



**UNIVERSITI PUTRA MALAYSIA**

***CONVECTION BOUNDARY LAYER FLOW OVER  
VARIOUS SURFACES IN NANOFLUID***

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# CONVECTION BOUNDARY LAYER FLOW OVER VARIOUS SURFACES IN NANOFLUID

By

NOOR SYAMIMI BINTI OMAR

Thesis Submitted to the School of Graduate Studies, Universiti  
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December 2016

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the Master of Science

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December 2016

**Chair: Norfifah binti Bachok @ Lati, PhD**

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The study of convection boundary layer flows over various surfaces in nanofluids is considered. Conventional heat transfer fluids, such as water, ethylene glycol and engine oil, have limited capabilities, hence, limiting the enhancement of the performance of the thermal applications. On the other hand, most solids, such as metals, oxides, carbides or carbon nanotubes, have unique physical properties (nano-sized particles and high thermal conductivity). Thus, an innovative idea has been introduced, which, uses a mixture of nanoparticles and base fluid in order to develop advanced heat transfer fluids with substantially higher conductivities. In this study, mathematical models are derived for three different problems of boundary layer flow and heat transfer over a moving plate, stretching or shrinking cylinder and stretching or shrinking sheet in nanofluids. The governing nonlinear partial differential equations are transformed into a system of nonlinear ordinary differential equations using a similarity transformation. The resulting system of equation are solved numerically for three different nanoparticles, namely copper (Cu), alumina ( $\text{Al}_2\text{O}_3$ ) and titania ( $\text{TiO}_2$ ) in a water-based fluid with Prandtl number ( $\text{Pr} = 6.2$ ) by using shooting technique with MAPLE software. The numerical results are presented in tables and graphs for the skin friction coefficient, the local Nusselt number and the local Sherwood number as well as the velocity, temperature and concentration profiles for a range of various parameters such as nanoparticles volume fraction,  $\varphi$ , velocity ratio parameter,  $\lambda$ , stretching or shrinking parameter,  $\varepsilon$ , curvature parameter,  $\gamma$ , stratification parameter,  $St$ , magnetic field parameter,  $M$ , and suction parameter,  $s$ . It is observed that the skin friction coefficient, the local Nusselt number which represents the heat transfer rate at the surface and the local Sherwood number are signifi-

cantly influenced by these parameters. Moreover, dual and unique solutions are obtained for a certain range of the parameters involved and inclusion of nanoparticles in water-base fluid does affects the skin friction coefficient as well as the heat transfer rate.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Sarjana Sains

## **ALIRAN LAPISAN SEMPADAN OLAKAN TERHADAP PELBAGAI PERMUKAAN DALAM BENDALIRNANO**

Oleh

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Kajian aliran sempadan olakan terhadap pelbagai permukaan dalam nanobendalir dipertimbangkan. Pemindahan haba bendalir klasik, seperti air, etilena glikol dan minyak enjin, mempunyai kebolehan yang terbatas, jadi, menghadkan peningkatan dalam pengendalian aplikasi terma. Sebaliknya, kebanyakan pepejal, seperti logam, oksida, karbait, atau karbon nanotiub, mempunyai ciri fizikal yang unik (zarah bersaiz-nano dan kekonduksian terma yang tinggi). Oleh itu, idea inovasi diperkenalkan, iaitu, menggunakan campuran nanozarah dan bendalir asas untuk menghasilkan bendalir pemindahan haba yang baik dengan kekonduksian haba yang tinggi. Dalam kajian ini, model matematik diterbitkan bagi tiga masalah yang berbeza bagi aliran sempadan olakan dan kadar pemindahan haba terhadap plat bergerak, silinder yang meregang atau mengecut dan lapisan yang meregang atau mengecut dalam nanobendalir. Persamaan pembezaan separa tak linear dijemakan kepada sistem persamaan pembezaan biasa tak linear menggunakan penjelmaan keserupaan. Sistem persamaan tersebut diselesaikan secara berangka bagi tiga nanobendalir yang berbeza, dinamakan kuprum(Cu), alumina ( $Al_2O_3$ ) and titania ( $TiO_2$ ) dalam bendalir asas air dengan nombor Prandtl ( $Pr = 6.2$ ) menggunakan teknik luruan dengan perisian MAPLE. Keputusan berangka dipersembahkan dalam bentuk jadual dan graf bagi pekali geseran kulit, nombor Nusselt setempat, dan nombor Sherwood setempat serta profil halaju, suhu dan kelikatan, bagi pelbagai parameter dengan julat yang tertentu, seperti pecahan isipadu nanozarah,  $\varphi$ , parameter pecahan halaju,  $\lambda$ , parameter regangan atau kecutan,  $\varepsilon$ , parameter lengkungan,  $\gamma$ , parameter stratifikasi,  $St$ , parameter medan magnetik,  $M$ , and parameter sedutan,  $s$ . Didapati bahawa pekali geseran kulit, nombor Nusselt setempat yang mewakili kadar peminda-

han haba pada permukaan dan nombor Sherwood setempat dipengaruhi oleh parameter tersebut. Selain itu, penyelesaian dual dan unik terhasil dengan julat tertentu bagi parameter yang terlibat dan penambahan nanozarah dalam bendalir asas air memberi kesan terhadap pekali geseran kulit beserta kadar pe-ningkatan haba.



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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
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## LIST OF ABBREVIATIONS

$a, A$	constants
$B_0$	uniform magnetic field strength
$C$	concentration
$C_f$	skin friction coefficient
$C_\infty$	concentration of the nanofluid far from the sheet
$C_w$	concentration at the surface
Cu	Copper
$D$	specific heat diffusivity
$D_1$	coefficient of mass flux
$Ec$	Eckert number
$f$	non-dimensional stream function
$J_w$	mass flux
$k$	thermal conductivity
$K_0$	chemical reaction parameter
$l$	characteristic length
$L$	constant reference length
$M$	magnetic field parameter
$Nu_x$	local Nusselt number coefficient
$p$	pressure
Pr	Prandtl number
$q_w$	heat transfer flux
$r$	coordinate along radial direction
$R$	constant radius of cylinder
$Re_x$	Reynold's number
$s$	constants, suction parameter
$Sc$	Schmidt number
$Sh_x$	Sherwood number
$Sr$	Soret number
$St$	stratification parameter
$t$	constants
$T$	temperature
$T_\infty$	free stream temperature
$T_w$	surface temperature
$u$	constant, velocity component along x-axis
$u_w$	stretching or shrinking velocity
$U$	composite velocity
$U_\infty$	free stream velocity
$U_w$	surface velocity
$v$	constant, velocity component along y-axis
$V_w$	prescribed distribution of wall mass suction
$x$	Cartesian coordinates
$y$	Cartesian coordinates

### Greek Symbols

$\alpha$	thermal diffusivity
$\beta$	scaled chemical reaction parameter
$\eta$	similarity variables
$\varepsilon$	stretching or shrinking parameter
$\gamma$	curvature parameter
$\lambda$	moving plate velocity
$\mu$	viscosity
$\nu$	kinematic viscosity
$\varphi$	nanoparticles volume fraction
$\phi$	non-dimensional concentration
$\psi$	stream function
$\rho$	fluid density
$\rho C_p$	heat capacity
$\theta$	non-dimensional temperature
$\tau_w$	surface shear stress

### Subscripts

$c$	critical value
$f$	fluid
$nf$	nanofluid
$s$	solid
$w$	condition at the surface
$\infty$	free stream condition

### Superscript

$'$	differentiation with respect to $\eta$
-----	--

# CHAPTER 1

## INTRODUCTION

### 1.1 Introduction

Mathematical modeling are used by researchers in any field for numbers of different reasons. It is a representation for behavior of a real devices or problems in mathematical term. Modeling may help the researchers to explain a process or a system and to study the effects of certain component and to make prediction on the behavior. However, in certain case where some model might not be valid or not consistent with its principle or assumptions. Hence, the model can be modified to satisfy a specify situation or problems will be discussed.

The science of fluid dynamics deals with the motion of fluid and forces that act on it. Boundary layer flow is one of the phenomenon that very prominent in research study since 1904 where professor Ludwig Prandtl, who first discover the concept of boundary layer in fluid flow over a surface. The theory was he assumed that there was no slip condition at the surface because of the effect of friction force that caused the fluid immediately adjacent to the surface and the frictional effects were experienced only in boundary layer which is the tiny region near the surface. The systematic calculation produces the flow variables in the boundary layer, including the velocity gradient at the surface. By those velocity gradients, the skin-friction drag on the surface or called wall shear stress can be determined (Anderson, 2005).

Heat transfer is a process that we experienced in our daily life. For example, by putting a canned of warm soft drink in a refrigerator to let it cool down and putting it back to warm it up in room temperature. The temperature differences in a system are reduced by heat exchange from region of higher temperature to the lower temperature. This process can occur in all matter which are found in nature including solid bodies, liquids and gases. There are three different mechanism of heat transfer which are conduction, convection and radiation (Eckert and Drake, 1959). Conduction is a process of a energy transfer from more energetic particles to less energetic one when both interact with each other. Convection is a mode heat transfer between a solid surface and the adjacent liquid or gas that is in motion. While, radiation is energy that emitted in matter in the form of electromagnetic waves which result in the changes in electronic configuration or the molecules.

Mass transfer is the movement of a chemical species from high concentration to lower concentration. Mass transfer is distinct from bulk motion (fluid flow). The main driving force for fluid flow is temperature difference, while mass transfer is concentration difference (Pang et al., 2015). Mass transfer happens in many processes, such as evaporation, drying, precipitation, adsorption, membrane filtration and distillation. Besides that, mass transfer is being used for different processes and mechanisms. Those terms are commonly

used in engineering and biological process. Some common examples of mass transfer processes are the evaporation of water from a pond to the atmosphere, the distillation of alcohol and the purification of blood in the kidneys and liver. On the other hand, the engineering processes that involves mass transfer operation such as separation of chemical components in distillation columns, absorbers i.e scrubbers, activated carbon beds and liquid extraction.

## **1.2 Research Background**

The study of boundary layer flow and heat transfer characteristic have been carried out by many researchers over the past few decades. In this thesis, a laminar flow over a moving plate and stretching or shrinking cylinder will be taken into consideration. There are also some factors that will affect the boundary layer flow and heat transfer characteristic.

### **1.2.1 Types of Fluids**

Working fluid has involved in various engineering and industrial process, which is used in cooling processes and heat transfer. Water, oil and ethylene glycol mixture are some of the conventional low thermal conductivity fluid, which limits the heat transfer enhancement (Khanafer et al., 2003). This motivates Choi (1995) to develop an innovative idea, which suspending nano-sized particles into a base fluid, called nanofluid. The materials that mostly used as nanoparticles are oxides such as alumina, silica, titania and copper oxides. Besides that, some metals such as copper and gold are also included (Buongiorno, 2006). Nanofluids have nanometer-sized particles which are smaller than 100 nm. This unique property gives them the advantage to have the ability to flow smoothly through the microchannels easily, hence, enhance the thermal conductivity and convective heat transfer coefficient compared to the base fluid only. Therefore, it is well known to be used as coolants, lubricants, heat exchangers and micro-channel heat sinks (Rohini et al., 2012).

The nanofluid model proposed by Buongiorno (2006) and Tiwari and Das (2007) are most common model that has been used widely by other researchers. Buongiorno model stated that the nanoparticle absolute velocity can be viewed as the sum of the base fluid velocity and a relative velocity (slip velocity). He considered seven slip mechanisms which are inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage and gravity settling. Differently, the Tiwari and Das model taking into accounts of the solid volume fraction of the nanofluid (Bachok et al., 2013a; Nield and Kuznetsov, 2009). In this thesis, the nanofluid model by Tiwari and Das (2007) will be used.

### **1.2.2 Types of Surfaces**

There are two types of surfaces that will be considered in this thesis. They will be moving flat plate and cylinder surface. Moreover, a stretching or shrinking case are also taken into consideration.

#### 1.2.2.1 Plate

The pioneering study on boundary layer over a plate was by Blasius (1908). Blasius considered the boundary layer flow on a semi-infinite fixed flat plate, without considering the heat transfer characteristics. However, the Blasius equations has never gives an exact analytical solutions (Ishak et al., 2009a). Later, Sakiadis (1961) considered the boundary layer flow over a moving flat plate in a quiescent fluid. The equation that he found was the same with previous study by Blasius (1908) but with different boundary conditions (see Ahmad et al., 2011). There are many researchers that extended the problem of Blasius and Sakiadis problem to different type of flow, effects and fluid such as Bachok et al. (2010a), Das and Jana (2015), Ja'fari and Rahimi (2013) and Rosca et al. (2014). The function of flow over a moving surface are important in engineering application such as paper production, extrusion of plastic sheet, drawing plastic films and many more (see Lok et al., 2012; Jat et al., 2012).

#### 1.2.2.2 Cylinder

Flow over cylinder is considered to be two-dimensional if the body-radius is large compared to the boundary layer thickness. Meanwhile, for a thin or slender cylinder, the radius of the cylinder may be of the same order as that of the boundary layer thickness. Hence, the flow may be considered as axisymmetric instead of two-dimensional (Vajravelu et al., 2012; Mukhopadhyay, 2013b; Najib et al., 2014b).

#### 1.2.2.3 Stretching or Shrinking

The production of drawing plastic films, the strips is sometimes stretched. Besides that, the production of sheeting material arises in a number of industrial manufacturing processes and includes both metal and polymer sheets (Ishak et al., 2008b; Mukhopadhyay, 2013a). Crane (1970) was the first to investigate the steady two-dimensional flow caused by the stretching of a sheet and reported an exact solution (Ishak et al., 2008b). This problem has later been extended by many researches such as Anderson (2002), Mahapatra and Gupta (2003), Wang (2007), Liu and Anderson (2008) and Ishak et al. (2009b).

Some cases occur when the sheet is stretched, it will move back to the origin point (shrinking). In other words, the velocity of the boundary layer flow is towards a fixed points. Micklavcic and Wang (2006) were the first who investigated the flow in shrinking sheet and found that the flow is unlikely to exist either an adequate suction on the boundary is imposed (Micklavcic and Wang, 2006) or stagnation point is considered (Wang, 2008). It is regarding on the vorticity generated due to the shrinking of the sheet is not confined within the boundary layer and so the steady flow over a shrinking sheet is not possible unless some opposite force is used to prevent the vorticity diffusion and to maintain boundary layer structure (Bhattacharyya, 2011; Bhattacharyya, 2013). Some of the applications in several industries and environments, include the extrusion of plastic sheets, thinning and annealing of

copper wires, making electronic chips, cooling of metallic sheets, hot rolling, metal extrusion, glass fiber production, wire drawing, paper productions, etc (Loganthan and Vimala, 2015).

### **1.2.3 Types of Effects**

There are a number of aspects of problems could be further investigated. In particular, the consideration of the simultaneous effects of transpiration through the sheet raises the number of parameters to three and therefore substantial extra numerical difficulties appear. Here are some of the effects that has been considered in this study.

#### **1.2.3.1 Stratification**

The ambient medium of natural convection flow sometime is non-isothermal (Jaluria and Gebhart, 1974). Hence, most of the free convection processes occur in environment with temperature stratification. Some of the perfect physical explanation are closed containers and environmental chambers with heated walls, a room which is heated by electrical wires embedded in the ceiling and a room fire with an open door or window through which fresh air is supplied near the bottom offers another example of a thermally stratified situation. The atmosphere itself is thermally stratified, such as the ocean (Yang et al., 1972; Jaluria and Gebhart, 1974; Kulkarni et al., 2011). In addition, stratification of fluids occurs not only with temperature differences but also variation of concentration and arises due to the presence of different fluids so that a stable situation arises when the lighter fluid overlies the denser one Mukhopadhyay and Ishak (2012).

#### **1.2.3.2 MagnetoHyDromagnetic (MHD)**

The study of magneto-hydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal-working processes. The uses of magnetic field to control the flow and heat transfer processes in fluids close to different types of boundaries has led considerable interest in the study of boundary layer flows subjected to an externally applied magnetic field (Chandran et al., 1996). The presence of magnetic field in the flow produces drag force, called Lorentz force which will increase the boundary layer thickness and decrease the heat transfer (Elbashbeshy et al., 2012). Despite of that, by producing Lorentz force, the magnetic field is able to transport liquids in the mixing processes as an active micromixing technology method. Since most biological transportation applications based on magnetic fields are in the micro or nano system (Yazdi et al., 2011a; Yazdi et al., 2011b). Besides that, in influence of magnetic field, one has to consider the effect of viscous dissipation on the boundary layer flow since it has a direct influence on the heat transfer rate (Salem et al., 2014).

### 1.2.3.3 Suction or Injection

Suction or injection are applied to a permeable surface. Suctions or injections of a fluid through the bounding surface, as, for example, in mass transfer cooling, can significantly change the flow field and, as a consequence, affects the heat transfer rate from the surface. It is often necessary to delay the separation of the boundary layer to reduce drag and attain high lift values (Pop and Watanabe, 1992). In general, suction tends to increase the skin friction and heat transfer coefficients, whereas injection acts in the opposite manner. Suction is applied in chemical processes to remove reactants while injection is used to add reactants, cool the surface, prevent corrosion or scaling and reduce the drag. Injection or withdrawal of fluid through a porous bounding heated or cooled wall is of general interest in practical problems involving boundary layer control applications such as film cooling, polymer fiber coating, coating of wires, etc (Ishak et al., 2008d).

### 1.2.4 Types of Solutions

In mathematical field, boundary layer problem is a differential equation together with a set of additional constraints, called boundary conditions. Solution to a boundary layer problem is a solution to the differential equation that satisfy the boundary conditions. The study of flow over a flat and cylinder surface highlighted many important features, including the situation where the governing differential equations may admit zero, one or two solutions. No solutions is when the boundary layer separates from the surface where the solution for the boundary layer approximation cannot be obtained. To obtain the solutions, the full version of the Navier-Stokes equations has to be used. A solution is exists when it satisfies the boundary layer approximations and the physical meaning can be obtained. Plus, the solution is claimed to be stable than the second solution (see Ishak et al., 2009a; Weidman et al., 2006). However, although the second solutions are not stable which deprived of physical significance, the existence of the solutions can not be neglected. Besides, similar equations may arise in other situation where the corresponding solutions could have more realistic meaning (Ridha, 1996).

## 1.3 Problem Statement

A lot of studies have been done in boundary layer flow of nanofluids towards different surfaces. In this study, plate and cylinder surface are the two types of cases that will be covered. Some of the issues regarding the boundary layer in nanofluids are:

1. What are the effects of moving plate in nanofluid on skin friction coefficient and local Nusselt number surface as well as velocity and temperature profiles?
2. How does the effect of stratification influence the flow over a moving plate in nanofluid?

3. How does stretching or shrinking cylinder parameter affects the skin friction coefficient and local Nusselt number in nanofluids?
4. How does the solid volume fraction affects the flow and heat and mass transfer?
5. How do presence of suction and chemical reaction affects the flow over a stretching or shrinking sheet?
6. What happen to skin friction coefficient and local Nusselt number on stretching or shrinking sheet in MHD flow?

#### 1.4 Objective and Scopes

The objectives of this study are to extend the mathematical formulations of the problems :

1. Boundary layer flow over a moving plate in nanofluid with presence of stratification effect.
2. Stagnation-point flow over a stretching or shrinking sheet in nanofluid to stretching or shrinking cylinder.
3. MHD flow over a stretching sheet in a nanofluid with chemical reaction and viscous dissipation to stretching or shrinking sheet and Joule effect.

and investigate the numerically the behavior of the boundary layer flow and heat transfer characteristic of nanofluid.

This study takes into consideration the steady two-dimensional laminar fluid flow of a flat surface and cylinder surface which based on nanofluids model by Tiwari and Das.

#### 1.5 Significant of Study

The enhancement of heating or cooling in an industrial process may create a saving in energy, reduce process time, raise thermal rating and lengthen the working life of equipment. The development of high performance thermal systems for heat transfer enhancement has become popular nowadays. A number of work has been performed to gain an understanding of the heat transfer performance for their practical application to heat transfer enhancement. Thus the advent of high heat flow processes has created significant demand for new technologies to enhance heat transfer.

The current advancement in heat transfer fluids, is an engineered colloidal mixture of the base fluids and nano-sized metallic particles (1-100 nm). The earlier versions of colloidal fluids such as micro-fluid substances tend to precipitate and cause erosion in the moving component. Anyhow, nanofluids are claimed to be a non-bundled mono-dispersed particles in the base fluids, which proved to be enhancing the heat transfer more than 50% in realtime

applications even when the volume ratio of the nanoparticle to base fluid is less than 0.3% (Asirvatham, 2015; Hwang et al., 2006).

## 1.6 Outline of Thesis

This thesis covers seven chapters with the following contents:

Chapter 1 is the brief introduction of this thesis which covers the background of this study together with the objectives and scopes. Then, Chapter 2 discuss the previous studies on scope that mention in the earlier chapter. Chapter 3 shows the details mathematical formulation and the numerical method that has been used to solved different problems which be presented in Chapter 5 and 6.

In Chapter 4, we study the boundary layer flow and the characteristic of heat transfer towards moving plate in a stable stratified nanofluids. Chapter 5 discusses on the stagnation point flow and heat transfer characteristic on stretching and shrinking cylinder in nanofluids. Then, Chapter 6 focused on MHD flow and heat transfer characteristic due to a stretching or shrinking sheet with chemical reaction, viscous dissipation and Joule effects in nanofluids.

Lastly, Chapter 7 contains the summary of the study, conclusions and possible studies that can be done in the future.

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## LIST OF PUBLICATIONS

### Published in Journal:

- Omar, N. S. and Bachok, N. and Arifin, N. M. (2015) Stagnation point flow over a stretching or shrinking cylinder in a copper-water nanofluid, In *Indian Journal of Science and Technology*. 8(31):1-7.
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- Omar, N. S. and Bachok, N. and Arifin, N. M. (2014) Boundary layer flow and heat transfer characteristics over a moving plate in a stable stratified nanofluid, In *Proceedings of the Symposium Kebangsaan Sains Matematik ke- 22 (SKSM 22)*, Shah Alam, Selangor, November 24-26, 2014.
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- Bachok, N., Omar, N. S., Arifin, N. M. and Ishak, A. (2015) Stagnation-point flow over a stretching/ shrinking cylinder in a alumina-water nanofluid, In *Proceeding of CHT-15. 6<sup>th</sup> International Symposium on Advances in Computational Heat Transfer*, Rutgers University, Piscataway, USA, 25-29 May 2015.



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