UNIVERSITI PUTRA MALAYSIA

CHARACTERIZING TWO- AND THREE-QUBIT ENTANGLEMENT CLASSES BY THEIR TENSORS

CHOONG PAK SHEN

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CHARACTERIZING TWO- AND THREE-QUBIT ENTANGLEMENT CLASSES BY THEIR TENSORS

By

CHOONG PAK SHEN

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Master of Science

December 2016

DEDICATIONS

To my beloved ones.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Master of Science

CHARACTERIZING TWO- AND THREE-QUBIT ENTANGLEMENT CLASSES BY THEIR TENSORS

By

CHOONG PAK SHEN

December 2016

Chairman : **Assoc. Prof. Hishamuddin Zainuddin, PhD**
Faculty : **Institute For Mathematical Research**

In this study, local unitary (LU) properties of two- and three-qubit quantum systems are studied. Specifically, the Schmidt decomposition approach to LU classification for two qubits is re-examined using a more general approach, i.e. higher order singular value decomposition (HOSVD). Later, HOSVD is used to classify three-qubit pure states by LU operations. We found that due to HOSVD, it is possible to characterize the entanglement classes of three qubits by the eigenvalues distribution of its one-particle reduced density matrices. This finding generalized the similar classification results in the literature and it is hoped that this work will fill in the gap of LU classification from bipartite to multipartite quantum states by using HOSVD.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

PENCIRIAN KELAS KETERBELITAN BAGI KEADAAN KUANTUM TULEN DUA DAN TIGA QUBIT DENGAN TENSOR MEREKA

Oleh

CHOONG PAK SHEN

Disember 2016

Pengerusi : **Prof. Madya Hishamuddin Zainuddin, PhD**
Fakulti : **Institut Penyelidikan Matematik**

Penyelidikan ini mengkaji sifat-sifat unitari setempat (LU) dalam sistem kuantum dua dan tiga qubit. Khususnya, pengelasan LU bagi sistem kuantum dua qubit yang menggunakan huraian Schmidt akan diperiksa dengan satu huraian yang lebih umum, iaitu huraian nilai singular tertib tinggi (HOSVD). Kemudian, HOSVD juga digunakan untuk mengelaskan keadaan tulen tiga qubit dengan operasi LU. Kita mendapati daripada HOSVD bahawa kelas keterbelitan dalam sistem kuantum tiga qubit boleh dicirikan dengan taburan nilai eigen bagi matriks ketumpatan terturun satu partikel. Hasil penyelidikan ini membolehkan kita mengitlak hasil pengelasan yang serupa dengan rujukan lain. Dengan ini, kami berharap penyelidikan kami akan mengisi kekosongan dalam rujukan berkaitan dengan pengelasan LU dari keadaan kuantum dwibahagian sehingga keadaan kuantum multi bahagian.

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I certify that a Thesis Examination Committee has met on 1 December 2016 to conduct the final examination of Choong Pak Shen on his thesis entitled "Characterizing Two- and Three-Qubit Entanglement Classes by their Tensors" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Zuriati binti Ahmad Zukarnain, PhD
Associate Professor
Faculty of Computer Science and Information Technology
Universiti Putra Malaysia
(Internal Examiner)

Jesni bin Shamsul Shaari, PhD
Assistant Professor
Universiti Islam Antarabangsa Malaysia
Malaysia
(External Examiner)

NOR AINI AB. SHUKOR, Ph.D.
Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 28 February 2017

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Master of Science.

The members of the Supervisory Committee were as follows:

Hishamuddin Zainuddin, PhD
Associate Professor
Institute for Mathematical Research
Universiti Putra Malaysia
(Chairperson)

Isamiddin S. Rakhimov, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

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School of Graduate Studies
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Signature: ______________________________
Name of
Member of
Supervisory
Committee: Prof. Isamiddin S. Rakhimov, PhD

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LIST OF SYMBOLS

$\mathscr{H}$	Hilbert space
$\mathbb{C}$	Complex vector space
$\lvert\varphi\rangle$, $\lvert\psi\rangle$, $\lvert\vartheta\rangle$	Ket vectors of one, two and three qubits
$\langle\varphi\rvert$, $\langle\psi\rvert$, $\langle\vartheta\rvert$	Bra vectors of one, two and three qubits
$\mathbf{\Phi}$, $\mathbf{\Psi}$, $\mathbf{\Theta}$	Tensors of one, two and three qubits
φ_i, ψ_{ij}, ϑ_{ijk}	Probability amplitudes (tensor elements) of one, two and three qubits
$\overline{\text{Overhead bar}}$	Complex conjugate
^{T}	Matrix transpose
†	Conjugate transpose
ρ^{φ}, ρ^{ψ}, ρ^{ϑ}	Density matrices of one, two and three qubits
$(\rho^{\psi})^A$, $(\rho^{\psi})^B$	One-particle reduced density matrices of two qubits
$(\rho^{\psi})^A_d$, $(\rho^{\psi})^B_d$	Diagonalized one-particle reduced density matrices of two qubits
$(\rho^{\vartheta})^{AB}$, $(\rho^{\vartheta})^{BC}$, $(\rho^{\vartheta})^{CA}$	Reduced density matrices of three qubits
$(\rho^{\vartheta})^A$, $(\rho^{\vartheta})^B$, $(\rho^{\vartheta})^C$	One-particle reduced density matrices of three qubits
$(\rho^{\vartheta})^A_d$, $(\rho^{\vartheta})^B_d$, $(\rho^{\vartheta})^C_d$	Diagonalized one-particle reduced density matrices of three qubits
$\otimes$	Tensor product
$U(n)$	Unitary group of degree n
$SU(n)$	Special unitary group of degree n
V^{I_n}	n-th vector space of dimension I_n
$\mathscr{X}$	Arbitrary tensor
$X_{(n)}$	n-th matrix unfolding of an arbitrary tensor
$\chi_{i_1 i_2 \ldots i_N}$	N-th order tensor elements
$\mathscr{T}$	Core tensor
$t_{i_1 i_2 \ldots i_N}$	Elements of the core tensor $\mathscr{T}$
$\mathscr{T}_{i_n=\alpha}$	Subtensor of the core tensor $\mathscr{T}$ when the n-th index is fixed to α
$\sigma_i^{(n)}$	i-th n-mode singular value

LIST OF ABBREVIATIONS

EPR	Einstein, Podolsky, Rosen
HOSVD	Higher Order Singular Value Decomposition
LOCC	Local Operation and Classical Communication
LU	Local Unitary
SLOCC	Stochastic Local Operation and Classical Communication
SVD	Singular Value Decomposition

CHAPTER 1

INTRODUCTION

1.1 Overview

Quantum entanglement is one of the distinctive features of quantum mechanics. Coined by Schrödinger (1935), it is interpreted to be the non-classical correlation found between the subsystems of a composite quantum system. This correlation is non-local (Bell, 1964) and can be a resource in quantum information science (Horodecki et al., 2009). Therefore, it is only natural to ask whether one can effectively quantify entanglement just like other physical quantities. Meanwhile, one would also want to be able to manipulate and characterize entanglement so that this resource could be used in quantum information science (Meznaric, 2012).

To answer the first question, several entanglement measures have been proposed, the common ones are the Shannon entropy (Nielsen and Chuang, 2000), concurrence and entanglement of formation (Wootters, 1998, 2001). Meanwhile, the qualitative studies of entanglement revolves around three different types of operations, namely the local unitary (LU), local operation and classical communication (LOCC) and stochastic local operation and classical communication (SLOCC). These operations are of special importance in quantum information science. Elements of these three operations will form a group on their own and divide the quantum states into equivalence classes when acting on them. These equivalence classes are sometimes called *entanglement classes* or *entanglement types*. In this way, one has classified the quantum states according to the group action of one of these operations.

Between the quantitative and qualitative studies on entanglement, our interest lies in the latter. Specifically, we would like to understand the LU properties of two- and three-qubit pure states using a tensor decomposition introduced by Lathauwer et al. (2000), called the higher order singular value decomposition (HOSVD).

1.2 Motivation

For the past decade, classification of quantum states by LU, LOCC and SLOCC operations has been studied extensively because of their importance in quantum information science. From the well-known example of LU classification of two-qubit pure states due to Schmidt decomposition, recent literature revealed that this problem has been extended to multipartite mixed states by various means.

Among all the approaches introduced in the literature, our work was inspired from the idea proposed by Carteret and Sudbery (2000). In short, Carteret and Sudbery showed that it is possible to classify two and three qubits pure states by LU operation through the tensor components of their respective quantum states. Due to Schmidt decomposition, the tensor components of two qubits is readily transformed into the

Schmidt form, so that the LU classification of two qubits is greatly simplified. On the other hand, a series of matrix operations and decompositions is needed to classify three-qubit pure states by LU properties. One would anticipate that Carteret and Sudbery's three-qubit LU classification can be considerably simplified if Schmidt decomposition could be generalized so that it is applicable in three-qubit case.

As a matter of fact, a suitable candidate for the generalized version of Schmidt decomposition has been introduced by Lathauwer et al. back in 2000. As a whole, their work generalized the Schmidt decomposition, which is a restatement of singular value decomposition (SVD), into higher order singular value decomposition (HOSVD) that can be applied to tensors of arbitrary orders. Recent studies also revealed that HOSVD can be used to compare the LU equivalence of two given multipartite states, regardless of pure or mixed states (Liu et al., 2012; Li and Qiao, 2013; Li et al., 2014).

To date, a scheme to completely classify multipartite states by LU operation using HOSVD is not explicitly given yet. Before we try to propose such a scheme, we would like to understand HOSVD and its role in quantum information science. Therefore, this work emphasizes on studying the specific example of two- and three-qubit pure states using HOSVD so that comparison between known results can be performed in the later stage.

1.3 Problem Statement

Stated as one of the postulates of quantum mechanics, the Hilbert space of a composite quantum system is constructed by the tensor product of its subsystems' Hilbert spaces. Therefore, the properties of tensors is automatically incorporated in this tensor product space. While the problem of LU classification of quantum states and the properties of tensors are separately well-studied, they are not strongly linked together.

Initiated by Carteret and Sudbery (2000), they had shown that the properties of tensors can be helpful in studying the LU classification of three-qubit states. Later, (Liu et al., 2012; Li and Qiao, 2013; Li et al., 2014) also showed that by using HOSVD, one can determine the LU equivalence between two arbitrary states. While the LU classification of multipartite states using HOSVD might have been suggested, to our knowledge explicit examples are not sufficiently provided in the literature. The implications of decomposing tensors using HOSVD are not discussed anywhere else.

Given such a scenario, this research is dedicated to tackle the following problems:-

1. What is the role of matrix unfolding and HOSVD in quantum information science?

2. How will the generalization of SVD to HOSVD affect the LU classification results of two- and three-qubit pure states?

3. Is HOSVD sufficient to completely classify two- and three-qubit pure states by LU operation?

1.4 Objectives

This research aims to study the LU classification of two- and three-qubit pure states through the method of tensors. Specifically, the objectives of this work are to:-

1. demonstrate the method of matrix unfolding and HOSVD by using two- and three-qubit tensors
2. classify two- and three-qubit pure states by LU operation using HOSVD
3. compare the classification results of this work with well-known results and study the effects of HOSVD on the two- and three-qubit tensors

1.5 Scope Of Present Work

This thesis is divided into five chapters, each chapter revolving around the central objectives defined in this chapter. In detail, Chapter 1 introduces our work, the purposes and the motivation behind it. In Chapter 2, we provide a review of the historical development and latest discoveries in this topic. Chapter 3 contains the technicalities and fundamentals required to understand this work. We will present and discuss our results in Chapter 4, while concluding remarks and suggested future works will be included in Chapter 5.

Within the constraints of time and resources, we have to limit ourselves to consider only the LU classification of two- and three-qubit pure states.

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BIODATA OF STUDENT

The student of this thesis, Choong Pak Shen, was born in Sungai Siput, Perak in the year of 1988. He enrolled to the Universiti Sains Malaysia in 2008 and graduated his bachelor of science with first class honours. He was awarded with the Honourable Dato' Professor Chatar Singh Gold Medal upon graduation. Currently, he is pursuing his master degree in Institute for Mathematical Research, Universiti Putra Malaysia in the field of quantum information science.

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