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MODIFIED SEQUENTIAL FENCES FOR IDENTIFYING UNIVARIATE OUTLIERS

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By

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The existence of outliers in data set can bring some impacts on statistical data analysis and affect decision making. Thus, it is vital for researcher to identify the outliers. Sequential fences is a graphical method which was proposed by Schewertman and de Silva (2007). Besides its simplicity, this method is also effective in detecting multiple outliers while maintaining the approximate specific outside rate at each stage as the series on number of outlier fences. This research focuses on the modification of sequential fences to improve its efficiency.

Sequential fences method is modified by replacing interquartile range with various robust scales such as semi-interquartile range, Qn, Sn, median absolute deviation (MAD) and Gini's mean difference (GMD) in order to improve outlier detection in symmetric distribution. Ultimately, the utilisation of GMD in sequential fences seems to demonstrate a comparable accuracy in detecting the contaminated data. We have shown that GSF approach effectively reduce the masking and swamping problems in identifying the outliers.

Furthermore, a new approach is proposed by considering the skewness of underlying distribution to increase efficiency of sequential fences in skewed distribution. Conclusively, based on the numerical examples and simulation study, newly proposed method has been adjusted according to the skewness of the underlying distribution of data. The results show that the new approach performed better in reducing swamping effect which is misclassifying non-contaminated observation as outlier in asymmetric distribution.

Moreover, we proposed a new method with modified algorithm and methodology namely bootstrap sequential fences. The proposed method involves initial screening of data and bootstrap technique to improve the performance of sequential fences. The modified sequential fences method is found can accurately detect the outliers in positively skewed distribution. In addition, this proposed method also estimates trimmed mean and trimmed standard deviation with smaller bias and smaller root of mean squares error. Thus, proposed method proves its superiority over the existing techniques.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk Ijazah Master Sains

PAGAR BERURUTAN TERUBAHSUAI BAGI MENGENAL PASTI DATA UNIVARIAT TERPENCIL

Oleh

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Kewujudan data terpencil dalam data boleh membawa kesan negatif terhadap analisis data statistik dan menjejaskan kesimpulan. Oleh itu, ini adalah penting bagi penyelidik untuk mengenal pasti data terpencil. Pagar berurutan adalah satu kaedah grafik yang dicadangkan oleh Schewertman dan de Silva (2007). Selain mudah, kaedah ini juga berkesan dalam mengesan pelbagai data terpencil disamping mengekalkan kadar luar tertentu yang sesuai pada setiap peringkat sebagai siri pada bilangan pagar titik terpencil. Kajian ini memberi tumpuan kepada pengubahsuaian pagar berurutan untuk meningkatkan kecekapannya.

Kaedah pagar berurutan telah diubahsuai dengan menggantikan julat antara kuartil dengan pelbagai skala teguh seperti julat semi-antara kuartil, Qn, Sn, sisihan mutlak median (MAD) dan perbezaan min Gini (GMD) untuk meningkatkan pengecaman data terpencil dalam taburan simetri. Penggunaan GMD dalam pagar berurutan menunjukkan ketepatan yang setanding dalam mengesan data yang tercemar. Kami telah menunjukkan bahawa pendekatan GSF berkesan dalam mengurangkan masalah litupan dan limpahan dalam mengenal pasti titik terpencil.

Selain itu, satu pendekatan yang baru telah dikemukakan dengan mempertimbangkan kepencongan taburan dasar untuk meningkatkan kecekapan pagar berurutan dalam taburan pencongan. Kesimpulannya, berdasarkan contoh-contoh berangka dan simulasi kajian, pendekatan baru yang dicadangkan telah disesuaikan mengikut kepencongan taburan pendasar data. Keputusan menunjukkan bahawa pendekatan baru memberikan prestasi yang lebih baik dalam mengurangkan kesan limpahan yang tersilap mengklasifikasikan titik bukan tercemar sebagai titik terpencil dalam taburan bukan simetri.

Di samping itu, kami juga mencadangkan satu kaedah baru dengan algoritma dan kaedah yang diubahsuai iaitu bootstrap pagar berurutan. Kaedah yang dicadangkan melibatkan pemeriksaan awal data dan teknik bootstrap untuk mepertingkatkan prestasi pagar berurutan. Kaedah pagar berurutan yang diubahsuai didapati bahawa boleh mengesan titik terpencil dengan tepat dalam lengkung pencong positif. Tambahan pula, pendekatan baru ini juga menunjukkan kecenderungan dan punca kuasa dua min ralat yang lebih kecil dalam penganggaran min terpangkas dan sisihan piawai terpangkas. Oleh yang demikian, terbuktilah keunggulan pendekatan baru berbanding dengan teknik-teknik yang sedia ada.

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LIST OF ABBREVIATIONS

ASF	Adjusted sequential fences
ESD	Generalised extreme studentized deviation test
GCD	procedure that is incorporating the GSF approach into the algorithm to generate a set of clean data
GMD	Gini's mean difference
GSF	Sequential fences using GMD
IQR	interquartile range
LF	Lower fence
MAD	Median absolute deviation
MADSF	Sequential fences using MAD
MS	Moment of skewness
Q1	First quartile
Q3	Third quartile
QnSF	Sequential fences using Qn
RMSE	Root of mean square error
SDSF	Sequential fences proposed by Schewertman and de Silva (2007)
SDSFB	Sequential fences using the proposed bootstrapping techniques
SIQR	semi-interquartile ranges
SIQRSF	Sequential fences using SIQR
SnSF	Sequential fences using Sn
SSFB	Bootstrap technique is used to estimate the cut off points of sequential fences
TB	Tukey's boxplot
TBB	Tukey's boxplot using the proposed bootstrapping techniques
TEB	Trimmed estimators based on bootstrap resampling
UF	Upper fence

CHAPTER 1

INTRODUCTION

1.1 Outlier Definitions

An outlier is an observation that appears discrepant with the other values of the sample. Outliers can also be defined as those observations that look different from other members in the data **(**Beckman & Cook, 1983). Another definition of an outlier is a value which appears inconsistent to the researcher (Iglewicz & Hoaglin, 1993). In other words, inconsistent observations with respect to the remaining data are defined as outliers. An outlier is also defined as an observation which deviates away from the other data values and this outlying observation is suspected that it was created by other mechanism (Hawkins, 1980).

From the historical definitions, these can be illustrated that an outlier is a subjective and post-data concept. Methods for dealing with outliers are applied to the data for checking the existence of the outliers after the contaminated observations are detected via a visual examination of the data (Beckman & Cook, 1983; Grubbs, 1969). In short, an observation that comes from a distribution that is different from that for all the other remaining observations is determined as a contaminated observation.

1.1.1 Causes and Influences of Outliers

The occurrence of outliers in the data set can be caused by mistake in recording or due to the malfunction of measuring instrument. Besides, the existence of discordant observations might be due to the natural variability which comes from the outside of the sample. These outliers may have great influence on the parametric data analyses and resulted in misleading results. During the estimation of parameters, the presence of outliers may cause high errors variance and low power of test (Zimmerman, 1994, 1995, 1998). When there are outliers in the errors, the normality in univariate case and sphericity and multivariate normality become low and lead to type I and type II errors. In linear regression, the effect of outlier is at least distorting the parameter estimation (Osborne & Overbay, 2004).

1.1.2 Swamping and Masking

Outlier identification plays a vital role in statistical inference, data processing and modeling. The presence of outliers might result in biased parameter, poor forecasting and misspecification in modeling (Tsay et al., 2000; Fuller, 1987). There are many literatures on the outliers detection methods. Some methods might classify clean observations as outliers and fail to detect the real outliers. Thus, swamping and masking effects emerge. The swamping and masking effects can cause mistake in

making decision during the regression analysis. (Chatterjee & Hadi, 2006). The characteristics of these effects are defined by Iglewicz and Martinez (1982) and Ben-Gal (2009).

Swamping effect occurs when a second observation is labeled as an outlier in the presence of first outlier. After discarding the first outlying observation, the second observation is detected as clean observation. This phenomenon is classified as the swamping effect. Swamping effect happens when outlier shifts the mean and the covariance estimates toward it and away from other inliers on another side of distribution tail. Hence, this causes the gap between these observations to the mean is large and make them look similar to outliers.

Another phenomenon is that the second observation is classified as outlier without the existence of the first outlier. After eliminating the first outlier, the second observation is appeared as an outlier. This occurrence is denoted as the masking effect. Masking effect occurs when mean and covariance estimates are skewed towards a group of outliers, and the resulting distance of the outlier from the mean is decreased.

1.2 Tukey's Boxplot

Traditional boxplot is one of the most frequently and widely used techniques for studying the shape of the distribution and analyzing some characteristics of the distribution such as location and spread. In addition, the boxplot technique also can be used to identify the potential outliers which deviate markedly from the remaining data.

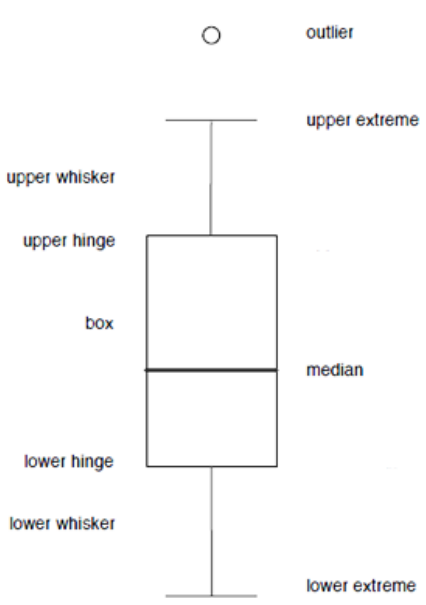

Figure 1.1: Construction of boxplot

The boxplot consists of five components which give a robust statistical summary of the distribution of a dataset. These components are illustrated in Figure 1.1 The components used to construct the boxplot are two hinges which are first quartile (q_1) and third quartile (q_3), median, observations which lie 1.5 constant from the interquartile range measured from median, two whiskers that connect to the lower and upper hinges and potential outliers which lie apart from median and exceed extreme. The first quartile and third quartile are equivalent to 25^{th} percentile and 75^{th} percentile respectively while the interquartile range is the difference between the third and first quartile.

Inner fences in a boxplot are positioned at an interval of 1.5 IQR beneath first quartile and above third quartile which can be denoted as

$$[q_1 - 1.5\text{IQR}\,, q_3 + 1.5\text{IQR}] \tag{1.1}$$

whereas the outer fences are located at a distance of 3 IQR less than first quartile and more than third quartile which are presented as

$$[q_1 - 3\text{IQR}\,, q_3 + 3\text{IQR}]. \tag{1.2}$$

Any observation that falls outside the inner fences is labeled as a mild outlier while value that falls beyond outer fences is marked as extreme outlier.

1.3 Sequential Fences

Schewertman and de Silva (2007) has modified the boxplot and introduced sequential fences as another useful technique to detect the outliers in the data. In this study, the sequential fences proposed by Schewertman and de Silva is henceforth referred as SDSF. The technique proposed by Schewertman and de Silva (2007) identifies outliers sequentially based on the specific sample size and the pre-specified outside rate attained which is the probability that an uncontaminated observation falls beyond the fences.

For the construction of sequential fences, the sample sizes are adjusted using Poisson model in order to decrease the tail probabilities. The adjustment is similar to the adjustment done in Davies and Gather (1993) and Gather and Becker (1997). This SDSF increases the accuracy to identify the outliers, reduces the swamping effect and less likely to misclassify an uncontaminated observation as an outlier in large sample size. In the procedure of outlier identification, this method allows the researchers to have flexibility in setting the level of confidence. The fences are constructed continuously until there is no extra outlier detected.

1.4 Problem Statement

Although there are a lot of literatures on outlier identification methods, most of the existing methods are suitable only for symmetric distributions as discussed in detail in Chapter 2. The popular boxplot method (Tukey, 1977) is too liberal and cause many unusual observations to be overlooked. The sequential fences method which was proposed by Schwertman and de Silva (2007) allows flexibility in setting the outside rate to detect the extreme and mild outliers. This method uses interquartile range to measure the dispersion of the data. A natural question comes to our mind is whether it can be developed using an alternative robust scale that can measure the dispersion of the data in the sequential fences method. Thus, it is important to find out the suitable robust scale in the replacement of interquartile range in order to improve the performance of sequential fences approach in detecting the outliers.

Furthermore, the major problem of the existing outlier detection techniques is too conservative in which these techniques work well in symmetric distribution and have low performance in asymmetric distribution. Some methods obey normality assumptions while most of the real data do not follow normal distribution. Some authors proposed outliers techniques for skewed data, but the performance of these techniques needs improvement. Therefore, the modification of the sequential fences method which was proposed by Schwertman and de Silva (2007) is needed to be improved by making some adjustments to the approach for detecting outliers in skewed data with the consideration of the skewness of the distributions.

Moreover, procedure of screening for the data before further analysis of data is important (Tabachnick & Fidell, 2001). Identification of outliers is a part of the data screening procedure which should be done regularly before starting a statistical analysis (Beckman & Cook, 1983; Ahmad et al., 2011). Simulated univariate data may contain outlying observations. When the data is from symmetric distribution, the extreme values that are located at the left or right tail may be suspected as outliers. For skewed distribution data, it is suspected that the extreme observation at the longer tail might be outlying observation.

In order to know whether the outliers present in the data, initial screening of the data is necessary. In the boxplot method, the data which are used to obtain the central tendency and spread of data such as mean and standard deviation are assumed normal. Test statistics are greatly affected when the data is non-normal. The critical values of sequential fences technique (Schwertman & de Silva, 2007) depends on the calculation of median and interquartile range. In Chapter 4, it can be observed that the existing sequential fences perform well in the symmetric distributions but capture too much outliers on the long tail of skewed distribution. Thus, the fences should be adjusted to allow a better coverage of the centre of the data especially when the data are skewed.

1.5 Objectives of Study

Since the study is focused on modification of sequential fences, the objectives of this study are i) to propose method for outlier identification in symmetric distribution with higher accuracy and lower misclassification of non-contaminated observations as outliers ii) to increase the accuracy in detecting the real outliers in asymmetric distribution with minimum swamping and masking effect and iii) to provide an efficient sequential fences in identifying outliers in wider types of distributions with new algorithm and parameters estimation.

1.6 Limitation of Study

SAS review 9.3 is selected as our research tool which helps in simulation, bootstrapping and computing the results. Due to the long computation time in large replications, the number of observations contamination is set up to three outliers. The sample size of the simulation is limited to $n = 100$ only, because simulations involve 10,000 replications. The procedures of sequential fences take some time because this method has to keep constructing the fences sequentially and checking for the presence of outliers continuously until there is no additional outlying observation being captured.

1.7 Overview of Thesis

Since this study is related to modification of existing sequential fences method (SDSF) which was proposed by Schwertman and de Silva (2007), it is important to improve its performance in outlier detection in symmetric and asymmetric distribution data.

Chapter 3 provides a review of sequential fences method of detecting the outliers in the normally distributed data. Instead of using interquartile range (IQR), this study modifies the existing sequential fences technique for identifying outliers by using different robust scales such as semi-interquartile range (SIQR), median absolute deviation (MAD), Qn, Sn and gini's mean difference (GMD). This study also compares proposed methods with the existing sequential fences method and generalized extreme studentized deviate (ESD) test. Two empirical examples are used to illustrate the efficiency of the methods. Simulation study is conducted with different number of outliers. The performance of all outlier detection techniques has been compared by evaluating the proportion of correctly identifies the outliers and the proportion of misclassifies the uncontaminated observation as outliers. Superiority of the proposed technique has been validated by simulation results.

In Chapter 4, SDSF method is extended to form a new technique based on the skewness of underlying distribution data to identify outliers in skewed distributions. Adjustment of the fences construction has been made using moment measure of skewness to measure the skewness of the data. Similarly, the proposed method and

existing SDSF are applied to a real data set as illustration and comparison. Besides Normal distribution, simulation study has been conducted on skewed distributions such as Lognormal, Chi-square, Gamma and Weibull with different parameters, and the simulation results are compared with existing SDSF method. The proposed technique shows its outstanding performance compared to SDSF technique in detecting outliers in the different distributions and also in real data set at different nominal outside rates.

In Chapter 5, a modification of algorithm and formulation are proposed based on the SDSF method which can identify outliers in the symmetric and asymmetric distributions. Instead of using Monte Carlo simulation, a new methodology involving bootstrapping technique has been developed. Before contamination of the data, a clean simulated data is generated and verified using Gini Sequential Fences (GSF) method which is proposed in Chapter 3. For the performance study, bootstrap resampling study has been done on the symmetric and skewed distribution, such as normal, chi-square with different degrees of freedom and lognormal distribution with different parameters. The performance of the newly proposed technique is compared with SDSF method and Tukey's boxplot by matching number of outliers detected with the contaminated observations for different sample sizes. Apart from that, based on the outliers detected, trimmed mean and trimmed standard deviation adopting bootstrap resampling technique has been calculated. The comparison of the estimation of parameters based on bias and mean square errors have been done. From the result, the supremacy of proposed modification method over existing SDSF technique and boxplot is proven.

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Wong, H. S., & Fitrianto, A. (2016) A Comparative Study of Outliers Identification Methods in Univariate Data Set. *Journal of Advanced Science Letters*.

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