is possible. For this distance the evader E spends time equal to

$$t_i = \frac{E_iE_{i+1}}{1} = E_iE_{i+1}.$$

We obtain from the right triangles $OE_1E_2, OE_2E_3, \ldots, OE_nE_{n+1}$ (see Figure 1.4)

$$t_1 = E_1E_2 = \sqrt{\left(R-\frac{r}{2}\right)^2 - \left(R-r\right)^2}$$

$$t_2 = E_2E_3 = \sqrt{\left(R-\frac{r}{3}\right)^2 - \left(R-\frac{r}{2}\right)^2}$$

.

.

.

$$t_n = E_nE_{n+1} = \sqrt{\left(R - \frac{r}{n+1}\right)^2 - \left(R - \frac{r}{n}\right)^2}. \quad (1.3)$$

We show that $t_n \geq \frac{r}{n+1}$. Indeed, substituting the equation (1.3) into this inequality then taking square, yields

$$R^2 - \frac{2Rr}{n+1} + \frac{r^2}{(n+1)^2} - R^2 + \frac{2Rr}{n} - \frac{r^2}{n^2} \geq \frac{r^2}{(n+1)^2}$$

or after simplification

$$\frac{2R}{n} - \frac{2R}{n+1} \geq \frac{r}{n^2}.$$

From here

$$2R(n+1) - 2Rn \geq \frac{r(n+1)}{n},$$

$$2Rn \geq rn + r.$$

As $R > r, n \geq 1$, then

$$Rn + Rn \geq rn + r.$$

Now we obtain

$$t_1 + t_2 + \ldots \geq \frac{r}{2} + \frac{r}{3} + \frac{r}{4} + \ldots = r \bullet \sum_{n=2}^{\infty} \frac{1}{n} = \infty,$$

since the series $\sum_{n=2}^{\infty} \frac{1}{n}$ is divergent.

Thus for the time $t_1 + t_2 + \ldots + t_n$ the evader will not be captured. Moreover $t_1 + \ldots + t_n \to \infty$ as $n \to \infty$. Therefore in the game of Lion and Man, evasion is possible.

1.3 Objective of Thesis

In the plane $\mathbb{R}^2$, motions of the pursuers $x_1, \ldots, x_m$ and evader y are subscribed by the equations

$$\begin{aligned} &\dot{x}_i = u_i, \quad x_i(0) = x_{i0}, \quad i = 1, \ldots, m, \\ &\dot{y} = v, \quad y(0) = y_0. \end{aligned}$$

On controls of the players, coordinate-wise integral constraints are imposed:

$$\begin{aligned} &\int_0^\infty u_{i1}^2(s)ds \le \rho_{i1}^2, \int_0^\infty u_{i2}^2(s)ds \le \rho_{i2}^2, \qquad i = 1, \ldots, m, \\ &\int_0^\infty v_1^2(s)ds \le \sigma_1^2, \qquad \int_0^\infty v_2^2(s)ds \le \sigma_2^2. \end{aligned}$$

Evasion is said to be possible if the state of the evader does not coincide with that of any pursuers, that is $x_i(t) \ne y(t)$ for all $t \ge 0$, and $i = 1, \ldots, m$. The objectives of the research are:

- to obtain sufficient conditions of evasion
- to construct the evasion strategy.

1.4 Outline Of Thesis

This thesis will be presented in six chapters which attempts mainly to construct optimal strategies of players in a linear differential game in plane $\mathbb{R}^2$. Chapter 1 consists of introduction to differential game problems. Here we present lion and man game problem and construct strategy for the evader which guarantees evasion from the pursuer.

In Chapter 2, we give a literature review which are some references have related work with this research.

For chapter 3, we study control and trajectory of an object, construction of P-Strategy.
Next in Chapter 4, we study a pursuit-evasion differential game of two players with integral constraints, one pursuer and one evader. In this chapter, we introduce some basic methods to prove the main theorem. In pursuit (respectively, **evasion**) games we:

1) construct a strategy for the pursuer (**evader**),
2) show admissibility of this strategy,
3) show that pursuit can be completed **(evasion is possible**).

In Chapter 5, we give some results which are used to prove our main theorem. Here we give sufficient conditions of evasion game in simple motion differential game. We consider an evasion differential game of many pursuers and one evader with integral constraints in the plane $\mathbb{R}^2$. The game is described by simple equations. Each component of the control functions of players is subjected to integral constraint. Evasion is said to be possible if the state of the evader does not coincide with that of any pursuer. Strategy of the evader is constructed based on controls of the pursuers with time lag. A sufficient condition of evasion from many pursuers is obtained and an illustrative example is provided.

The last but not least is Chapter 6 where general conclusion and future research are presented.

BIBLIOGRAPHY

Azimov, A.Y. (1974). A linear differential evasion game with integral constraints on the controls. *USSR Computational Mathematics and Mathematical Physics*. 14(6): 56-65.

Azamov, A.A. and Samatov, B. (2000). *π-Strategy. An Elementary Introduction to the Theory of Differential Games*. NUU press, Tashkent, Uzbekistan.

Berkovitz, L. D. (1967). Necessary Conditions for Optimal Strategies in a Class of Differential Games and Control Problems. *SIAM Journal on Control*. 5: 1-24

Chernous' ko, F.L. (1976). A problem of evasion from many pursuers. *Journal of Applied Mathematics and Mechanics.* PMM pp. 14–24. 40(1): 11-20.

Chikrii, A.A. and Belousov, A.A. (2009). On linear differential games with integral constraints. *Trudy Instituta Matematikii Mekhaniki.* 15(4): 290-301. Russian.

Friedman, A. (1971). *Differential Games*. John Wiley & Sons, New York, USA.

Gusiatnikov, P.B. and Mohon'ko, E.Z. (1980). On $l_\infty -$escape in a linear many-person differential game with integral constraints. *Journal of Applied Mathematics and Mechanics*. 44(4): 436-440.

Ibragimov, G.I. (2005). Optimal Pursuit with Countably Many Pursuers and One Evader. *Differential Equations*. 41(5): 627-635.

Ibragimov, G.I. and Salimi, M. (2009). Pursuit-Evasion differential game with many inertial players. *Mathematical Problems in Engineering*. 2009.

Ibragimov, G.I. and Hasim, R.M. (2010). Pursuit and evasion differential games in Hilbert space. *International Game Theory Review*. 12(3): 239-251.

Ibragimov, G.I., Azamov, A.A. and Khakestari, M. (2010). Solution of a linear pursuit-evasion game with integral constraints. *ANZIAM Journal*. 52: E59-E75.

Ibragimov, G.I., Salimi, M. and Amini, M. (2012). Evasion from many pursuers in simple motion differential game with integral constraints. *European Journal of Operational Research*. 218(2): 505-511.

Ibragimov, G.I. and Salleh, Y. (2012). Simple Motion Evasion Differential Game of Many Pursuers and One Evader with Integral Constraints on Control Functions of Players. *Journal of Applied Mathematics*. 2012.

Isaacs, R. (1965). *Differential Games*. John Wiley & Sons, New York, USA.

Ivanov, R.P. (1980). Simple pursuit-evasion on the compact. *Dokl. USSR*. T. 254(6): 318-1321.

Ivanov, R.P. and Ledyaev, Yu.S. (1981). Optimality of pursuit time in a simple motion differential game of many objects. *Collected papers of the Mathematical Institute named after V. A. Steklova*. 158: 87-97.

Krasovskii, N.N. (1968). *The Theory of Motion Control*. Nauka, Moscow, Russia.

Krasovskii, N.N. and Subbotin, A.I. (1988). *Game-theoretical control problems*. Springer, New York.

Kuhn, H.W. and Szego, G.P. (1971). *Differential games and related topics*. Amsterdam: North-Holland Pub. Co.

Lee, E.B. and Marcus, L. (1985). *Foundations of optimal control theory*. John Wiley and Sons Inc., New York, London.

Levchenkov, A.Y. and Pashkov, A.G. (1990). Differential game of optimal approach of two inertial pursuers to a noninertial evader. *J. Optimization Theory and Applications.* 65(3): 501-517.

Lewin, J. (1994). *Differential Games*. Springer, Berlin, Germany.

Littlewood, J.E. (1986). *Littlewood's Miscellany*. Cambridge University Press.

Mesencev, A.V. (1974). A sufficient conditions for evasion in linear games with integral constraints. *Doklady Akademii Nauk SSSR*. 218: 1021-1023.

Nikolskii, M.S. (1969). The direct method in linear differential games with integral constraints. in *Control Systems*. *IM, IK, SO AN SSSR*. 49-59.

Pashkov, A.G. and Teorekhov, S.D. (1983). On a Game of Optimal Pursuit of One Object by Two Objects. *Prikl. Mat. Mekh*. 47(6): 898-903.

Petrosyan, L.A. (1977). *Differential Games of Pursuit*. Leningrad: Izd-vo LGU.

Pontryagin, L.S. (1981). Linear differential games of pursuit. *Mathematics of the USSR-Sbornik*. 40(3): 285-303.

Pshenichnii, B.N. and Onopchuk, Y.N. (1968). Linear differential games with integral constraints. *Akademii Nauk SSSR, Tekhnicheskaya Kibernetika*. 1968(1): 13-22.

Pshenichnii, B.N. (1976). Simple pursuit by several objects, Cybernetics. 3: 145-146.

Rikhsiev, B.B. (1989). *Differential games with simple moves*. Tashkent: Fan.

Satimov, N.Y. (1981). Problems of pursuit and evasion in differential games. *Mat. Zametki*. 29(3): 455-477.

Satimov, N. and Mamatov, M.S. (1983). The problem of pursuit and encounter avoidance in differential games between groups of pursuers and evaders. *Dokl. Akad. Nauk UzbSSR*. 4: 3-6.

Satimov, N.Y. and Rikhsiev, B.B. (2000). *Methods of Solving of Evasion Problems in Mathematical Control Theory*. Fan, Tashkent, Uzbekistan.

Satimov, N.Y., Rikhsiev, B.B. and Khamdamov, A.A. (1982). On a pursuit problem for n person linear differential and discrete games with integral constraints. *Mathematics of the USSR-Sbornik*. 46(4): 456-469.

Sinitsyn, A.V. (1993). Construction of the Cost Function in the Game of Pursuit by Several Objects. *Prikl. Mat. Mekh*. 57(1): 52-57.

Soravia, P. (1993). Pursuit-evasion problems and viscosity solutions of Isaacs equation. *SIAM J. Control and Optimization* 31: 604-623.

Subbotin, A.I. and Chentsov, A.G. (1981). *Optimization of guaranteed result in control problems*. Moscow.

Turetsky, V. and Shinar, J. (2003). Missile guidance laws based on pursuit-evasion game formulations. *Automatica*. 39: 607-618.

Ushakov, V.N. (1972). Extremal strategies in differential games with integral constraints. *Journal of Applied Mathematics and Mechanics*. 36(1): 15-23.

Vagin, D.A. and Petrov, N.N. (2001). Simple pursuit of rigidly connected evaders. *Izv. Ross. Akad. Nauk. TSU*. 5: 75-79

Zak, V.L. (1978). On a problem of evading many pursuers. *PMM* 43(3): 456-465.