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MATHEMATICAL ANALYSIS OF CHAOTIC BEHAVIOR IN MONETARY POLICY RULES

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MATHEMATICAL ANALYSIS OF CHAOTIC BEHAVIOR IN MONETARY POLICY RULES

By

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Chaotic behavior which is based on nonlinearity and deterministic process has captured the attention of many mathematical economists in recent years. This behavior can put limits of predictability on the future behavior of a series. Thus, detecting the chaos in economics can help the policy designer to have a better understanding about the impact of monetary policy on real economy. This thesis tries to find the chaotic behavior in an economic dynamical system by performing both theoretical and empirical investigation. For the theoretical part Hopf bifurcation theorem is used. The Existence of bifurcation ensure us the availability of being limited cycles. The results of this part show that there exists the Hopf bifurcation between the parameters of the model. Empirically this thesis used the Brock, Dechert and Scheinkman (BDS) test to detect the chaotic behavior in the

system. The BDS is a portmanteau test which can be used against a variety of possible independence including nonlinearity and chaos. Generally, this test is based on the correlation dimension, and the null hypothesis is tested that a time series data came from a data generation process that is independent and identical distribution. Asymptotically, this test statistic has a standard normal distribution under the pure whiteness hypothesis. If we remove the linear dependency and conditional heteroscedasticity from the time series data this test can show whether that data come from a chaotic data generation process. We used the quarterly data of the United State of America and the period is from 1980:1-2010:4. We employed the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test for checking the unit root and Generalized Method of Moments (GMM) estimator for estimating the coefficients of the system of equations. Autoregressive Moving Average (ARMA) is used for removing the linear relations. The main finding of the BDS test confirms the existence of chaotic behavior in the simulated outcomes (output gap) obtained from the estimated system of equations. The general conclusion drawn from this thesis indicates that the rules with feedback may create the chaotic behavior in the real world, and seems the rules without feedback can be a better choice for conducting monetary policy.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

ANALISIS MATEMATIK BAGI KELAKUAN KALUT DALAM KAEDAH DASAR KEWANGAN

Oleh

REZA MOOSAVI MOHSENI

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Kalut tingkah laku yang berdasarkan tak-linear dan proses ketentuan, telah menarik perhatian ahli ekonomi matematik banyak dalam tahun ini. Tingkah laku ini boleh meletakkan had ramalan pada siri tingkah laku masa depan. Oleh itu, mengesan keadaan kalut dalam bidang ekonomi boleh membantu pereka dasar untuk mempunyai pemahaman yang lebih baik tentang kesan dasar kewangan ke atas ekonomi sebenar. Tesis ini cuba untuk mencari tingkah laku yang kalut dalam sistem ekonomi yang dinamik dengan melaksanakan kedua-dua penyiasatan teori dan empirik. Bagi sebahagian teori Hopf, teorem dua pencabangan digunakan. Sedia ada dua pencabangan memastikan kita dengan kesediaan tentang menjadi kitaran terhad. Keputusan bahagian ini menunjukkan bahawa wujud pencabangan dua Hopf antara parameter model. Empirik tesis ini menggunakan ujian Brock, Dechert dan Scheinkman (BDS) untuk

mengesan tingkah laku yang kalut dalam sistem. BDS adalah ujian singkatan yang boleh digunakan terhadap pelbagai kebebasan mungkin termasuk tak-linear dan kalut. Secara amnya, ujian ini adalah berdasarkan dimensi korelasi, dan hipotesis yang seimbang diuji bahawa data siri masa dari proses penjanaan data yang adalah bebas dan sama rata. Secara asimptot, ujian statistik ini mempunyai taburan normal piawai di bawah hipotesis keputihan tulen. Jika kita menghapuskan pergantungan linear dan heteroskedastisiti bersyarat dari data siri masa ujian ini boleh menunjukkan sama ada data tersebut datang dari proses generasi data kalut. Kami menggunakan data suku tahunan Negeri Amerika Syarikat dan tempoh dari 1980:1 ke 2010:4. Kami mengerjakan ujian Kwiatkowski, Phillips, Schmidt dan Shin (KPSS) untuk memeriksa akar unit dan Kaedah Teritlak Moments (GMM) penganggar bagi menganggar pekali sistem persamaan. Purata Autoregresi Bergerak (ARMA) digunakan untuk menghapuskan hubungan linear. Penemuan utama ujian BDS mengesahkan kewujudan tingkah kalut dalam hasil simulasi (jurang keluaran) yang diperolehi daripada sistem anggaran persamaan. kesimpulanya daripada tesis ini menunjukkan bahawa kaedah-kaedah dengan maklum balas boleh mewujudkan tingkah laku yang kalut dalam dunia sebenar, dan seolah-olah peraturan tanpa maklum balas boleh menjadi pilihan yang lebih baik untuk menjalankan dasar kewangan.

I certify that a Thesis Examination Committee has met on **13 May 2013** to conduct the final examination of **Reza Moosavi Mohseni** on his thesis entitled "**Mathematical Analysis of the Chaotic Behavior in Monetary Policy Rules**" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master in Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledge. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

REZA MOOSAVI MOHSENI

Date: 13 May 2013

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y_t	Gap between actual from potential outputs (steady state)
π_t	Domestic inflation rate
e_t	Real exchange rate
π_t^*	World inflation rate
i_t	Short run interest rate (Policy rate)
E_t	Mathematical expectation conditioned on period t information
ξ^D	Demand side shock
ξ^S	Supply side shock
ξ^F	Foreign shock

CHAPTER 1

OVERVIEW OF THE STUDY

1.1.Introduction

Chaotic behavior has captured attention of many mathematical economists in recent years. This theory is based on two main assumptions that the system is a nonlinear and deterministic process which looks random. In other words, the process will be determined if the system has no disturbance (H. Nagashima and Y. Baba; 2002). However, the chaos theory revolutionary has shown that the external disturbance or noise may not be the only source of randomness and nonlinearity that can lead to complexity. Robert M. May (1976) in his semantic paper argued that a very simple mathematical nonlinear model can possess extraordinary rich dynamical behavior. As we know, this behavior can put limits of predictability on the future behavior of the process from past history. In such a system, history can be irrelevant. In other words, our process is not time-dependent. But if we can discover the chaotic behavior in a system, it could be possible to predict the system at least in the short run. Due to the instability of chaotic process the long prediction is impossible. If the dynamics of the process is chaotic the long run prediction is impossible even if the structure of the model is completely known (W. J. Baumol and R. E. Quandt; 1985, W. J. Baumol and J. Benhabib; 1989, G. P. Decoster and D. W. Mitchell; 1991). For instance, it warns the monetary policy maker that the random behavior may not be random at all.

Historically, the chaos theory dates back over 120 years to the work of Henri Poincare (R. Devaney; 1992). But it started in 1960 When Edward Lorenz created his numerical atmospheric turbulence model at the MIT. The Lorenz works showed that complex system often seems to run through some kind of cycle. The basic theoretical explanation on chaotic dynamics and strange attractor were provided by Smale (1967) and in the early 1970s by Ruelle and Takens (1971). Nowadays, this theory is a branch of mathematics that studies the complex dynamic systems. The complex systems are the systems that contain so many variables that motion, and computers are needed to determine all the various possibilities. That is why the chaos theory could not have emerged before the second half of the 20^{th} century.

In 1989 Devaney presented a definition of chaos which is probably the most popular in mathematics textbooks. He proposed three properties for a chaotic behavior (J. Banks, V. Dragan and A. Jones; 2003)[2]:

Definition: Let V be an interval. We say that f: $V \to V$ is chaotic on V if:

i. f has sensitive dependence on initial condition.

ii. f is transitive.

iii. Periodic points are dense in V.

Devaney (1992) argued that many of the most important processes in nature are nonlinear, and dynamical system has a long history as a branch of mathematics. On the other hand, chaos theory as an emerging theory has been considered in various field of study such as biology, physics and economics. Broadly speaking, chaos can be defined

[2] This thesis will consider the definition of chaos in the sense of Devaney.

as “stochastic behavior occurring in a deterministic system” (Royal Society, London, 1986).

1.2.Problem Statement

In the most of economic models we accepted that the external noise is the source of the randomness and volatility in the behavior of the system, but the chaos revolution has shown that we can have another source for this behavior. In other words, in econometric analysis with a linear systems, the stochastic disturbances at least in some cases be inadequate and may be nonlinear system can be more appropriate. Where chaos occurs economic forecasting becomes extremely difficult (Baumol and Quandt; 1985), and puts limits of predictability of the future behavior from the past history. This theory can provide difficulties for both policy designer and economic analyst.

In recent years there has been a growing interest in the search for evidence of nonlinear dynamics, and in particular chaos, in economic data (Airaudo and Zanna; 2012, Park and Whang; 2012, Barkulas; 2008, Barnett and Duzhak; 2008, Grandmont; 2008, Serletis and Shintani; 2006, Kyrtsou and Serletis; 2006, Shintani and Linton; 2003, Benhabib et al; 2002, Michener and Ravikumar; 1998, Kaas; 1998, Serletis; 1996, Decoster and Mitchell; 1992, Scheinkman; 1990 and many others). As we know if the dynamics are chaotic, then at least the long term prediction is impossible even if the correct economic system is known. On the other hand, only short-term predictability is possible. If this nonlinearity is indeed present, it is important to identify the source of this nonlinearity (DeCoster and Mitchell; 1991, DeCoster and Mitchell; 1992).

However, the existence of the chaotic behavior can provide serious difficulties and probable misunderstanding about the conducting and effectiveness of monetary policy for both policy designer and the public, so discovering the possibilities of occurrence of this behavior can help to the policy maker.

All in all, the evidence from the past research for the existence and importance of nonlinearity and chaos in economic behavior may suggests further research in this field of study especially the policy outcomes.

1.3.Purpose of the Study

After formulating a stochastic macroeconomic dynamic system, the main objective of this study is to detect the chaotic behavior between two different important monetary policy rules called Taylor rule and inflation targeting rule.

Therefore the specific purpose of this study can be stated in two parts:

i. Theoretically finding the Hopf bifurcation in an open economic model

ii. Discover the possible occurrence of chaotic behavior in the outcome of economic model.

1.4.Significance of the Study

Searching for chaotic behavior in monetary data is important for three reasons. First, it is important to know if the outcome's behaviors of different types of monetary policy are chaotic. Second, if it shows nonlinearities, the behavior of linear stochastic models that

we use in economics cannot be employed for forecasting. Third, the results of this study can help the monetary policy designer to have a better understanding about the effectiveness of monetary policy.

1.5.Scope of the Study

The empirical scope country of this study is the United State of America. The data set used in this study is quarterly and from the first quarter of 1980 to the fourth quarter of 2010. The data are collected from the data stream software.

1.6.Organization of the Study

The rest of this thesis is organized as follows. The next chapter describes the academic background. On the other words, this chapter analyzes the semantic previous academic literatures of this subject. Chapter three presents a model with two different types of monetary policy rule: Taylor rule and inflation targeting. The next chapter describes the econometric and mathematical methodology. Estimating and evaluating the macroeconomic model, simulating of the outcomes of different types of monetary policy rules and detecting Hopf bifurcation and chaotic dynamics with the Brock, Dechert and Scheinkman's independence test are presented in chapter five. Finally, chapter six expresses the summery and the conclusion remarks.

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