UNIVERSITI PUTRA MALAYSIA

NUMERICAL SOLUTIONS OF LINEAR FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS OF THE SECOND KIND USING QUADRATURE-DIFFERENCE METHODS

CHRISCELLA BINTI JALIUS

IPM 2016 14

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By

CHRISCELLA BINTI JALIUS

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Master of Science

December 2016

DEDICATIONS

Special dedicated to;
Mama
Charles, Clarier, Tyronce & Rinzo
Beloved lecturers
Family & Friends

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the Degree of Master of Science

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December 2016

Chairman:Zanariah binti Abdul Majid, PhD
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Fredholm integro-differential equation (FIDE) is an equation which is the unknown functions appears under the sign of derivative and also integral sign. Therefore, the formulation of numerical quadrature rules and finite difference method are applied for solving first-order and second-order linear FIDE of the second kind. The finite difference method is used for ordinary differential equations part, while composite quadrature rules are applied for the integral part of FIDE. Numerical solutions of linear FIDE by using quadrature-difference methods are proposed in this thesis.

There are four types of formulation proposed in this thesis which are composite Simpsons 3/8 rule with first derivative of 5-point finite difference, composite Simpsons 3/8 rule with second derivative of 5-point finite difference, composite Booles rule with first derivative of 7-point finite difference and composite Booles rule with second derivative of 7-point finite difference. These formulations will be used to produce an approximation equations in order to discretize the FIDE into a system of linear algebraic equation. The system of linear algebraic equation will be solved by using Gauss elimination method. An algorithm and a coding of the proposed methods are developed in this thesis. The source of the coding for solving linear FIDE is developed by using C programming with constant step size.

The four types of formulation which based on quadrature rules and finite difference method are implemented for solving Type 1 and Type 2 of first-order and second-order linear FIDE. In this thesis, the boundary condition will be considered in solving the second-order linear FIDE. Moreover, the order of accuracy of the proposed

method are studied in this thesis.

Finally, the numerical experiments were carried out in order to examine the accuracy of the proposed method. The results indicated that the proposed methods are suitable for solving first-order and second-order linear FIDE of the second kind.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk Ijazah Sarjana Sains

PENYELESAIAN BERANGKA BAGI PERSAMAAN PEMBEZAAN-KAMIRAN FREDHOLM LINEAR JENIS KEDUA MENGGUNAKAN KAEDAH BEZA-KUADRATUR

Oleh

CHRISCELLA BINTI JALIUS

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Persamaaan pembezaan-kamiran Fredholm (PPKF) adalah persamaan yang mempunyai fungsi yang tidak diketahui muncul di dalam bentuk pembezaan dan dalam bentuk pengamiran. Oleh itu, formulasi aturan kuadratur berangka dan kaedah beza terhingga digunakan untuk menyelesaikan peringkat pertama dan kedua bagi PPKF linear jenis kedua. Kaedah beza terhingga digunakan untuk menyelesaikan bahagian persamaan pembezaan biasa, manakala aturan kuadratur gubahan digunakan untuk menyelesaikan bahagian kamiran pada PPKF. Penyelesaian berangka bagi linear PPKF dengan menggunakan kaedah beza-kuadratur yang dicadangkan di dalam tesis ini.

Terdapat empat jenis formulasi yang diusulkan di dalam tesis ini iaitu aturan Simpson 3/8 gubahan dengan kaedah beza terhingga 5-titik terbitan pertama, aturan Simpson 3/8 gubahan dengan kaedah beza terhingga 5-titik terbitan kedua, aturan Boole gubahan dengan kaedah beza terhingga 7-titik terbitan pertama dan aturan Boole gubahan dengan kaedah beza terhingga 7-titik terbitan kedua. Formulasi-formulasi itu digunakan untuk mendapatkan persamaan hampiran bagi tujuan mendiskritkan PPKF kepada sistem persamaan aljabar linear. Sistem persamaan aljabar linear akan dapat diselesaikan dengan menggunakan kaedah penghapusan Gauss. Algoritma dan kod bagi kaedah yang diusulkan dibina di dalam tesis ini. Sumber kod bagi menyelesaikan PPKF linear dibina dengan menggunakan pengaturcaraan C dengan saiz langkah yang malar.

Keempat-empat jenis formulasi itu adalah berdasarkan peraturan kuadratur dan

kaedah beza terhingga yang dilaksanakan untuk menyelesaikan Jenis 1 dan Jenis 2 bagi PPKF linear peringkat pertama dan peringkat kedua. Dalam tesis ini, syarat sempadan akan dipertimbangkan bagi menyelesaikan PPKF linear peringkat kedua. Seterusnya, peringkat bagi kejituan kaedah yang dicadangkan adalah turut dikaji di dalam tesis ini.

Akhir sekali, ujikaji bagi masalah berangka telah dijalankan untuk mengkaji kejituan kaedah yang dicadangkan. Hasil kajian menunjukkan bahawa kaedah yang dicadangkan adalah sesuai untuk menyelesaikan PPKF linear bagi peringkat pertama dan kedua.

ACKNOWLEDGEMENTS

Firstly, praise to God for giving me patience, strength and courage in completing this research. This research would not possible to be completed without the support of many people who always with me during my study. I would like to express my sincere and deepest gratitude to the Chairman of the Supervisory Committee, Prof. Dr. Zanariah Abdul Majid for her priceless encouragement, excellent guidance and golden advice throughout my study.

Moreover, I would like to extend my gratitude to my co-supervisor, Dr. Faranak Rabiei for her valuable suggestions and comments throughout this research. My deepest appreciation goes to my helpful friends and staffs of INSPEM. I would like to thank the Graduate Research Fellowship (GRF) from Universiti Putra Malaysia and MyMaster from the Ministry of Higher Education for the financial support.

Finally, my special thanks and appreciation goes to my lovely parents and siblings for their continuous motivation, encouragement and unconditional love during the past two years.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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Declaration by Members of Supervisory Committee

This is to confirm that:
- the research conducted and the writing of this thesis was under our supervision;
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LIST OF ABBREVIATIONS

IVP	Initial value problem
BVP	Boundary value problem
FIDE	Fredholm integro-differential equation
GMRES	General minimal Residual Gauss Seidel
HSCG	Half-Sweep conjugate gradient
FSCG	Full-Sweep conjugate gradient
FSGS	Full-Sweep Gauss Seidel
HSGS	Half-Sweep Gauss Seidel
QSGS	Quarter-Sweep Gauss Seidel
QSCGNR	Quarter-Sweep conjugate gradient normal residual
FSCGNR	Full-Sweep conjugate gradient normal residual
HSCGNR	Half-Sweep conjugate gradient normal residual
GE	Gauss elimination
CS1FD(GE)	Composite Simpson's $\frac{3}{8}$ rule with first derivative of Finite Difference (Gauss Elimination) method proposed in this thesis
CS2FD(GE)	Composite Simpson's $\frac{3}{8}$ rule with second derivative of Finite Difference (Gauss Elimination) method proposed in this thesis
CB1FD(GE)	Composite Boole's rule with first derivative of Finite Difference (Gauss Elimination) method proposed in this thesis
CB2FD(GE)	Composite Boole's rule with second derivative of Finite Difference (Gauss Elimination) method proposed in this thesis

CHAPTER 1

INTRODUCTION

1.1 Introduction

The area of integral equations has attracted many researchers. There are various of research works that have been done by the researchers which contributed towards the development of the field. An integral equation is an equation in which the unknown function appears under one or more integral signs (Wazwaz, 2011). The general type of integral equation is given in the following form

$$y(x) = f(x) + \lambda \int_a^b K(x,t)y(t)\,\mathrm{d}t, \tag{1.1}$$

where a and b are the limits of integration, λ is the constant parameter and $K(x,t)$ is called the *kernel* of the integral equation. The function y is the unknown function which will be determined and it appears inside the integral sign and also outside the integral sign. While, the functions $f(x)$ and $K(x,t)$ are given in advance. The limits of integration a and b may be both fixed or at least one of the limit is variable. If at least one of the limits of integration are variable, then the equation will be called Volterra equation, whereas when the limits of integration is fixed, it is called Fredholm equation. Therefore, we can say that, this is one significant different between Fredholm equation and Volterra equation. In addition, this type of integral equations can be divided into two groups which is first kind and the second kind.

Nowadays, integro-differential equations has been emerged in many scientific and engineering applications which mostly appeared in an electrical circuit analysis. An integro-differential equations appeared especially when the initial value problem (IVP) or boundary value problem (BVP) are converted to integral equations. Both differential and integral operators appeared together in integro-differential equations and the differential operator may appear in any order depending on the problems studied. Thus, the general equation of integro-differential equation appear in the form of

$$y^{(n)}(x) = f(x) + \lambda \int_a^b K(x,t)y(t)\,\mathrm{d}t, \tag{1.2}$$

where n is represent the order of the derivative of the unknown function. There are three types of integro-differential equation which are regularly studied by many researchers such as Fredholm, Volterra and Fredholm-Volterra because of its various used either in practical or in real life applications. Fredholm-Volterra is a combination of Fredholm and Volterra where the general form will have two integral operators.

Integral equations or integro-differential equations are usually difficult to solve analytically. Thus, by choosing numerical techniques, it is much easier to obtain an approximation to the solution of the problem. In addition, the numerical approaches can obtain accurate approximation solution and the algorithm can be developed which will be used in the computation of the approximation solution.

1.2 Fredholm Integro-Differential Equations

The Fredholm Integro-Differential Equation (FIDE) contains both differential and integral operators in the same equation (Delves and Mohamed, 1985). FIDE appear when the differential equations were converted into integral equations. Thus, FIDE is given in the following form

$$y^{(n)}(x) = f(x) + \lambda \int_a^b K(x,t)y(t)\,\mathrm{d}t, \tag{1.3}$$

where $y^{(n)}$ is the nth derivative of $y(x)$. The value of n refer to the order of the equation. In addition, this general equation for linear FIDE is for second kind problem. Thus, first-order FIDE and second-order FIDE are focused in this study.

It is important to understand the concept of linearity in FIDE. The FIDE is classify as nonlinear if the unknown function of $y(t)$ contains nonlinear function such as $y^2(t)$, $sin(y(t))$ and $e^{y(t)}$. In this thesis only linear problems will be solved. FIDE can be classified into two types for first-order linear FIDE and four types for second-order linear FIDE.

1. First-order linear FIDE of the second kind

$$y'(x) = p(x)y(x) + g(x) + \lambda \int_a^b K(x,t)y(t)\,\mathrm{d}t, \tag{1.4}$$

with the initial condition

$$y(a) = y_a$$

(i) Type 1 [$p(x) = 0$]

$$y'(x) = g(x) + \lambda \int_a^b K(x,t)y(t)\,\mathrm{d}t, \tag{1.5}$$

(ii) Type 2 [$p(x) \neq 0$]

$$y'(x) = p(x)y(x) + g(x) + \lambda \int_a^b K(x,t)y(t)\,\mathrm{d}t, \tag{1.6}$$

2. Second-order linear FIDE of the second kind

$$y''(x) = q(x)y'(x) + p(x)y(x) + g(x) + \lambda \int_a^b K(x,t)y(t)\,\mathrm{d}t, \qquad (1.7)$$

with the boundary condition

$$y(a) = y_a \quad \text{and} \quad y(b) = y_b$$

(i) Type 1 [$p(x) = 0$ and $q(x) = 0$]

$$y''(x) = g(x) + \lambda \int_a^b K(x,t)y(t)\,\mathrm{d}t, \qquad (1.8)$$

(ii) Type 2 [$p(x) \neq 0$ and $q(x) = 0$]

$$y''(x) = p(x)y(x) + g(x) + \lambda \int_a^b K(x,t)y(t)\,\mathrm{d}t, \qquad (1.9)$$

(iii) Type 3 [$p(x) = 0$ and $q(x) \neq 0$]

$$y''(x) = q(x)y'(x) + g(x) + \lambda \int_a^b K(x,t)y(t)\,\mathrm{d}t, \qquad (1.10)$$

(iv) Type 4 [$p(x) \neq 0$ and $q(x) \neq 0$]

$$y''(x) = q(x)y'(x) + p(x)y(x) + g(x) + \lambda \int_a^b K(x,t)y(t)\,\mathrm{d}t, \qquad (1.11)$$

In this thesis, Type 1 and Type 2 of the first-order linear FIDE and second-order linear FIDE will be solved. In addition, the inital conditions will be used in solving first-order linear FIDE, while boundary conditions will be used for second-order linear FIDE. In this thesis, it is important to know the type of FIDE before solve the problem in order to choose the right method that will be applied. The FIDE of the second kind appear in a various of scientific applications such as the theory of signal processing and neural networks (Wazwaz, 2011).

During this several decades, there are many researches that introduced variety of efficient numerical methods and analytical methods for solving FIDE. In order to solve the integral term in (1.4) - (1.11), numerical quadrature rules will be used. Therefore, in this thesis direct quadrature method will be considered for solving the integral term in FIDE problems. The examples of direct quadrature method are Newton-Cotes quadrature rule, Gregory rule and Gaussian quadrature formula. In this thesis, the Newton-Cotes quadrature rule will be considered to solve the integral part.

1.3 Objective of the Thesis

The main objective of this thesis is to solve Type 1 and Type 2 for first-order and second-order linear FIDE of second kind by using quadrature-difference method. Therefore, the main objective of this thesis can be achieved by

1. Constructing the formulation of the first derivative of 5-point finite difference together with composite Simpson's $\frac{3}{8}$ rule to generate a system of linear algebraic equations for solving first-order linear FIDE.

2. Constructing the formulation of the second derivative of 5-point finite difference together with composite Simpson's $\frac{3}{8}$ rule to generate a system of linear algebraic equations for solving second-order linear FIDE.

3. Constructing the formulation of the first derivative of 7-point finite difference together with composite Boole's rule to generate a system of linear algebraic equations for solving first-order linear FIDE.

4. Constructing the formulation of the second derivative of 7-point finite difference together with composite Boole's rules to generate a system of linear algebraic equations for solving second-order linear FIDE.

5. Solving the system of linear algebraic equations using Gauss elimination method in C programming.

1.4 Scope of the Study

This research will focused on solving Type 1 and Type 2 of first-order and second-order linear FIDE of the second kind. In solving second-order linear FIDE, the boundary condition will be considered. The quadrature-difference methods are implemented for solving linear FIDE. The quadrature rules that used in this research are composite Simpson's $\frac{3}{8}$ rule and composite Boole's rule, while for finite difference method, forward, central and backward difference of the 5-point finite difference and 7-point finite difference will be applied. The quadrature rules and finite difference method were discretized and a system of linear algebraic equation are generated. The Gauss Elimination method will be used to solve the system of linear algebraic equation.

1.5 Outline of the Thesis

This thesis will cover five chapters such in the following contents:

Chapter 1 is a brief introduction of this thesis. FIDE is introduced in this chapter together with the objective of the thesis and the scope of the study. In Chapter 2, the reviews of previous work done by other researchers which related to FIDE are given.

Besides that, the mathematical concepts on FIDE are included. The basis definitions and properties of quadrature rules and finite-difference method are presented.

In Chapter 3, the first-order and second-order linear FIDE of the second kind are solved by using quadrature-difference methods which are formulation of composite Simpson's $\frac{3}{8}$ rule together with first derivative and second derivative of 5-point finite difference method. Firstly, the derivation of composite Simpson's $\frac{3}{8}$ rule and the derivation of 5-point finite difference method for first derivative and second derivative are shown. The order of accuracy for the proposed methods are investigated. Next, the formulation of composite Simpson's $\frac{3}{8}$ rule with 5-point finite difference method for first-order and second-order linear FIDE together with the algorithms are given. Several numerical problems for first-order and second-order linear FIDE are tested and the numerical results are presented in order to show the accuracy of the proposed method. Lastly, the discussion on numerical results are presented.

In the next chapter, the quadrature-difference methods which based on the formulation of composite Boole's rule together with first derivative and second derivative of 7-point finite difference method are presented for solving first-order and second-order linear FIDE of the second kind. First, the methods are derived and the order of accuracy of the methods are determined. Next, the proposed methods for solving first-order and second-order linear FIDE are formulated in order to produce the approximation equations. The algorithms of the proposed methods are presented. In order to show the accuracy of the proposed method, some numerical experiments are carried out.

Lastly, Chapter 5 contains the summary of the thesis and the future work that can be suggested or extended from this research.

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LIST OF PUBLICATIONS

Publications that arise from the study are:

Jalius, C. and Majid, Z. A. 2015. Application of the Quadrature-Difference Method for Solving Fredholm Integro-Differential Equations. In *IEEE Conference Proceedings*. pp. 66–70.

Chriscella Jalius and Zanariah Abdul Majid. 2017. Numerical Solution of Second-Order Fredholm Integro-differential Equations with Boundary Conditions by Quadrature-Difference Method. *Journal of Applied Mathematics*. Vol. 2017 : pp. 1–5.

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