UNIVERSITI PUTRA MALAYSIA

ADAPTIVE CONTROL SCHEME FOR ANTI- SYNCHRONIZATION OF CHAOTIC DYNAMICAL SYSTEMS

MAHMOUD MAHERI

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By

MAHMOUD MAHERI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

November 2016

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DEDICATIONS

This thesis is dedicated to:
The soul of my late grandmother Roghayyeh
My mother Ashraf
My uncle Hossein
My dear son Arash

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

ADAPTIVE CONTROL SCHEME FOR ANTI- SYNCHRONIZATION OF CHAOTIC DYNAMICAL SYSTEMS

By

MAHMOUD MAHERI

November 2016

Chairman: Norihan Md Arifin, PhD
Faculty: Institute for Mathematical Research

Due to physical limitation in most real-world applications, the assumption on chaotic dynamical systems with identical drive and response systems is somehow unrealistic. Hence, synchronization of two different chaotic systems in the presence of unknown parameters is more essential and useful in real-world applications. Recently, several techniques have been proposed in the literature to synchronize chaotic dynamical systems. Therefore, this thesis presents chaos synchronization of a couple of chaotic systems. Based on the exponential and Lyapunov stability theory, the controller with the corresponding parameter update rules is designed such that the different chaotic systems can be synchronized asymptotically. The proposed function control is composed of both variable proportional and adaptive control actions for guaranteeing the convergence of the residual synchronization error to zero in the presence of disturbances. Three proposed chaos synchronization techniques are considered. The first technique considered chaos synchronization of two different chaotic systems with the same and different parametric perturbation by nonlinear control functions. Second technique studied an adaptive synchronization, phase synchronization and functional phase synchronization of two different chaotic systems with nonlinear control functions and the third technique, a robust adaptive nonlinear feedback controller technique is proposed to realize the synchronization between two different fractional order chaotic systems with fully unknown parameters, external disturbance and uncertainties. The proposed techniques are applied to achieve chaos synchronization for the chaotic dynamical systems We demonstrate that a coupled chaotic dynamical systems can be synchronized and numerical simulations show the effectiveness of the proposed control techniques. Moreover, as an application, a new technique for transmitting digital signals was proposed based on chaos masking using chaotic dynamical system. Also, simulation results verify the proposed technique's success in the communication application.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

SKIM KAWALAN MUDAH SUAI UNTUK ANTI PENSINKRONIAN SISTEM DINAMIK KALUT

Oleh

MAHMOUD MAHERI

November 2016

Chairman: Norihan Md Arifin, PhD
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Disebabkan oleh batasan fizikal dalam kebanyakan aplikasi dunia sebenar, andaian sistem dinamik kalut dengan sistem-sistem pemacu dan tindak balas itu walaubagaimanupun tidak realistik. Oleh itu, pensinkronian dua sistem kalut yang berbeza dengan adanya parameter tidak diketahui adalah lebih penting dan berguna dalam aplikasi dunia sebenar. Terkini, beberapa teknik telah dicadangkan dalam literatur untuk sinkroni sistem dinamik kalut. Oleh yang demikian, tesis ini mengkaji pensinkronian kalut untuk sistem kalut berpasangan. Berdasarkan teori kestabilan Lyapunov, pengawal dengan aturan terkini parameter yang sepadan direkabentuk supaya sistem kalut yang berbeza boleh disinkroni secara asimptot. Kawalan fungsi yang dicadangkan ini dibentuk untuk kedua-dua pembolehubah berkadaran dan tindakan kawalan sesuai bagi menjamin memastikan penumpuan ralat pensinkronian reja kepada sifar dengan kehadiran gangguan. Tiga cadangan teknik pensinkronian kalut dipertimbangkan. Teknik pertama mempertimbangkan pensinkronian kalut dua sistem kalut berbeza dengan gangguan parameter yang sama dan berbeza oleh fungsi kawalan tak linear. Teknik kedua mengkaji pensinkronian mudah suai, pensinkronian fasa dan pensinkronian fasa berfungsi dua sistem kalut yang berbeza dengan fungsi kawalan tak linear dan teknik ketiga, teknik kawalan suapbalik teguh tak linear mudah suai dicadangkan untuk merealisasikan pensinkronian antara dua sistem kalut peringkat pecahan berbeza dengan parameter tidak diketahui sepenuhnya, gangguan luar dan ketidaktentuan. Teknik-teknik yang dicadangkan ini diaplikasi untuk mencapai pensinkronian kalut untuk sistem dinamik kalut. Kami tunjukkan sistem dinamik kalut berpasangan boleh disinkroni dan simulasi berangka menunjukkan keberkesanan teknik kawalan yang dicadangkan. Selain itu, sebagai applikasi, satu teknik baru untuk penghantaran isyarat digital telah dicadangkan berdasarkan pada penopengan kalut dengan menggunakan sistem dinamik kalut. Juga, keputusan simulasi mengesahkan kejayaan teknik yang dicadangkan dalam aplikasi komunikasi.

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I certify that a Thesis Examination Committee has met on 29 November 2016 to conduct the final examination of Mahmoud Maheri on his thesis entitled "Adaptive Control Scheme for Anti-Synchronization of Chaotic Dynamical Systems" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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LIST OF ABBREVIATIONS

FOS	Fractional Order System
ODEs	Ordinary Differential Equations
DS	Dynamical System
ABM	Adams-Bashforth-Moulton
RK4	Fourth-order Runge-Kutta Method
OPF	Occasional Proportional Feedback
TDFB	Time Delay Feedback
OGY	Ott- Grobgi- Yorke
OPF	Occasional Proportional Feedback
RKF	Runge-Kutta-Fehlberg
TV	Time Variant
LTI	Linear Time Invariant
FPS	Functional Phase Synchronization
RSA	Rivest- Shamir- Adlemen
DES	Data Encryption Standard
GPS	Generalized Projective Synchronization
CS	Complete Synchronization
LMI	Linear Matrix Inequality

CHAPTER 1

INTRODUCTION

Dynamical system is certain rule that explains the position of each point in the phase space over the time which is used in the modeling of both natural and technological sciences. If a model of a practical problem has been explained by a dynamical system, it is possible to predict future status of it by knowing the current position of the system in a particular moment.

Chaotic systems are special cases of nonlinear dynamical systems that are highly sensitive to their initial conditions. In the recent years, chaos synchronization is widely used as an important topic in the discussion of nonlinear dynamical systems. Control and synchronization of chaotic dynamical system are playing a significant role in the study of applied sciences such as communications, cryptography, biology, economics and so on.

This chapter first gives the basic concepts of continuous dynamical systems such as fixed point, eigenvalues, linear stability, phase space in linear systems, chaos, bifurcation and Lyapunov exponent. Then, after a brief description of chaos in the classical form by an example, objectives and outline of this thesis is discussed.

1.1 Basic concepts of dynamical systems

Generally a dynamical system is described by:

$$\dot{x} = f(x), \tag{1.1}$$

or

$$\dot{x} = f(x,t), \tag{1.2}$$

where $x = (x_1, x_2, \cdots, x_n) \in R^n$ is state variables vector. Systems (1.1) and (1.2) are called autonomous and non-autonomous respectively (Guckenheimer and Holmes, 2013).

A suitable method for describing solution of the differential equation is to find an explicit formula for the answer. However, generally it is not possible to find such solutions. But there are other methods to describe the solution such as study of the systems behavior near fixed points.

1.1.1 Fixed points and stability

Assume the following nonlinear differential equation:

$$\dot{x} = \frac{dx}{dt} = f(x). \tag{1.3}$$

Definition 1.1 *Suppose $D \subseteq \mathbb{R}^{n+1}$, it is said that $f \in C(D)$ if f is a continuous function and if f have continuous derivatives in order $k > 0$, it is called $f \in C^k(D)$ (Guckenheimer and Holmes, 2013).*

Theorem 1.1 *[Existence and uniqueness condition:] Suppose D is an open subset in R^{n+1} include x_0. If $f \in C(D)$ then initial value problem*

$$\dot{x} = f(t,x), \quad ;x(0) = x_0,$$

has solution and if $\frac{\partial f}{\partial x} \in C(D)$, then the solution is unique on I (Perko, 2013).

Remark 1.1 *In theorem 1.1, I is the biggest interval that the initial value problem has solution in it. Interval I is called maximal interval.*

Definition 1.2 *In system (1.3), a point x_0 is called fixed point (critical point) if*

$$\dot{x} = f(x_0) = 0,$$

which plays an important role in the survey of system's behavior. If a trajectory starts from these points, remains at there forever (Perko, 2013).

Definition 1.3 *A fixed point x_0 in system (1.3) is called singular if there exist a neighborhood of point x_0 so that x_0 is only fixed point in that neighborhood (Perko, 2013).*

Definition 1.4 *A fixed point x_0 of system (1.3) is called stable if for any neighborhoods N, there exist smaller neighborhood $N' \subseteq N$ of x_0 so that, when each trajectory of system entered to N', remain in N for all time t, Figure 1.1 (Perko, 2013).*

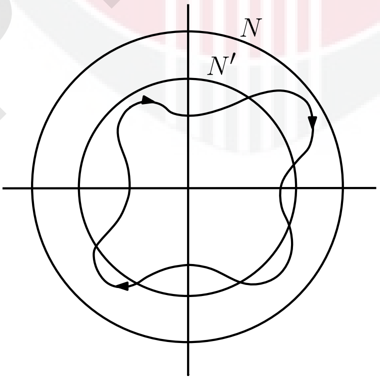

Figure 1.1: Neighborhoods N and N' of definition 1.4 in origin at R^2.

Definition 1.5 *A fixed point, x_0 of system (1.3), is called asymptotically stable if (Guckenheimer and Holmes, 2013):*

(i) It is stable.

(ii) There exists a neighborhood N' of x_0 where for each trajectory entered to N', tends to x_0 by increase of t, see Figure (1.2).

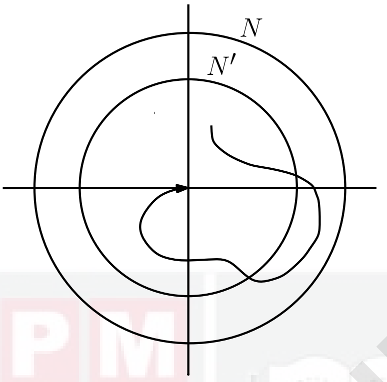

Figure 1.2: Neighborhoods N and N' in definition (1.5) in origin at R^2.

Definition 1.6 *A fixed point x_0 of system (1.3) is called center stable if is stable but does not satisfy definition 1.5 (Guckenheimer and Holmes, 2013).*

Definition 1.7 *A fixed point x_0 of system (1.3) is called unstable if it not is stable, see Figure (1.3) (Guckenheimer and Holmes, 2013).*

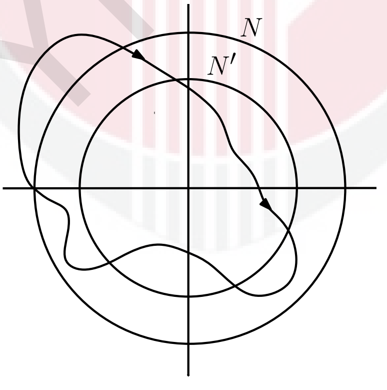

Figure 1.3: Neighborhoods N and N' at definition 1.7 in origin at R^2.

Remark 1.2 *Fixed points of system (1.3) based on the behaviors near these points are classified in three types, attractive, repulsive and transit points.*

Theorem 1.2 *If x_0 be fixed point of system (1.3) then (Teschl, 2012)* *(i) It is called attractive point if* $\frac{df}{dx}(x_0) < 0$.

(ii) It is called repulsive point if $\frac{df}{dx}(x_0) > 0$.

(iii) If $\frac{df}{dx}(x_0) = 0$ *it is necessary to use higher order derivatives to determine the nature of the point.*

1.1.2 Phase space of linearized system

Most of the nonlinear systems locally, are equivalent to their linear systems, so by knowing the characteristics of linear system it is possible to identify properties of a wide range of nonlinear systems (Teschl, 2012).

Qualitative behavior of linear systems leads to the classification finite numbers of phase space for the corresponding non-linear systems. Extension of these techniques is useful in the study of qualitative behavior of nonlinear systems. Consider linear system,

$$\dot{x} = f(x) = Ax, \tag{1.4}$$

where A is a nonsingular matrix. This system, by changing variable $x = Py$, it's canonical system

$$\dot{y} = Jy,$$

where A is coefficient matrix and J is the Jordan canonical form of matrix A such that $J = P^{-1}AP$. In fact, the columns of P form a basis in R^n and y is the coordinates of x respect to this new base. Since the matrices A and J are similar, the solutions of the system $\dot{x} = Ax$ and $\dot{y} = Jy$ are related together by relations

$$x = Py, \quad P^{-1}AP = J. \tag{1.5}$$

Thus, if one of these systems is solved, the solutions of the other class is achieved (Teschl, 2012).

Definition 1.8 *Each answer of the system (1.4), namely* $\phi(t) = (x(t), y(t))$, *displays by a curve in plane. These curves is called phase space, orbit or trajectory (Guckenheimer and Holmes, 2013).*

Definition 1.9 *The phase portrait is a two dimension figure that shows system qualitative behavior respect to* x_1 *and* x_2 *by changing variable t (Guckenheimer and Holmes, 2013).*

Theorem 1.3 *Jordan matrix that appear in the transformation of system (1.4) to its canonical form has one of these four cases,*

$$J_1 = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \quad \lambda_1 > \lambda_2; \qquad J_2 = \begin{bmatrix} \lambda_0 & 0 \\ 0 & \lambda_0 \end{bmatrix},$$

$$J_3 = \begin{bmatrix} \lambda_0 & 1 \\ 0 & \lambda_0 \end{bmatrix}; \qquad J_4 = \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix}, \quad \beta > 0$$

where α, β, λ_0, λ_1 and λ_2 are real numbers (Arrowsmith and Place, 1992).

Consider system (1.4) where λ_1 and λ_2 are roots of the characteristic polynomial of matrix A. Assume $\dot{y} = Jy$ is the corresponding canonical system for (1.4). The nature of fixed point $(0,0)$ of the simple canonical form is depended to the nature of eigenvalues λ_1 and λ_2. For classification of eigenvalues see appendix A.

Proposition 1.1 *The fixed point $(0,0)$ of system $\dot{x} = Ax$:*

- *is asymptotically stable if the real part of the eigenvalues in the characteristics polynomial A is negative.*
- *is center stable if the eigenvalues are pure imaginary.*
- *is unstable if at least one the eigenvalues are positive (Brockett, 2015).*

Definition 1.10 *Assume that the origin is the fixed point of system (1.4). In this case, the stable and unstable manifold of the origin respectively are shown by $E_s(0)$ and $E_u(0)$ which are determined by eigenvectors of matrix A (Brockett, 2015).*

1.1.3 Linearization nonlinear systems at fixed points

Phase space of a linear system is determined by the nature of the fixed points. In most cases, it is possible to approximate nonlinear systems with their corresponding linear system near the fixed points. Nonlinear systems, can have more than a fixed point and it is possible to obtain local phase space of all of them. However, local phase spaces, always can not display the global phase spaces nature. In other words, unlike linear systems, phase spaces of non linear systems can not specify by the nature of their fixed points. in this part, the linearization process is explained on two-dimensional systems that can extend to higher orders systems easily (Sastry, 2013).

Assume (ξ, η) are the fixed points of the nonlinear system

$$\begin{cases} \dot{x_1} = f_1(x_1, x_2), \\ \dot{x_2} = f_2(x_1, x_2) \end{cases} \tag{1.6}$$

and suppose the local coordinate $y_1 = x_1 - \xi$ and $y_2 = x_2 - \eta$. As a result, we have

$$\begin{aligned} \dot{y_i} &= f_i(y_1 + \xi, y_2 + \eta) \\ &= f_i(\xi, \eta) + y_1 \frac{\partial f_i}{\partial x_1}(\xi, \eta) + y_2 \frac{\partial f_i}{\partial x_2}(\xi, \eta +) + R_i(y_1, y_2), \;\; i = 1, 2, \end{aligned} \tag{1.7}$$

where the function R_i satisfies the relation

$$\lim_{(y_1, y_2) \to (\xi, \eta)} \frac{R_i(y_1, y_2)}{\sqrt{y_1^2 + y_2^2}} = 0.$$

Since (ξ,η) is the fixed point of system (1.6), so $f_i(\xi,\eta)=0$. By ignoring nonlinear expressions, the resulting linearization of system (1.6) at point (ξ,η) is given by $\dot{y}=Jy$ where

$$J=\begin{bmatrix}\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2}\\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2}\end{bmatrix}_{(x_1,x_2)=(\xi,\eta)}$$

is the Jacobian matrix of system (1.6).

Theorem 1.4 (Hartman linearization theorem) *Assume that (ξ,η) is a hyperbolic fixed point of nonlinear system $\dot{x}=f(x)$. In this case, in a neighborhood of this point, the phase space of nonlinear system and the linearized system are qualitative equivalence (Hartman, 1964).*

Remark 1.3 *Stable and unstable manifolds of a non linear system is given respectively by W_s and W_u. Hartman theorem guarantees that W_s and W_u are tangent respectively on E_s and E_u at fixed point.*

1.2 Chaos theory

Chaos chaos theory, refers to an apparent lack of arrangement in a system that nevertheless obeys particular law of rules. This concept of chaos is synonymous with dynamical instability, a condition discovered by the physicist Henri Poincare (Miller and Ross, 1993) in the early 20th century that refers to an inherent lack of predictability in some physical systems.

The main components of chaos theory is the idea that very simple or small changes and events can cause very complex behaviors or events. This latter idea in known as sensitive dependence on initial conditions, a condition discovered by Edward Lorenz (who is generally credited as the first experimenter in the area of chaos) in the early 1960s (Lorenz, 1993). Chaos theory successfully can describe many phenomena in various disciplines branches such as engineering, economics, science, biology, meteorology, astrophysics, management and so on.

This part, first explains the detection methods of a chaotic system including bifurcation, sensitivity to initial conditions and Lyapunov exponent. Then, these concepts are applied to the Lorenz system.

1.2.1 Bifurcation

Most of mathematical models in addition to variables have parameters. Due to various applications of a model, parameters can have different values. For example, suppose the exponential growth model represented by $\dot{x}=ax$, where a is the parameter for growth. Value of a for rabbits is much higher than that for humans.

Study on changes in the behavior of a dynamic system by changing its parameters, is one of the topics studies in the dynamical systems. If the behavior of the system is changed suddenly by changing the parameters, it is called a bifurcation occurs in the system.

The point where the bifurcation occurs, is called the bifurcation point. At one bifurcation point the number of fixed points or its stability can change. In most applied problems, there do not exist exact values of the parameters and there are only an approximation of them. For having useful modeling for such problems, it is necessary to evaluate the effect of small changes in the value of parameters on the behavior of the system. In a chaotic system, by changing the parameters of bifurcation, different behaviors occur in the system (Morin et al., 2003).

Definition 1.11 *A vector field f, $f \in C^1(D)$, is called structurally stable if a small change in the system $\dot{x} = f(x)$, does not change the qualitative behavior of it. If a small change in the system change the qualitative behavior of the system, it is called structurally unstable (Kuznetsov, 2013).*

Theorem 1.5 (Peixoto theorem) *Assume that the vector field f is continuous differentiable on the compressed set $D \subseteq R^2$. In this case, f on D is structurally stable if and if (Peixoto, 1962)*

- *The number of fixed point and limit cycles are finite, and all fixed points are hyperbolic;*
- *There do not exist any orbit to connect two saddle points to each other.*

Definition 1.12 *Assume that a system as $\dot{x} = f(x,\mu)$, where $f \in C^1(D)$, $x \in D \subseteq R^2$ and $\mu \in R$. The value μ_0, where $\dot{x} = f(x,\mu_0)$ is structurally unstable, is called bifurcation value (Kuznetsov, 2013).*

These concepts are explained on the Lorenz system in the next section.

1.2.1.1 Sensitivity to initial conditions

When a chaotic system is in the chaos area, the smallest change in the initial value case system show quite different behavior after a long time. Consider two different answers in a certain system that are in the chaotic region and the difference is only in the initial values. These two solutions, over time diverge from each other, while both are remain in the bounded area of the phase space. In the chaotic systems two orbits are separated exponentially from each other with close initial conditions (Willems and Polderman, 2013). This behavior also is shown in the Lorenz system in the next section.

1.2.2 Lyapunov exponent

Lyapunov exponents have played a key role in the studying of the behaviors in chaotic systems. These exponents measure the mean rate of divergence and convergence of orbits which started together from a very closed initial points. Hence, the Lyapunov exponents can apply to analysis the stability of a system and for the assessment of the sensitivity to initial conditions to find chaotic behavior or existing strength attractors.

Assume the points x_0 and $x_0 + u_0$ that are on two closed paths of phase space of a continuous system where u_0 is a small perturb on x_0 as shown in Figure 1.4. After time t,

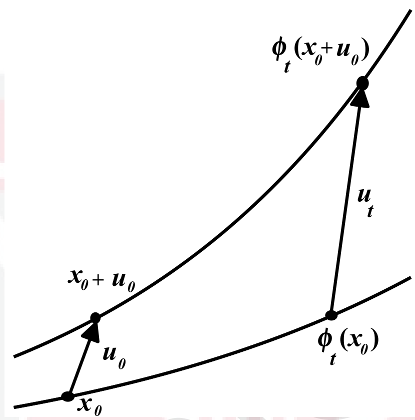

Figure 1.4: Divergence or convergence of two close path

projection of these points will change to $\phi_t(x_0)$ and $\phi_t(x_0 + u_0)$ under the effect of flow. Difference u_t is

$$u_t = \phi_t(x_0 + u_0) - \phi_t(x_0) = D_{x_0}\phi_t(x_0).u_0,$$

where the right hand expression comes from the linearization of ϕ_t around x_0. Assume after the time t, distance of two paths is

$$\| u_t \| = \| u_0 \| e^{Lt}.$$

In this case, the exponent rate of divergence is defined as

$$L = \lim_{t\to\infty} \frac{1}{t} \ln \frac{\| u_t \|}{\| u_0 \|},$$

where $\| . \|$ is the length of vector. Determination of L (Lyapunov exponent) is possible when the right hand of equation (1.8) is exist. Selecting base e is an appropriate choice but arbitrary.

The divergence rate of orbits that represents the chaotic behavior, is only measurable locally. When a system is bounded, over time t, u_t can not tend to infinity. So, to have an appropriate criteria for measure of the divergence of these paths, it is necessary to use average of numerous points for the calculation of it. Using the results of these exponents can help to discern between the fixed points, quasi periodic or chaotic motions. If axis coordinates and theirs correspond Lyapunov exponent classify in descending order as $\varepsilon_1 \geq \cdots \geq \varepsilon_n$ and $\lambda_1, \geq, \cdots, \lambda_n$, each $\lambda_i,\ i = 1, 2, \ldots, n$, shows average exponential rate of divergence for axis ε_i. So, the number of Lyapunov exponents have to be equal with the phase space dimension (Teschl, 2012).

1.3 Chaotic Lorenz system

The first numerical investigation which led to the introduction of chaos is presented by Edward Norton Lorenz in 1960 (Marsden and McCracken, 2012). He was a meteorologist and tried to model and solve weather convention as shown in Figure 1.5. He introduced

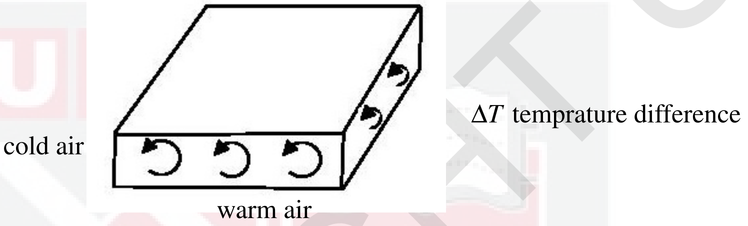

Figure 1.5: A layer of material fluid is heated from below.

the nonlinear autonomous systems which is called the Lorenz system:

$$\begin{cases} \dot{x}_1 = \sigma(-x_1 + x_2), \\ \dot{x}_2 = rx_1 - x_2 - x_1x_3, \\ \dot{x}_3 = -bx_3 + x_1x_2, \end{cases} \tag{1.8}$$

where the parameters σ, r and b are positive real number. The variable x_1 is the intensity of fluid motion and x_2 and x_3 respectively show the temperature changes horizontally and vertically. Parameters σ and b depend on the geometric and material properties of fluid layer. Lorenz observed that simulations result is quite different due to the very small changes in initial conditions. He published a paper in a meteorology journal that paved the way to start new investigation on the chaotic dynamical system. For $\sigma = 10$, $b = \frac{8}{3}$ and $r = 28$ the system is chaotic as shown in Figure 1.6.

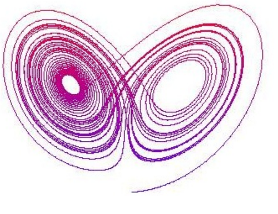

Figure 1.6: Lorenz chaotic dynamical system

For of detection chaotic behavior in a system, there are some known methods such as bifurcation, invariant manifold, Poincare section and more importantly Lyapunov exponent. In the next part bifurcation and Lyapunov exponent are explained on the Lorenz system.

1.3.1 Bifurcation in the Lorenz system

In this part the qualitative behaviors on the Lorenz system (1.8) are explained based on the concept of bifurcation (Arnold et al., 2013).
Lorenz system (1.8) properties are:

- **Symmetry**: There is no changes in the equations of the Lorenz system by transformation $(x_1,x_2,x_3) \to (-x_1,-x_2,x_3)$, so if (x_1,x_2,x_3) be a solution, then $(-x_1,-x_2,x_3)$ is also a response.

- **Invariant manifold x_3:** When $x_1 = x_2 = 0$, then $\dot{x}_1 = \dot{x}_2 = 0$ and $\dot{x}_3 = -bx_3$. So on axis x_3, all trajectories go to origin that means x_3 is an invariant manifold.

- **Damping:** Lorenz system is damped in other words, the volume of phase space is shrink towards zero over time. By using divergence theorem, where v is for volume:

$$\dot{V} = \int_V \triangledown .XdV = \int_V \text{div}XdV \tag{1.9}$$

  For Lorenz system, $\text{div}X = -(\sigma+b+1)$ is fixed and

$$\dot{V} = -(\sigma+b+1)\int_V dV = -(\sigma+b+1)V.$$

  So $V(t) = V(0)e^{-(\sigma+b+1)t}$ results in phase space all volume shrink to $V(0)e^{-(\sigma+b+1)t}$, and all paths in the Lorenz system go to a set with zero volume. In a damped system, orbits go to fixed points, limit cycles or strong attractors.

- **Fixed points:** For $0 \leq r < 1$, the origin is the only fixed point of the Lorenz system. For $r > 1$, in addition of the origin there are two other fixed points,

$$C_{\pm} = (\pm\sqrt{b(r-1)}, \pm\sqrt{b(r-1)}, r-1).$$

  That when $r \to 1^+$, $C_{\pm}$ go to the origin and there is a bifurcation in $r = 1$.

- **Linear stability of origin:** linearized Lorenz system is given by

$$\begin{cases} \dot{x}_1 = \sigma(x_2 - x_1), \\ \dot{x}_2 = rx_1 - x_2, \\ \dot{x}_3 = -bx_3. \end{cases} \tag{1.10}$$

System (1.10) shows that x_3 over the time will go to zero and the eigenvalues that come from the system from the:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -\sigma & \sigma \\ r & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \tag{1.11}$$

are $\lambda_{\pm} = \frac{1}{2}\Big(-(\sigma+1) \pm \sqrt{(\sigma+1)^2 - 4\sigma(1-r)}\Big)$. For $0 \le r < 1$ the eigenvalues are negative so the origin is a stable node. For $r > 1$, $\lambda_+ > 0$ and $\lambda_- < 0$ so the origin is saddle point.

- **Global stability:** For $r < 1$, tha Lorenz path converges to the origin so it is stable and there is no limit cycle or strong attractor, using Lyapunov function, $V(x_1,x_2,x_3) = x_1^2 + \sigma x_2^2 + \sigma x_3^2$, $\sigma > 0$, derivative along Lorenz trajectory is

$$\begin{aligned} \dot{V} &= -2\sigma(x_1^2 + x_2^2 - (1+r)x_1x_2) - 2\sigma b x_3^2 \\ &= -2\sigma(x_1 - \frac{r+1}{2}x_2)^2 - 2\sigma(1 - \Big(\frac{r+1}{2}\Big)^2)x_2^2 - 2\sigma b x_3^2, \end{aligned}$$

  that is strictly negative, so it is stable asymptomatically.

- $C_{\pm}$ **stability:** For $r > 1$ the origin is unstable. Linearizion at two other fixed points $C_{\pm}$ described by

$$\begin{aligned} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} &= \begin{pmatrix} -\sigma & \sigma & 0 \\ 1 & -1 & \mp\sqrt{b(r-1)} \\ \pm\sqrt{b(r-1)} & \pm\sqrt{b(r-1)} & -b \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \\ &= A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \end{aligned} \tag{1.12}$$

  and the characteristic polynomial is:

$$\lambda^3 + (1+b+\sigma)\lambda^2 + b(\sigma+r)\lambda + 2b\sigma(r-1) = 0.$$

  Suppose $\sigma - b - 1 > 0$, so the fixed point is stable if

$$1 < r < r_H = \sigma\left(\frac{\sigma+b+3}{\sigma-b-1}\right),$$

where the parameters values are $\sigma = 10$, $b = \frac{8}{3}$ and $r_H \approx 24.74$. For $1 < r < r_1 = 1.3456$ all eigenvalues are real and negative. For $r_1 < r < r_H$, there are one real negative and two complex eigenvalue with negative real part, therefore, for $1 < r < r_H$ real parts is negative and fixed points $C_{\pm}$ are stable. In $r = r_H$ there exist one negative eigenvalue and two complex. For $r > r_H$ there are one negative real number and two complex with positive real part. So for $r > r_H$ fixed points $C_{\pm}$ are unstable that result to have bifurcation in $r = r_H$.

Chaos in the Lorenz system: For $r > r_H$ all fixed points are unstable so the Lorenz trajectory goes to infinity over the time. But the Lorenz system is damping therefore for $r > 0$ finally it approaches to zero volume, for $r > r_H$ and it has strong attractor. Anyway all trajectories stay at limit area of phase space (Marsden and McCracken, 2012).
It is possible to show the behavior of Lorenz system using bifurcation graph. Figure 1.7 shows the result for $\sigma = 10$, $b = \frac{8}{3}$, $0 < r < 30$ while initial condition is $(x_1(0), x_2(0), x_3(0)) = (2, 3, 5)$.

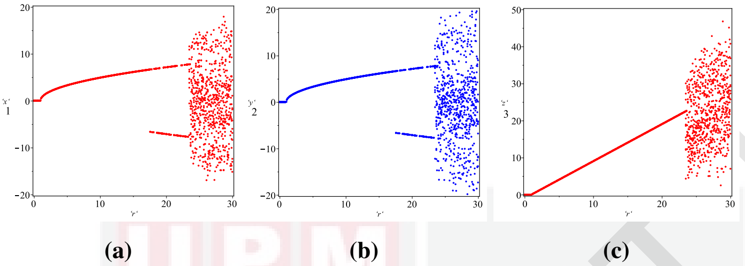

Figure 1.7: Bifurcation diagram of the Lorenz system for $0 \leq r \leq 30$: (a) variable x_1; (b) variable x_2 and (c) variable x_3.

1.3.2 Sensitivity to initial conditions

When a system is in the chaotic area, a slightest change in the initial value causes to completely different behavior in the system. For Lorenz system, the effect of small changes in initial condition confirms chaotic behavior of the system. For initial conditions $x_1(0) = x_2(0) = x_3(0) = 1$ and $x_1(0) = 1.0001, x_2(0) = x_3(0) = 1$, two different behavior is shown in Figure 1.8.

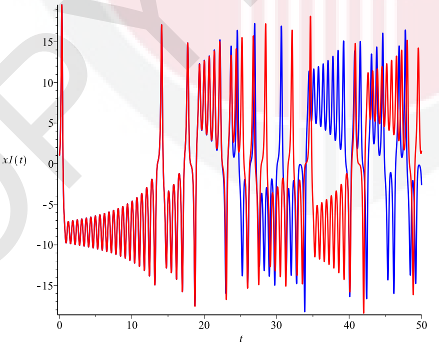

Figure 1.8: Time series of $x_1(t)$ for the Lorenz system, shows sensitivity to initial condition $x_1(0) = x_2(0) = x_3(0) = 1$ and $x_1(0) = 1.0001, x_2(0) = x_3(0) = 1$

1.3.3 Lyapunov exponent

For Lorenz system, for initial conditions $(x_1(0), x_2(0), x_3(0)) = (1,1,1)$ and parameters $\sigma = 10$, $b = \frac{8}{3}$, $r = 28$, Lyapunov exponents are

$$L_1 = 0.9057, L_2 = 0.0005, L_3 = -14.5729,$$

Figure 1.9 shows Lyapunov exponent for the Lorenz system.

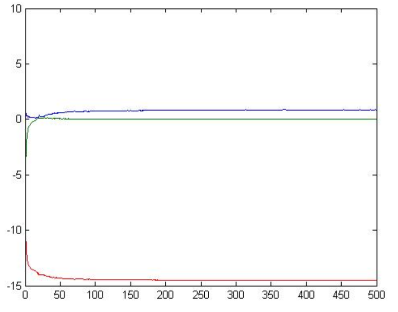

Figure 1.9: The Lyapunov exponent of Lorenz system for 0$\leq r \leq 30$.

1.4 Fractional order systems

Theory of Fractional order systems have been an attractive and important field over last decade. Recently a lot of forward movement have been done in both control and the calculus of fractional order systems. A called fractional order system means the order of system is no longer an integer. The operator D^{α} is used to show fractional order integration and derivations, where α can be positive, negative or zero.

The idea of derivative extension $\frac{d^p f(x)}{dx^p}$ to non-integral orders p, was proposed by Leibniz for $p = 1/2$ on 1695 and Euler introduced this derivative type to exponential function x^{α} on 1738. In 1832, Liouville introduced his first description for functions with exponential series which are indicated as

$$f(x) = \sum_{n=0}^{\infty} c_n e^{a_n x}, \tag{1.13}$$

in form

$$D^{\alpha} f(x) = \sum_{n=0}^{\infty} c_n a_n^{\alpha} e^{a_n x}, \tag{1.14}$$

and

$$D^{-\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t) dt, \tag{1.15}$$

which, then was known as Liouville formula to fractional integral (Aghababa, 2013).

In 1892, the idea of fractional derivative of analytical function was posed as (Herrmann, 2014)

$$D^{\alpha}f(z) = \sum_{k=0}^{\infty} \frac{\Gamma(k+1)}{\Gamma(-\alpha+1+k)} c_k (z-z_0)^{k-\alpha}, \;\; c_k = \frac{f^k(z_0)}{k!}. \tag{1.16}$$

Taylor series in the paper of Hadamard who described it as

$$I^{\alpha}f(x) = \frac{z^{\alpha}}{\Gamma(\alpha)} \int_0^1 (1-\tau)^{\alpha-1} f(z\tau)d\tau. \tag{1.17}$$

Later, this subject changed into idea to describe fractional integral as $\int_0^1 \nu(t)f(z\tau)d\tau$. Of course, Hadamard could not expand it and then on 1968, this work was performed by Dzherbashyan (Gorenflo et al., 2014).

By development of mathematic analysis and functions theory, as mathematician, Wiley, described fractional integral for periodic functions and defined it by (Herrmann, 2014)

$$I^{\alpha}_{\pm}\varphi = \frac{1}{2\pi} \int_0^{2\pi} \psi^{\alpha}_{\pm}(x-t)\varphi(t)d(t) \tag{1.18}$$

and then demonstrated as

$$I^{\alpha}_{+}\varphi = \frac{1}{\Gamma(\alpha)} \int_{-\infty}^{x} \frac{\varphi(t)dt}{(x-t)^{1-\alpha}} d(t), \tag{1.19}$$

$$I^{\alpha}_{-}\varphi = \frac{1}{\Gamma(\alpha)} \int_{x}^{\infty} \frac{\varphi(t)dt}{(x-t)^{1-\alpha}} d(t). \tag{1.20}$$

where $0 < \alpha < 1$, which nowadays is known as fractional right & left integrals. He also proved that, function $f(x)$ has continuous derivative by order α, if Lipschitz is in order $\alpha < \lambda$. The similar theory for non-periodic functions was posed by Montel (1918) and the theorem of mean value for fractional integral was described by Riese on 1922 as well as other theorems were proved by Hardy and Littlewood (1931) for fractional calculus. On 1938, the improper fractional integral was described by Love for functions which dont defined in infinite as

$$I^{\alpha}_{+}\varphi = \frac{1}{\Gamma(\alpha)} \lim_{n\to\infty} \int_0^{n} \varphi(x-t)^{\alpha-1} d(t). \tag{1.21}$$

And on same year, the partial method to fractional integral was posed by Love & Young as

$$\int_a^b (D^{\alpha}_{a} f)(x)g(x)dx = \int_a^b f(x)(D^{\alpha}_{b} g)(x)dx. \tag{1.22}$$

The novel definition was posed by Love for fractional derivative as

$${}^{c}D^{\alpha}_{t} f(t) = \frac{1}{\Gamma(\alpha-n)} \int_a^t \frac{f^{(n)}(\tau)d\tau}{(t-\tau)^{\alpha-n+1}}, \tag{1.23}$$

for $n-1 < \alpha < n$ which nowadays is known as Caputo derivative (Baleanu et al., 2016).

1.5 Problem statement and objectives

Chaos control and synchronization have been studied extensively in recent years, but, there are many more problems in this area needed to be solved that motivated me for carrying research on this thesis and applying synchronization in secure communication.

Almost most recent studies have generally focused on classic chaotic dynamical systems such as the Lorenz system, Rossler and the Chua's circuit system, among that, very low has been achieved for exponential stability in synchronization. In addition, the study of chaos synchronization of fractional order systems are more complicated and have a lot of application in most interdisciplinary sciences such as electromagnetic with great application in secure communication.

Based on the literature, in synchronization, all mentioned previous works they just used Lyapunov stability theorem for synchronization as well as on classical non-linear systems. Also, in projective synchronization almost all of the mentioned methods are applied on partially linear systems. In this study we applied synchronization to control chaotic systems using exponential stability which used to synchronize on a chaotic system without any linear term. Also we applied phase synchronization in adaptive chaos while scaling factor is a arbitrary function.

To our best knowledge, most of the previous works in the literatures, which have been proposed to stabilize or synchronize fractional non-autonomous chaotic systems, either have not considered the effects of unknown nonlinear terms, model uncertainties, and external disturbances or are sometimes specific and multi-input. Motivated by the above discussions, this thesis proposes a novel fractional-order nonlinear mode controller for robust stabilization-synchronization of second-order fractional non-autonomous chaotic systems in the presence of both model uncertainties and external disturbances. After introducing a novel terminal fractional-order nonlinear controller, its stability is proven. Then, on the basis of fractional-order Lyapunov stability theory, a robust nonlinear control law is derived to guarantee the occurrence of the stability in a given finite time. The proposed control law is single and practical in real world applications. Also adaptive control schemes for synchronization fractional is applied to encryption both text and image signals.

In application of chaos, all studied method, for application in secure communication, just applied modulation and masking method on active systems in classical systems to sent and received text signal. We applied modulation and masking schemes for encryption text and image signal using adaptive control methods based on the exponential stability for fast recovery in classical and fractional order systems.

The main objectives of this thesis are to propose some methods of chaos synchronization between two different chaotic classical and fractional order systems and its application in secure communication based and the exponential and Lyapunov stability theorems. The analytical conditions for synchronization of these chaotic systems are derived and numerical simulations are used to verify the proposed methods for the following problems

1. To propose exponential stability for fast synchronization classical chaotic dynamical systems using nonlinear control functions.
2. To apply active exponential and Lyapunov synchronization of classical chaotic systems in secure communication using nonlinear control functions.
3. To design adaptive control schemes for functional phase synchronization chaotic systems with unknown parameter using nonlinear feedback controllers based on the Lyapunov stability.
4. To use adaptive synchronization in encryption context text and image signals using masking and modulation methods based on the exponential and Lyapunov stability theorems.
5. To establish the stability of fractional order systems and applying robust adaptive nonlinear feedback controllers in Caputo definition in presence of unknown parameters, disturbance and uncertainties.
6. To apply adaptive synchronization of fractional order systems in encryption text and images signals using masking methods.

Scope of the study is synchronization in the classic and fractional order systems. The design of the proposed method are based on the exponential, Lyapunov first and second stability theorems. Lorenz system, Chen system, Yang system, hyper chaotic Qi and hyper chaotic Lorenz system in the classic case and in the fractional order case Liu, Cenesio and Rossler chaotic system have been used to validate the suggested theoretical results.

1.6 Thesis outline

There are six chapters in this thesis. The first chapter, chapter 1 is an introductory chapter which gives general introduction on dynamical system. Then, the objectives and scopes of study will be stated.

In chapter 2, literature review on the earlier works will be presented. The review will be divided into several parts based on the relevant aspects to current investigation on chaotic dynamical system. The second Lyapunov, exponential stability and synchronization of different chaotic dynamical with certain parameters will be presented in chapter 3. Numerical simulation and application in the encryption are presented to illustrate the application of proposed method. In chapter 4, the adaptive synchronization is expansively to phase synchronization with uncertain parameters and the total possible states based on the exponential and Lyapunov stability theorem with application in secure communications of context and pictorial data. The fractional chaotic dynamic system, its descriptions and theorems are provided and reviewed then the synchronization of same and different systems presented using new stability theorems will be given in chapter 5. the application of fractional order systems in the encryption and secure communications are applied. Finally, chapter 6 will summarized all the results from the previous chapters and further work will be given in the section later.

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LIST OF PUBLICATIONS

Mahmoud Maheri and Norihan md Arifin, ”Advances in Difference Equations Application adaptive exponential synchronization of chaotic dynamical systems in secure communications.” Advances in Difference Equations 2017(1), 96.

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