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SYMMETRICAL NON-SEPARABLE WAVELET TRANSFORMS IN REMOTE SENSING IMAGE FUSION

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BY

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Master of Science

SYMMETRICAL NON-SEPARABLE WAVELET TRANSFORMS IN REMOTE SENSING IMAGE FUSION

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August 2012

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The research presented in this work is concerned with the problem of image fusion by Symmetric Non-Separable wavelet transforms. The main aim was to give a solid foundation for the application of certain types of Symmetrical Non-Separable wavelets considering their mathematical properties and to implement the Fusion of Remote Sensing Images using these wavelets. Two classes of Symmetric Non-Separable wavelets are considered in this thesis: spherically symmetric wavelets and wavelets with square symmetric spectrum. The first type of wavelet was known from the early 1990s while the wavelets with square symmetric spectrum are outlined and studied in this research for the first time.

The study of symmetrical non-separable wavelets started with the investigation of corresponding Continuous Wavelet Transforms in the context of different mathematical models of images. Spherically symmetric wavelets were studied in L_2 and distributional models and

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wavelets with square symmetric spectrum were studied in L_2 and general $L_p$models. For each case the conditions guaranteeing pointwise convergence and convergence in mean were formulated and proved as previously unknown theorems using the revealed relations between non-separable wavelet and Fourier integrals. These theorems constituted the theoretical framework for their subsequent application in the fusion of remotely sensed images. Obtained theorems guarantee that the usage of symmetrical wavelets from considered classes is correct and valid. Such theoretical background is not only crucial for further steps in current research but also considerably enhances the methodology of investigation of wavelet based image fusion for future researches. Indeed with proven theorems one can study not only single non-separable wavelets as it was done before but the whole classes of wavelets and obtain general results.

Numerical implementation of the fusion of Remote Sensing Images was carried out in MATLAB environment. Nine remote sensing images of different sceneries were chosen as source data for the numerical experiments and spherical Haar and square Shanon wavelets were selected as representatives of non-separable wavelets. The source data were degraded spatially using blurring filter and spectrally using 'RGB to monochrome' conversion function. Then the degraded images were fused using non-separable wavelets (spherical Haar and square Shanon) and separable wavelets (Daubechies 2, classical Haar and Symlet). For the fusion with non-separable wavelets we performed numerical calculations of integral transforms while for separable wavelets standard MATLAB functions were applied. The results of fusion based separable and non-separable wavelets were compared using five different quality measures: RMSE, RASE, ADV, ERGAS and SAM. Four metrics representing spatial precision accuracy (RMSE,

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RASE, ADV and ERGAS) showed that the best fusion outputs were produced by the application of non-separable spherical Haar wavelet and SAM quality measure attributing to spectral precision displayed the supremacy of non-separabe square Shanon wavelet over the others. Thus in all cases non-separable wavelets performed better than their separable counterparts.

Finally it was found out that the fusion based spherically symmetric non-separable wavelets showed better spatial precision in the processing of imageries of spherically symmetric objects and patterns rather than the approaches based on separable wavelets and wavelets with square symmetry. On the other hand the best spectrum accuracy was shown by image fusion based wavelets with square symmetric spectrum in the processing of heterogeneous images with sharp edges such as the images of urban areas.

Hereby obtained results not only supported known heuristic suggestion that image fusion based non-separable wavelets perform better accuracy but also revealed the correlation between the symmetry of used non-separable wavelets and the structure of processed images.

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Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

TRANSFORMASI WAVELET SIMETRI BUKAN DIASINGKAN BAGI FUSION IMEJ PENDERIAAN JAUH

Oleh

ALMAZ BUTAEV

Ogos 2012

Pengerusi: Biswajeet Pradhan, PhD

Fakulti: Institut Teknologi Maju

Penyelidikan yang dibentangkan dalam kerja-kerja ini adalah berkenaan dengan masalah fusionimej oleh ubahan wavelet bukan diasingkan simetrik. Tujuan utama ialah untuk memberi asas yang kukuh bagi mengaplikasikan beberapa jenis Wavelet Bukan Diasingkan Simetrik memandangkan ciri-ciri matematik mereka dan implementasi ke atas fusionimej Penderiaan Jauh menggunakan wavelet - wavelet ini. Sebanyak dua kelas Wavelet Bukan Diasingkan Simetrik dipertimbangkan dalam tesis ini: wavelet simetrik sfera dan wavelet dengan spektrum simetrik empat segi. Jenis wavelet pertama yang dikenalpasti pada awal tahun 1990-an di mana wavelet dengan spektrum simetrik segi empat adalah buat kali pertama digariskan dan dikaji dalam penyelidikan ini.

Kajian Wavelet Bukan Diasingkan Bukan Simetrik bermula dengan penyiasatan berkenaan Ubahan Wavelet Berturutan dalam konteks imej model-model matematikal yang berbeza.

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Wavelet simetrik sfera telah dikaji dalam L_2dan pengagihan model-model dan wavelet dengan spektrum simetrik empat segi dikaji dalam L_2 dan model-model L_p umum. Bagi setiap kes syarat-syarat yang menjamin penumpuan pointwise yang dan penumpuan dalam min telah digubal dan dibuktikan sebagai teorem sebelum ini tidak diketahui dengan menggunakan hubungan yang mendedahkan antara bukan diasingkan wavelet dan kamiran Fourier

Teorem-teorem ini dihasilkan daripada rangka kerja teoritikal untuk aplikasi berturutan mereka dalam fusionimej-imej Penderiaan Jauh. Mendapat teorem menjamin bahawa penggunaan riak simetri dari kelas yang dianggap betul dan sah. Latar belakang teori itu tidak hanya penting bagi langkah-langkah lanjut dalam penyelidikan semasa tetapi juga agak meningkatkan kaedah penyiasatan pelakuran imej berasaskan wavelet untuk kajian akan datang. Sememangnya dengan teorem terbukti seseorang boleh belajar bukan hanya satu riak bukan diasingkan sebagaimana yang telah dilakukan sebelum ini tetapi seluruh kelas daripada riak dan mendapat keputusan umum.

Impementasi berangka imej-imej Penderiaan Jauh telah dijalankan dalam persekitaran MATLAB. Sebanyak sembilan imej penderiaan jauh dalam keadaan yang berbeza telah dipilih sebagai sumber data untuk ujikaji –ujikaji berangka dan wavelet Haar sfera dan wavelet Shanon empat segi dipilih mewakili wavelet-wavelet bukan diasingkan. Keputusan fusionberasaskan wavelet diasingkan dan bukan diasingkan dibandingkan menggunakan lima pengukuran berkualiti yang berbeza: RMSE, Rase, ADV, ERGAS dan SAM. Sebanyak empat Metrik mewakili ketepatan spatial yang jitu (RMSE, Rase, ADV dan ERGAS) menunjukkan bahawa output fusionterbaik adalah dihasilkan melalui aplikasi wavelet Haar Sfera bukan

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Diasingkan dan pengukuran SAM berkualiti yang menunjukkan kejituan spektral yang dipamerkan terhadap keunggulan wavelet Shanon segi empat berbanding yang lainnya. Akhirnya ia mendapati bahawa Fusion berasaskan wavelet simetrik sfera bukan diasingkan menunjukkan kejituan spatial yang lebih baik dalam pemprosesan imej-imej objek sfera simetrik dan corak-corak berbanding pendekatan yang berasaskan wavelet wavelet diasingkan dan wavelet-wavelet dengan simetri segi empat.

Sebaliknya ketepatan spektrum terbaik telah ditunjukkan oleh Fusion Imej berasaskan wavelet-wavelet dengan spektrum simetrik empat rsegi dalam pemprosesan imej-imej yang heterogen dengan hujung tajam seperti imej kawasan bandar. Dengan ini keputusan yang didapati tidak hanya menyokong cadangan heuristic yang dikenali bahawa FusionImej berasaskan output wavelet –wavelet tidak diasingkan memberikan keputusan yang lebih tepat bahkan mendedahkan kolerasi antara simetri penggunaan wavelet-wavelet tidak diasingkan dan struktur imej yang telah diproses.

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This thesis is dedicated to those who always believed in me: my parents and mentors

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I certify that a Thesis Examination Committee has met on August 7, 2012 to conduct the final examination of Almaz Butaev on his Master Thesis entitled "Symmetric Non-Separable Wavelet Transforms in Remote Sensing image fusion" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirements for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

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ALMAZ BUTAEV

Date: 7 August 2012

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LIST OF ABBREVIATIONS

SAR	Synthetic Aperture Radar
VIR	Visible Infrared
PCA	Principle Component Analysis
HIS	Hue Intensity Saturation
ERGAS	Relative Dimensionless Global Error in Synthesis (from French Erreur Relative Globale Adimensionnelle de Synthese)
RMSE	Root Mean Square Error
RASE	Relative Average Spectral Error
SAM	Spectral Angle Mapper
ADV	Absolute Difference Value
CWD	Concealed Weapon Detection
MMW	Millimeter Wave
DWT	Discrete Wavelet Transform
CWT	Continuous Wavelet Transform

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CHAPTER 1

INTRODUCTION

1.1 Image Fusion

1.1.1 Background

Modern Earth observation missions provide huge amount of data covering various parts of the electromagnetic spectrum at different spatial, temporal and spectral resolutions. The full exploitation of this ever-growing complex multisource information requires the advanced analytical and numerical data fusion techniques. Multisensor information fusion merges data obtained from several different sensors recording an environment and results in an output with increased interpretation capabilities and more reliable content since information with different characteristics are combined.

Subject to particular type of information, information fusion techniques are classified into different groups. The subject matter of this thesis is image fusion methods. Image Fusion is a fusion of visual information of the same area captured by different sensors. Usually input data for image fusion presents a series of image signals processed by preliminary operations like correction of radiometric and geometric distortions and image co-registration. The aim of image fusion is to achieve better situation assessment and/or more rapid and accurate completion of a pre-defined task than would be possible using any of the sensors individually. In the literature, it has been defined as the synergistic combination of different sources of sensory information into a single representational format. In other words the goal of image fusion can broadly be defined as: to represent the visual information presented in any number

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of input images, in a single fused image without the introduction of distortion or information loss.

The idea of image fusion is illustrated in Figure 1.1. In this case an infrared camera along with digital camera obtains the pictures of the same scene and then their individual images are merged to get a fused one. The usage of this scheme allows one to capture daylight scenes as well as to get pictures in poor illumination cases. As one of the applications one can consider the fusion used in navigation systems showed in Figure 1.2

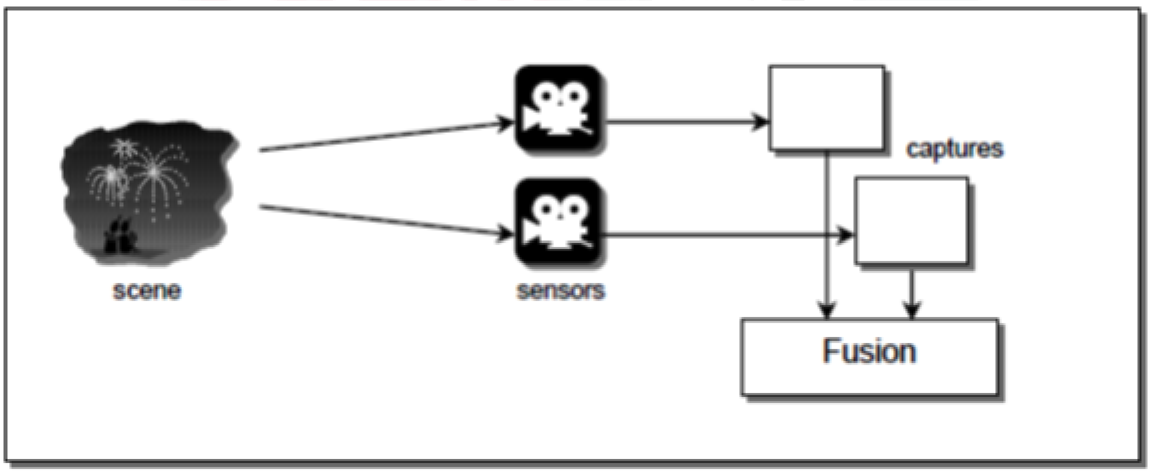

Figure 1.1: Multi-sensor imaging system and image fusion (Ballester et al 2009)

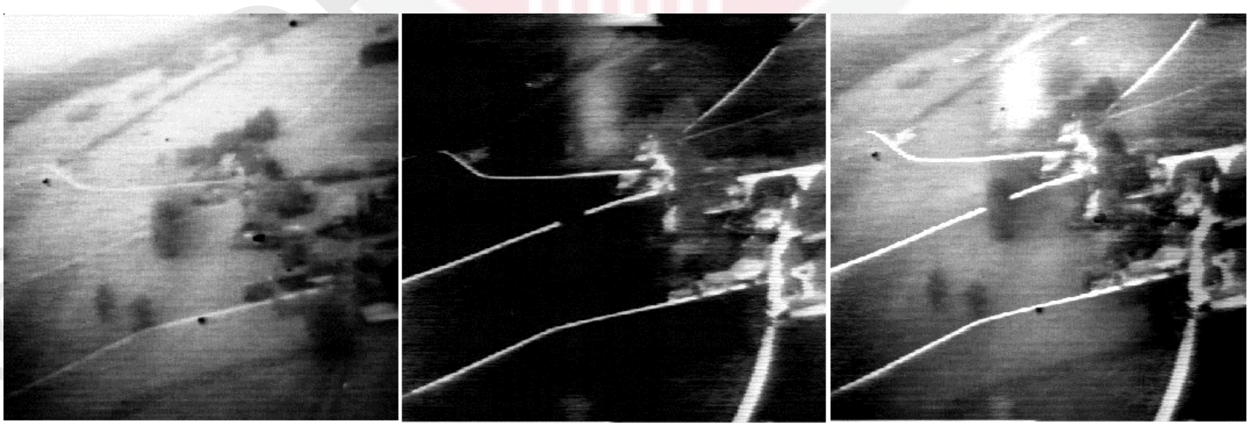

Figure 1.2: Image Fusion of digital camera (left image) and infrared camera (center image) with the output (right image) aimed for better navigation (Pajares and de la Cruz (2004)

The similar idea is employed in remote sensing image fusion in a fusion of visible/infrared (VIR) sensor images and active synthetic aperture radar (SAR) data. The information contained in VIR imagery usually is more suitable for visual analysis and interpretation however may suffer by the presence of clouds and shadows. At the same time SAR data is free from clouds/shadows defects but maybe less useful in data interpretation. The fusion of this disparate image types contribute to a better understanding of observing objects. (Figure 1.3 shows Canadian SAR satellite and optical GeoEye 2 satellites providing VIR and SAR data).

1.1.2 Motivation for research and benefits of image fusion.

The motivation in image fusion research observed over the last 20 years is the result of technological progress in sensor design and sensing methods. Availability of modern sensing systems due to acceptable cost and increasing performance of their usage have made the application of multiple sensors common in various areas such as medical imaging, night vision, autonomous vehicle navigation, remote sensing, concealed weapons detection and different security and surveillance systems.

On the one hand increased spatial, spectral resolution and faster scan rates offered by sensor suits equipped by different modern sensors, provide a more reliable and complete information of the scanned scene. However, this improved performance comes at the cost of a huge increase of input data to be processed. The growth of raw information coming from sensors depends on an increase of the number of sensors used as well as an increase in the size of the imaging arrays. In fact, while the increased number of sensors leads to the proportional rise in the amount of input data, a linear growth in the size of sensor arrays outputs in the quadratic increase of raw information that has to be processed.

Figure 1.3: Canadian SAR (left) and optical GeoEye 2 (right) satellites.

To handle described growth of raw information coming from the use of additional sensors, one can increase the processing power of the image system which will demand additional processing units and larger memory devices. However this approach may be quite ineffective for the following reasons. First of all it can be rather expensive. Secondly, if the image system is operated by a human then the analysis of several images with different modalities will place an additional demand on operator's expertise or will result in diminishing performance. And thirdly, in systems which use a group of operators integrating visual information across the group will be almost impossible (Toet et al. 1989).

As a secondary option one can consider the application of image fusion procedure to integrate all input complementary images prior to further processing. In the context of mentioned problems such approach turns out to be particularly helpful. It is stated that image fusion usage leads to more accurate processing of information obtained by different sensors (Keys et al 1990) and increased utility of its further analysis (Rogers and Wood 1990). Moreover preliminary fused data characterized by increased confidence, reduced ambiguity, improved reliability and improved classification (Rogers and Wood 1990). A summary of benefits resulting from preliminary fusion Is presented in the Table 1 (content is adapted from Sharma R., Pavel M., (1997))

Benefits	**Description/Example**
Extended range of operation	Multiple sensors that operate under different operating conditions can be deployed to extend the effective range of operation. For example different sensors can be used for day/nightoperation
Extended spatial and temporal coverage	Joint information from sensors that differ in spatial resolution can increase the spatial coverage. The same is true for the temporal dimension
Reduced uncertainty	Joint information from multiple sensors can reduce the uncertainty associated with the sensing or decision process
Increased reliability	The fusion of multiple measurements can reduce noise and therefore improve the reliability of the measured quantity
Robust system performance	Redundancy in multiple measurements can help in systems robustness. In case one or more sensors fail or the performance of a particular sensor deteriorates, the system can depend on the other sensors
Compact representation of information	Fusion leads to compact representations. For example, in remote sensing, instead of keeping the imagery from several spectral bands, it is comparatively more efficient to store the fused information

Table 1.1: Benefits of image fusion approach in multi-sensor image systems

1.1.3 Objectives of image fusion

Image Fusion can be considered successful if all visible information on the input images present in the fused image. Yet, although such a result is possible in theory, evidence suggests that the complete representation of all visual data into a single image rarely achieved in practice (Thomas et al. 2008). It can be explained by the fact that usually the same object has different signatures on images recorded by different sensors and as a result this redundant nature of

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multisensory information lead to uncertainty of fusion process and corresponding inevitable loss of data.

Thus along with ultimate ideal aim, image fusion has a practical goal which is modified to: the fusion with the preservation of the "most important" visual features of input images in the output fused image (Pajares et al. 2004). The main practical requirement then, is to identify the most valuable visual information and transfer it into the fused image without any loss. The criteria that define the importance of visual information are usually application dependent. For example in image fusion for display purposes the priority is given to perceptually important information.

An important problem associated with image fusion algorithms is related to a possible introduction of distortions into the fused image. Distortion refers to 'false' information that does not present in the input data and introduced by a fusion technique itself (Nunez 1999).

Distortions may occur in the form of new features which cannot be found in the input images or in the form of 'noise' superimposed over the correct input information. In both cases, distortion decreases the effectiveness and reliability of image fusion process providing an observer with incorrect data which leads to errors of consequent analysis, e.g. causing miss-classification of objects on studied area (Pajares et al. 2004).

As an example of the most common and most noticeable loss of information, one can consider the loss of contrast happening quite often during image fusion as well as during histogram equalization and radiometric correction (contrast in this context refers to the relative difference in the illumination level of each pixel and mean illumination level of the surrounding area) (see e.g. Gonzales, Woods 2007).

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1.2 Wavelets

This subchapter is devoted to a modern popular tool of signal processing know as wavelets. Wavelets play principally important role in present study and giving introduction to this topic below, we will cover technical part of the subject in the following chapters.

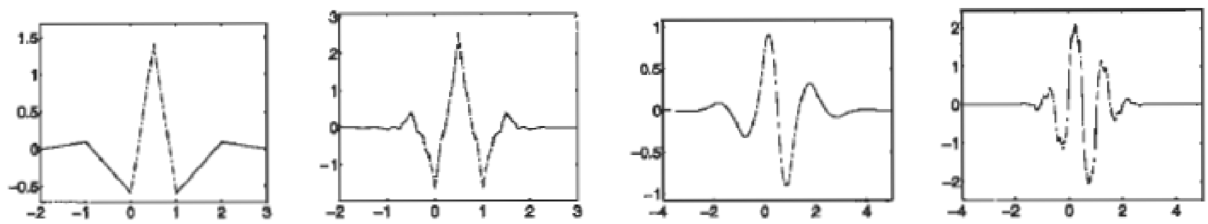

Figure 1.4: Examples of one-dimensional wavelets (Mallat, 2009)

1.2.1 General idea of wavelet

Wavelets are functions that are well localized simultaneously in time and frequency domains which are used to decompose a signal into components with different time and frequency resolutions (the examples of most popular wavelets are shown at Figure 1.4). Wavelets differ from other types of functional basis by the fact that the basis generated by wavelets is produced by dilation and translation of a single function called 'mother wavelet'. Wavelet theory has been developing since the mid 80's and although similar ideas had been employed much earlier in signal analysis with Fourier and Cosine Transforms, wavelet based techniques showed betted performance and suitability rather than traditional Fourier and Cosine Transforms.

One of the reasons that stimulated rapid development of this subject was a revealed relation between wavelets as continuous functions and discreet filter banks' theory. The possibility to

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use filter banks as a mean of wavelets' implementation turned them into a very attractive from a computational point of view tool. Figure 1.5 illustrates the implementation of wavelet transform in terms of high pass filter H_0 and low pass filter H_1.

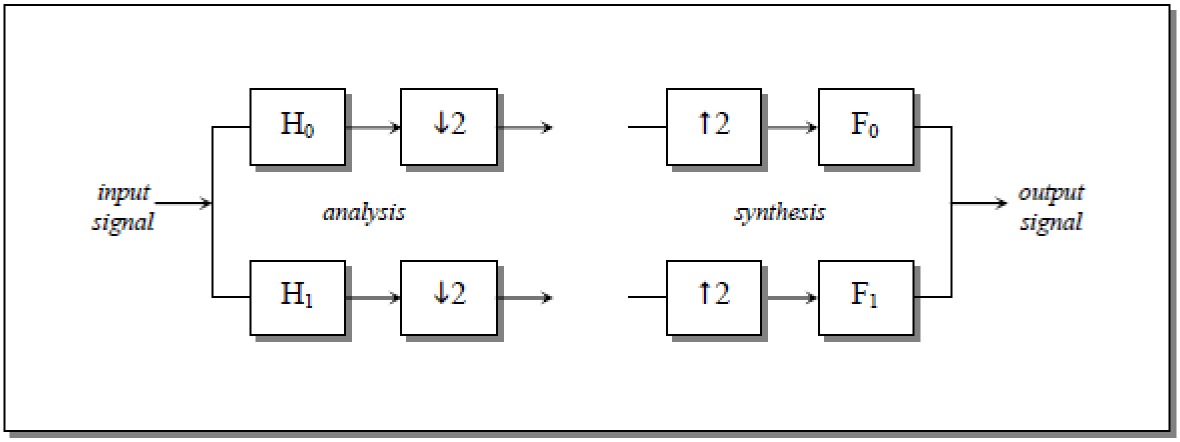

Figure 1.5: Wavelet transforms using filter banks

1.2.2 Image fusion using wavelets

There are many different approaches of the implementation of image fusion and unfortunately, it is impossible to claim absolute superiority of one particular method over all others. However in general, as it was shown by a number of independent studies, (see e.g. Karathanassi et al. 2007, Dong et al. 2009 and Thomas et al. 2008) Image Fusion based wavelet transforms shows the best results in comparison with well-established and widely accepted techniques like IHS, Principal analysis-based fusion etc.

Traditionally wavelet based image fusion is performed using separable wavelet transforms mentioned above. In this case image fusion contains three general steps: first two images are transferred into wavelet domain using separable wavelet transform resulting in the decomposition into approximation image (LL) and directional geometric details (HL, LH, HH) as shown on Figure 1.6. This process of decomposition may be repeated with the approximation image (LL), depending of the particulars of chosen Fusion technique. As a second step one chooses certain wavelet coefficients of each image decomposition and applies predefined fusion rule in order to obtain fused image in wavelet domain. Finally, inverse wavelet transform converts fused image into feasible spatial domain and completes fusion (see Figure 1.7)

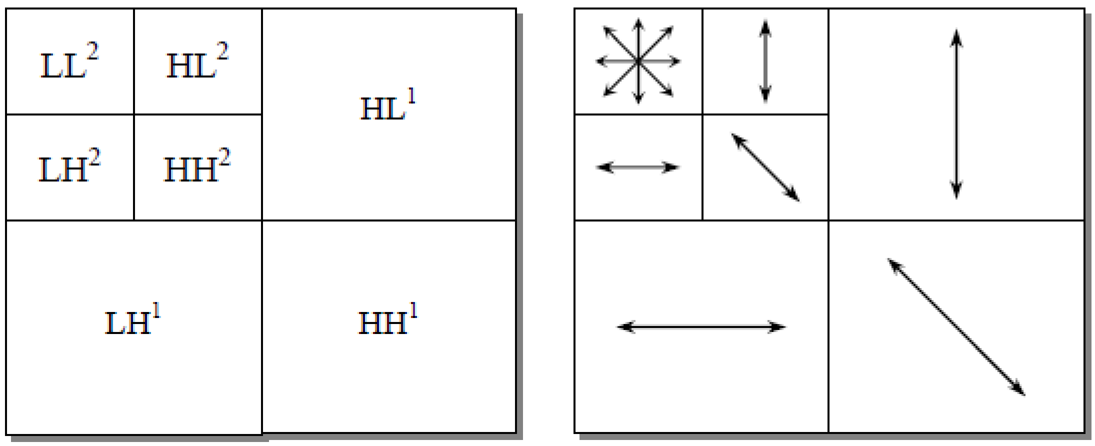

Figure 1.6: Image in wavelet domain after two iterations

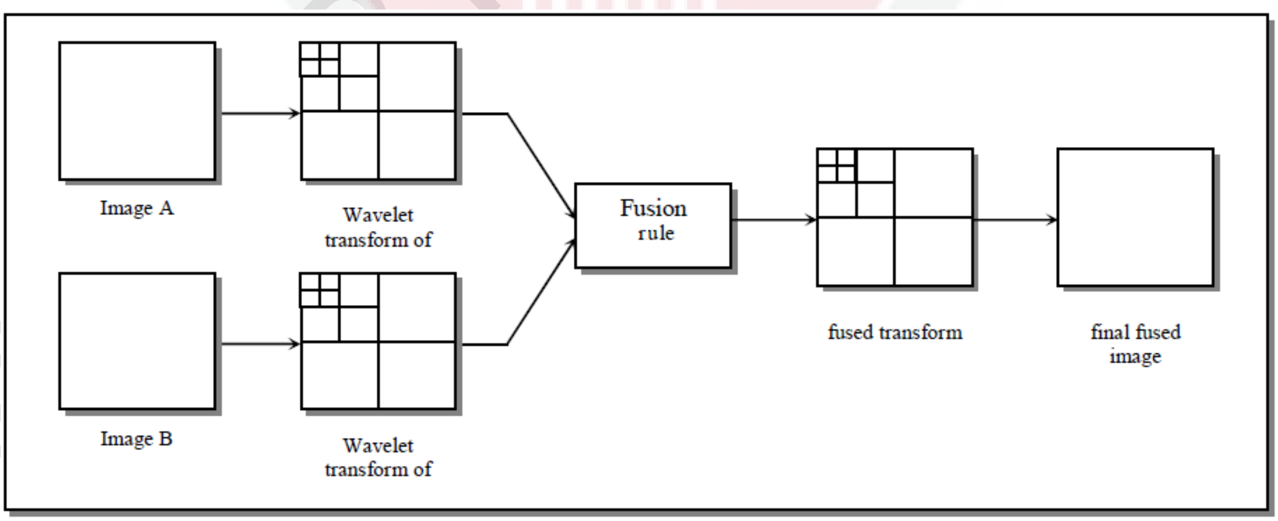

Figure 1.7: Image fusion via wavelet transform

1.3 Objectives and scope of study

As it was mentioned above, the decomposition using one-dimensional wavelets to extract the features of two dimensional images sometimes attracts certain distortions and visual artifacts to the processing of these images. The aims of research is to investigate wavelets based on non-separable transforms which uses two-dimensional wavelets as a tool of decomposition instead of one-dimensional functions.

Despite many researchers (see e.g. Vautrot et al (1998) and Kovačević et al (2012)) agree with the proposition that processing of two-dimensional signals by nonseparable wavelets must be more efficient than conventional separable approach, it is impossible to check this claim in its whole generality in view of the lack of proper classification for nonseparable wavelets so far (Kovačević et al (2012)).

As a possible restriction of scope, one can focus only on wavelets possessing certain type of symmetry in spatial or frequency domain and study image fusion performed with their help. Following this idea, the general approach can be divided into the following general milestones:

- to identify wavelet transform that provides a good reconstruction and possessing symmetry in the spatial domain,
- to identify wavelet transforms with good reconstruction and possessing symmetry in the frequency domain,
- to perform image fusion by means of both types of wavelets and compare their results with classical separable wavelet image fusion techniques.

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In order to meet above mentioned objectives, it is possible to estimate claimed superiority of non-separable wavelets over their separable counterparts. Moreover as a supplementary question, it is possible to ask whether the symmetry of non-separable wavelets can affect the performance of image fusion and possibly how.

In this thesis we will pursue the above mentioned objectives with the following technical specifications:

1) Since even the basic type of nonseparable transforms known as continuous wavelet transforms is not explored properly so far, in this thesis we will consider only image fusion based on continuous nonseparable wavelet transforms .
2) With the aim to give a rigorous definition of 'good reconstruction' property we will apply certain mathematical notions considering pointwise convergence properties for such wavelets
3) As an environment for numerical fusion implementation, we will consider one of the most powerful tools and recognized system – i.e. MATLAB. Implementation of different Fusion methods under the same software will guarantee correct comparison of these methods.

With emphasized specification, the objectives of the current study can be summarized as follows:

1) To formulate and prove the theorems establishing conditions to symmetrical wavelets that will guarantee mean-square and pointwise convergence for the corresponding Continuous wavelets transforms.

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2) To formulate and prove the theorems establishing conditions to the wavelets symmetrical in frequency domain that will guarantee mean-square and pointwise convergence for the corresponding Continuous wavelets transforms.

3) To implement numerical image fusion in MATLAB for nonseparable and separable wavelet transforms and compare the outputs using major error metrics with the aim to find out whether non-separable wavelets enhance separable-wavelet based image fusion.

4) To conclude whether the symmetry of non-separable wavelet transform can affect the output of image fusion and how

1.4 Thesis organization

The chapter next to the introduction provides background information on image fusion, Wavelet theory and the role of wavelets in modern fusion techniques. The content of this chapter is mostly based on the works reported in the past several years in various publications in the fields of remote sensing, information fusion and applied mathematics.

Chapter 3 is devoted to the theoretical analysis of nonseparable wavelet transforms. In this chapter, we define two types of symmetric non-separable wavelets: spherically symmetric wavelets and wavelets with square symmetric spectrum. We will study them in the context of three image models: such as L_2, L_p and distributional models. For each model, we establish the conditions that may guarantee correct reconstruction of any image from the model by means of continuous wavelet transforms based on studied wavelets. The formulation of such conditions will provide a solid foundation for further application of these symmetrical non-

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separable wavelets for the fusion of remotely sensed images. As a remark we note that a lot of mathematical formulas will appear in Chapter 3 and for the sake of compactness only the formulas which will be referenced later are numerated.

In chapter 4 we will implement the fusion based on a spherically symmetric wavelet and a wavelet with square symmetric spectrum and compare the results with the fusion based on classical separable wavelets. Nine numerical experiments will be conducted with the fusion of images obtained through airborne and satellite sensors. The fusion accuracy will be measured using five different metrics such as ADV, RMSE, RASE, SAM and ERGAS. The analysis of numerical calculations will show whether the non-separable approach can be considered as an efficient way of image fusion

Finally, chapter 5 outlines the results obtained in this study by evaluating the efficiency of nonseparable wavelets in comparison with separable wavelet transforms. This answers to the problems formulated at the beginning and pose the directions for possible future researches.

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