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BAYESIAN SURVIVAL AND HAZARD ESTIMATES FOR WEIBULL REGRESSION WITH CENSORED DATA USING MODIFIED JEFFREYS PRIOR

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By

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Thesis submitted to the school of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Doctor of Philosophy

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May 2013

Chair: Professor Noor AkmaIbrahim,PhD

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In this study, firstly, consideration is given to the traditional maximum likelihood estimator and the Bayesian estimator by employing Jeffreys prior and Extension of Jeffreys prior information on the Weibull distribution with a given shape under right censored data. We have formulated equations for the scale parameter, the survival function and the hazard functionunder Bayesian with extension of Jeffreys prior. Next we consider both the scale and shape parameters to be unknown under censored data. It is observed that the estimate of the shape parameter under the maximum likelihood method cannot be obtained in closed form, but can be solved by the application of numerical methods. With the application of the Bayesian estimates for the parameters, the survival function and hazard function, we realised that the posterior distribution from which Bayesian inference is drawn cannot be

obtained analytically. Due to this, we have employed Lindley's approximation technique and then compared it to the maximum likelihood approach.

We then incorporate covariates into the Weibull model. Under this regression model with regards to Bayesian, the usual method was not possible. Thus we develop an approach to accommodate the covariate terms in the Jeffreys and Modified of Jeffreys prior by employingGauss quadrature method.

Subsequently, we use Markov Chain Monte Carlo (MCMC) method in the Bayesian estimator of the Weibull distributionand Weibull regression model with shape unknown. For the Weibull model with right censoring and unknown shape, the full conditional distribution for the scale and shape parameters are obtained via Gibbs sampling and Metropolis-Hastings algorithm from which the survival function and hazard function are estimated. For Weibull regression model of both Jeffreys priors with covariates, importance sampling technique has been employed. Mean squared error (MSE) and absolute bias are obtained and used to compare the Bayesian and the maximum likelihood estimation through simulation studies.

Lastly, we use real data to assess the performance of the developed models based on Gauss quadrature and Markov Chain Monte Carlo (MCMC) methods together with the maximum likelihood approach. The comparisons are done by using standard error and the confidence interval for maximum likelihood method and credible interval for the Bayesian method.

The Bayesian model for Weibull regression distribution with known and unknown shape using right censored data for Jeffreys prior and modified Jeffreys priors obtained by Gauss quadrature method are better estimators compared to maximum likelihood estimator (MLE). Moreover, the extention of the Bayesian model for Weibull regression distribution using right censored data via Markov Chain Monte Carlo (MCMC) give better result than maximum likelihood estimator (MLE).

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

KELANGSUNGAN HIDUP BAYESIAN DAN ANGGARAN BAHAYA UNTUK REGRESSION WEIBULL DENGAN DATA DITAPIS MENGGUNAKAN DIUBAH SUAI JEFFREYS SEBELUM

Oleh

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Dalamkajianini, pertamanyadipertimbangkanpenganggarkebolehjadianmaksimumtradisionaldanpeng anggarBayesan yang menggunakan prior Jeffreysdankembanganmaklumat prior Jeffreysbagi data tertapissebelahkanan yang bertaburanWeibulldengan parameter bentukdiberikan. Kami telah merumuskan persamaan bagi parameter skala, fungsi mandirian dan fungsi bahaya dibawah Bayesan dengan kembangan prior Jeffreys. Seterusnya kami mempertimbangkan apabila kedua-dua parameter bentuk dan skala tidak diketahui bagi data tertapis ini. Diperhatikan bahawa bentuk tertutup tidak boleh diperolehi apabila kaedah kebolehjadian maksimum digunakan untuk menganggar parameter bentuk, walau bagaimanpun ianya boleh diselesaikan dengan menggunakan kaedah berangka. Bagi menganggar parameter, fungsi mandirian dan bahaya menggunakan kaedah Bayesan, taburan posterior dari mana inferens

Bayesan diperolehi tidak boleh diperolehi secara analitik. Yang demikian kami gunakan teknik penghampiran Lindley dan membandingkannya dengan pendekatan kebolehjadian maksimum.

Kami kemudiannya menggabungkan kovariat ke dalam model Weibull. Dibawah model regresi ini dengan Bayesan, kaedah biasa tidak boleh digunakan. Oleh itu, kami bangunkan suatu pendekatan untuk mengambilkira kovariat dalam Jeffreys prior dan mengubah suai Jeffreys prior dengan menggunakan kaedah kuadratur Gauss.

Seterusnya kami gunakan kaedah Rantai Markov Monte Carlo (RMMC) dalam anggaran Bayesan bagi taburan Weibull dan regresi Weibull dengan parameter bentuk tidak diketahui. Bagi model Weibull dengan tapisan sebelah kanan dan parameter bentuk tidak diketahui, taburan bersyarat yang penuh bagi parameter skala dan bentuk diperolehi melalui pensampelan Gibbs dan algoritma Metropolis-Hastings dari mana fungsi mandirian dan bahaya dianggar. Untuk model regresi Weibull menggunakan kedua-dua prior Jeffreys, teknik pensampelan kepentingan digunapakai. Ralat kuasadua min dan kepincangan mutlak diperolehi dan digunakan untuk membandingkan anggaran Bayesan dengan kebolehjadian maksimum melalui kajian simulasi.

Akhir sekali kami gunakan data sebenar untuk menilai prestasi model yang telah dibangunkan berdasarkan kaedah kuardratur Gauss dan Rantai Markov Monte Carlo bersama pendekatan kebolehjadian maksimum. Perbandingan dilaksanakan dengan menggunakan ralat piawai dan selang keyakinan bagi kaedah kebolehjadian maksimum dan selang kredibel bagi kaedah Bayesan.

Model Bayesian untuk taburan regresi Weibull dengan bentuk yang diketahui dan tidak diketahui menggunakan data tertapis kekanan untuk Jeffreys prior dan Jeffreys prior diubahsuai, yang diperoleh melalui kaedah kuadratur Gauss adalah penganggar yang lebih baik berbanding dengan penganggar kebolehjadian maksimum (maximum likelihood estimation, MLE). Selainitu, kembangan model Bayesian untuktaburanregresiWeibullmenggunakan data tertapis yang betulmelalui MCMC bolehmemberikanhasil yang lebihbaikdaripada MLE.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree atUniversiti Putra Malaysia or at any other institution.

AL OMARI MOHAMMED AHMED

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LIST OF ABBREVIATIONS

MSE	Mean Squared Error
pdf	Probability Density Function
cdf	Cumulative Distribution Function
BJ	Bayesian using Jeffreys prior
BE	Bayesian using extension of Jeffreys prior
MLE	Maximum Likelihood Estimator
MCMC	Markov Chain Monte Carlo
M-H	Metropolis-Hastings algorithm
CP	Percentage of censor

CHAPTER1

INTRODUCTION

1.1 Background

One of the most appealing classical statistical techniques used for fitting statistical models to data as well as providing estimates for the parameters of a model is the maximum likelihood estimation (MLE) method. It is for investigating the parameters of a model. There are two major points for which this method intends to achieve. The first point is that, it provides some sensible computational analysis in our quest to fitting statistical model to data. The second point is that it gives very good response in a computational point of view. The logic or reasoning behind maximum likelihood parameter estimation is to discover those parameters that grow up the probability of a sample data. Statistically, it is considered that maximum likelihood estimation gives good estimates and has very good statistical properties but with some few exceptions. Forthrightly, the maximum likelihood estimation method is considered as multifaceted as a result of which it has been employed in many models with different data sets. In addition to this, it provides very efficient ways of measuring uncertainty via confidence bounds. Maximum likelihood estimation contains distinctively deep mathematical implementation, although it has a simple methodology (Croarkin& Tobias, 2002).

On the other hand, Bayesian estimation approach has recently become a generally acceptable method in estimating parameters which is now in rivalry with other methods. Inthe past, the Bayesian approach was discouraging due to the necessity of numerical integration. However, as a result of the radical change in the computerintensive sampling methods of estimation, the Bayesian method is now vigorously pursued by researchers for its comprehensive approach to the estimation of complex models. In Bayesian, inference is based on the posterior estimate, and the posterior estimate is simply the combination of ones prior knowledge and the availability of the data (the likelihood). When the prior is well defined, the Bayesian approach tends to be very precise because the prior brings in more information and the posterior estimate is based on the combined sources of information (prior and likelihood estimation).

Bayesian analysis can be used as a substitute for hypothesis testing as it is applied in the classical stand point where *p*-values are constructed in the data space. The *p*-value is simply the measure of consistency by calculating the probability of which the results are observed from the data sample, with the assumption that the null hypothesis is true. Those who use this test, mostly interpret the *p*-values as being associated to the hypothesis space; which is observed as a range for the parameter and the data given. In interpreting probabilities of this nature, it is observed that this is more suitably interpreted using the Bayesian approach. The classical approach to confidence interval for the estimation of parameters is consciously perceived because in the analogy, say 95% confidence interval, we have that when the sample is repeated several times there is the likelihood that the true parameter will fall within the range approximately 95% of the time. We also perceived that the true parameter may not be observed after drawing only one sample data because the parameter under investigation is constant. This contradicts the Bayesian analogy in that we see the parameter as being random and can therefore conclude after having observed a sample data say 95%, of the Bayesian credible interval contain the true parameter with approximately 95% certainty (Congdon, 2001).

1.1.1. Right Censoring Data

One of the special features of survival data is censored observations. There are several types of censoring mechanisms and here we will consider right censoring which is made up of Type-I and Type-II. Type-I censoring is where a study is designed to end at some pre-specified given time and an event is said to have taken place if and only if the event occurs before or at the specified time. Censoring times vary according to individuals. We make use of the following notations for right censoring.Consider an individual under study, with the assumption that X represents the lifetime of the individual and C (C for "right" censoring time) the fixed censoring time. X is taken to be independent and identically distributed with probabilitydensity function $f(x)$ and survival function $S(x)$. In a situation where Xgoes beyond C, where C is the censored time, then the individual is said to have survived. The data described above can be represented by T andδ, whereδdenotesthe lifetime if the event occurs,that is$\delta = 1$ orif it iscensored,$\delta = 0$. The observed time T isthe minimum of the failure and censored times that is$T = \min(X, C)$.

Another type of right censoring is Type-II censoring. In this type of censoring the experiment continues till the r-th failure takes place or occurs where$r$ is a pre-specified integer with $r<n$with$n$ as the sample size. This censoring is mostly applied in engineering for testing durability of equipments. Suppose all n units are put on a life test at the same time, the test is terminated when the pre-specified number r out of the n units have failed. One of the advantages of this censoring is that it reduces cost and maximizes judicious use of time since testing all the n units may take a longer time for all to fail thereby resulting in high cost. Since Type-II censored data

consist of a specified r lifetimes out of n, it makes the statistical treatment very simple to deal with because the theory of order statistics is employed directly to determine the likelihood and other inferential techniques. Here, it should be noted that r is the number of failures and n - r the number of censored observations as in Klein and Moeschberger (2003).

1.1.2 Parametric Maximum Likelihood Estimation

The likelihood function of the sample data is simply the mathematical expression of maximum likelihood estimation and the likelihood of a set of data can be said to be the probability of acquiring that particular set of data with respect to the chosen probability model. This mathematical expression has in it the unknown distribution parameters. The parameter value that maximizes the likelihood is referred to as the Maximum Likelihood Estimate or MLE. (Croarkin& Tobias, 2002)

We introduce the concept of maximum likelihood estimation with probability density function (pdf), where we have set ofrandom lifetimes $t_1,\ldots,t_n$ and the vectors of the unknown parameters $\theta=(\theta_1,\ldots,\theta_n)$, then the likelihood function $L(\theta;t)$ is given as

$$L(\theta;t)=\prod_{i=1}^{n} f(\theta;t_i). \qquad (1.1)$$

In trying to determine the MLE's of the parameters that maximizes the likelihood function, we take the natural logarithm of the likelihood function, differentiate it with respect to the unknown parameters and set the resulting equation to zero. In the Weibull model, the scale parameter can easily be determined but with regards to the shape parameter there is the need to employ a numerical approach which in most cases is determined by Newton-Raphson method, as stated by Croarkin& Tobias (2002).

For a regression model with maximum likelihood estimation, we introduce the covariate parameters through the parameter as given below

$$\theta = \exp(\beta' x_i),$$

where, $\beta' = (\beta_0, \beta_1, \beta_2, \ldots, \beta_n)$ is the vector of the parameters of covariate and

$x_i = (1, x_{i1}, x_{i2}, \ldots, x_{in})$ is the vector of covariates.

Then the likelihood function of the covariates $L(\beta'; t)$, is given as

$$L(\beta'; t) = \prod_{i=1}^{n} f(\beta', t_i). \qquad (1.2)$$

The maximum likelihood estimation of the parameters of covariatecan be obtained in a similar manner in the estimation of the likelihood functionas given in equation (1.1). In dealing with the Weibull model, the scale parameter is replaced by the covariate. The parameter of the covariate with respect to the shape parameter cannot be determined analytically, therefore, there is the need to employ a numerical approach which in most cases is determined by Newton-Raphson method.

1.1.3 Survival and Hazard Functions

The essential or elementary measurable property that is employed to characterise time-to-event phenomena is the survival function. It is the probability that an individual will survive past time *t* (where an individual is experiencing the event after time *t*). It is defined as

$$S(t) = \Pr(T > t) . \tag{1.3}$$

With regards to equipment or items failure in a manufacturing industry, *S* (*t*) is known as the reliability function. If *T* is taken to be a continuous random variable, it implies that *S* (*t*) is also continuous, and an absolutely decreasing function. The survival function is a complement of the cumulative distribution function since *T* is a continuous random variable, which is, *S* (*t*) =1- *F* (*t*), where *F* (*t*) = *Pr* (*T* ≤*t*). The survival function is also the integral of the probability density function, *f* (*t*), where

$$S(t) = \Pr(T > t) = \int_t^\infty f(t)\, dt . \tag{1.4}$$

Consequently,

$$f(t) = \frac{-dS(t)}{d(t)} .$$

Observe that *f* (*t*) *dt*can be considered as the “approximate” probability which indicates that the event will occur at time *t* with *f* (*t*) taken as a nonnegative function where the area classified within *f* (*t*) is equal to one (Klein and Moeschberger, 2003).

An important measurable quantity that is central in survival analysis is the hazard function. The hazard function is called the instantaneous failure rate in reliability, the concentrated or intensity function in stochastic processes, in epidemiology it is known as the age-specific failure rate, in demography it is the force of mortality, the inverse of the Mill’s ratio is what it is known in economics, or simply as the hazard function. The hazard function is defined as

$$h(t) = \lim_{\Delta t \to 0} \frac{P[t \le T < t + \Delta t \mid T \ge t]}{\Delta t} \text{.} \tag{1.5}$$

Having considered T as the continuous random variable, then, the cumulative hazard function denoted by $H(t)$ is a relative quantity and given as

$$H(t) = \int_0^t h(u)\, du = -\ln[S(t)] \text{ .}$$

As a result, the continuous lifetime is

$$S(t) = \exp[-H(t)] = \exp[-\int_0^t h(u)\, du].$$

From (1.5), one may observe that $h\ (t)\ \Delta t$ can be expressed as the "approximate" probability about an individual with age t which is experiencing the event at the next moment in time. The hazard function is very useful in ascertaining the desired failure distribution to make use of substantial facts or information surrounding the technicalities of the failure and to explain accordingly the way certain occurrences change with time.

The hazard function has many shapes and that the only limitation is that $h\ (t)$ should be nonnegative, that is $h\ (t) \ge 0$.

1.1.4 Bayesian Estimation

If n items are put on test with the assumption that their recorded lifetimes form a random sample with size n chosen from a particular population and having $f(t\,|\,\theta)$ as the probability density function and density function is conditioned on the parameter, then the joint conditional density with respect to the sampling vector $T = (T_1, T_2, \ldots, T_n)$ is

$$f(t\,|\,\theta) = \prod_{i=1}^{n} f(t_i\,|\,\theta), \quad (1.6)$$

If $t = (t_1, t_2, \ldots, t_n)$, is the lifetime, then $f(t\,|\,\theta)$ can be considered as a function of θ and not of t. If the above condition is satisfied, then we can express this mathematically as $L(\theta; t)$ which is known as the likelihood function of t given θ. To emphatically establish the significance of $f(t\,|\,\theta)$ methodologically, we have

$$f(t\,|\,\theta) = L(\theta; t).$$

We can consider θ as an interpretation of a random vector ω which has $g(\theta)$ as the prior density known as the prior model. In Bayesian inference the prior model is of significance which the details about how $g(\theta)$ can be chosen will be discussed later in this section. The joint densities with respect to T and ω is found by simply applying the multiplication theorem of probabilities as

$$f(t, \theta) = g(\theta) f(t\,|\,\theta). \quad (1.7)$$

The marginal density of the lifetimecan be expressed as

$$f(t) = \int_D g(\theta) f(t\,|\,\theta)\, d\theta. \quad (1.8)$$

with the integral taken over D of θ, the admissible range. The conditional density ofω, given the data$t$, is found by using Bayes' theorem

$$g(\theta\,|\,t) = \frac{f(t,\theta)}{f(t)} = \frac{g(\theta)f(t\,|\,\theta)}{f(t)},\quad(1.9)$$

where $g(\theta\,|\,t)$ is known as the posterior density of ω. The posterior model is used in the Bayesian perspective to make inferences about the parameter $\theta$and for hypotheses testing on θ. We shall in most occasions henceforth refer to the posterior distribution simply as "posterior" and the prior distribution as the "prior". In Bayes theorem if $f(t\,|\,\theta)$ is regarded as the likelihood function that is $L(t\,|\,\theta)$, then (1.6) can be rewritten as

$$g(\theta\,|\,t) \propto g(\theta)L(t\,|\,\theta).\quad(1.10)$$

Equation (1.7) implies that there is a direct proportionality between the product of the prior distribution and the likelihood function against the posterior distribution. The necessity of the proportionality constant needs to be emphasised in that it ensures that the posterior density integrates to one, which is known as the marginal density of T.

In Bayesian estimation approach a loss function is always crucial since it gives an indication about the loss incurred in using $\hat{\theta}$when the true state of nature isθ. If $\hat{\theta} = \theta$then we have a zero loss. As a result of which the loss function $\ell(\hat{\theta},\theta)$is mostly taken to be

$$\ell(\hat{\theta},\theta) = h(\theta)\varphi(\theta - \theta),\qquad(1.11)$$

with $\varphi(.)$ been the non–negative function of the error $\theta-\hat{\theta}$ such that $\varphi(0)=0$ and $h(\cdot)$ is a non–negative weighting function that shows comparatively the seriousness of a given error for different values of θ. If we assume this loss function, the function $h(\theta)$ can be considered as a constituent of the prior $g(\theta)$ in Bayesian estimation. Due to the aforementioned reason, the function $h(\theta)$ in (1.8) is mostly seen as a constant. With one–dimensional parameter say θ, the loss function can be expressed mathematically as

$$\ell(\hat{\theta},\theta) = A\left|\theta-\theta\right|^{B}, \qquad (1.12)$$

where A, $B > 0$. This loss function is called the quadratic or squared-error loss if and only if $B = 2$, but with $B = 1$, (1.9) assumes a linear form and becomes proportional to the absolute value of the estimation error known as the absolute–error loss.

The Bayesian estimator, for any specified prior $g(\theta)$, will be the estimator that minimizes the posterior risk given by

$$E[A(\omega-\hat{\theta})^2 \mid t] = \int_D A(\theta-\theta)^2\, g(\theta \mid t)\, d\theta. \qquad (1.13)$$

provided this expectation exists. After adding and subtracting $E(\omega \mid x)$ and simplifying, we have

$$E[A(\omega-\hat{\theta})^2 \mid t] = A[\theta - E(\omega \mid t)]^2 + A\,Var(\omega \mid t), \qquad (1.14)$$

which is minimized when

$$\hat{\theta} = E(\omega \mid t) = \int_D \theta\, g(\theta \mid t)\, d\theta. \qquad (1.15)$$

With the squared–error loss function the Bayesian estimator is simply the posterior mean of ω given t.

Prior distributions can be divided into several distinct ways. The most common categorisation is by simply dichotomizing the prior into “proper” and “improper”. Proper prior is simply a prior that assumes a positive weight age of the values of the parameters to a total of one. Hence a proper prior is a weight function that meets the condition of probability mass function or a density function. Improper prior on the other hand any weight function that integrates or sums over the possible values of the parameter to a value other than one, say K. If we assume K to be a finite value, then an improper prior can persuade or influence a proper prior by normalizing the function. Other categorizations of priors according to properties, for instance, non–informative, or by distributional forms, e.g., beta, gamma or uniform distributions (Rinne, 2009).

When covariate is added to the Bayesian method, the survival function and hazard function will frequently depend both on time t and on covariates x_i, which may be fixed throughout the observation period or may be time varying,see Congdon(2001).

The Bayesian using prior estimator under loss function for survival and hazard functions with covariate is the integration over all parameters of covariate for the survival function of the regression model combining with the posterior as shown below,

$$S(t) = \int_D \cdots \int_D S_M(t) \prod (\beta' \mid x_i)\, d\beta_0 \cdots d\beta_n \text{ ,(1.16)}$$

$$h(t) = \int_D \cdots \int_D h_M(t) \prod(\beta' \mid x_i)\, d\beta_0 \cdots d\beta_n \,, \tag{1.17}$$

where the $S_M(t)$ and $h_M(t)$ is the survival function and hazard function respectively for the maximum likelihood estimation, and $\prod(\beta' \mid x_i)$ is the posterior density function of the Bayesian method.

1.1.5Jeffreys Prior and Extension of Jeffreys Prior.

Non-informative prior is one of the categories of the prior distribution. It refers to a situation where there is very limited knowledge or information available to the researcher. With non-informative prior there is little or no influential information that is added to the actual data available. What this means is that we have an occurrence of a set of parameter values in which the statistician believes that the choice of a parameter is equally likely.Jeffreys prior and Extension of Jeffreys prior are used to avoid any hyper parameter specification. Both areinvariant under reparametrization, because of the relation to the Fisher information, when we have large information, we minimize the influence of the prior such that it is as non-informative as possible. Priors like Jeffrey are considered a default procedure and in practice should be used if we have a lot of data and few parameters. Moreover, Jeffreys prior and Extension of Jeffreys prior are very useful for data that do not have any prior information available and give better result in many cases than classical estimation.

Another type of prior is theuniform prior distribution that is considered uniformly distributed over the interval of interest. Box &Tiao(1973) gives the following definition:

If $\phi(\theta)$ is a one–to–one transformation of θ, then a prior is locally proportional to $|d\phi(\theta)/d\theta|$ is non–informative for the parameter θif, in terms of ϕ, the likelihood curve is data translated; that is, the data $t$only serve to change the location of the likelihood $L(t\,|\,\theta)$.

A general rule to find a non–informative prior has been proposed by Jeffreys (1961), known as Jeffreys' rule:

$$g(\theta) = \text{constant } \sqrt{I(\theta)}, \tag{1.18}$$

for a one–dimensional prior, $I(\theta)$ is the Fisher information.

For a multi–dimensional prior, $|I(\theta)|$ is the determinant of the information matrix. Another type of prior is the conjugate prior distribution. For a given sampling distribution, say $f(t\,|\,\theta)$, the posterior distribution $g(\theta|\ t)$ and the prior $g(\theta)$ are members of the same family of distributions (Rinne, 2009).

Extension of Jeffreys prior is a non-informative prior distribution on parameter space that is proportional to the negative expectation of the determinant of the Fisher information in the power of a constant c. Consider c to be a positive real number, then the Jeffreys prior can be said to be a special case of extension of Jeffreys' prior information. As will be shown later, the extension of Jeffreys prior gives better results than Jeffreys prior for certain values of $c$see Al-Kutubi and Ibrahim (2009).

1.2 Problem Statements

Bayesian methods have become relatively common for analyzing survival data. The Bayesian approach has been employed in most areas today, like the medical, engineering, accounting, public health and many other fields. The common principle of Bayesian updating is to combine our prior knowledge on the parameters which is known as prior distribution. It also takes into consideration the observed data that is available to us. In survival analysis, we often encounter data that contain right censored observations; as a result, it is always imperative that the researcher identifies a method which can be used for the analyses so that inferences can be drawn. This makes the Bayesian approach attractive to many researchers.

Jeffreys prior and extension of Jeffreys prior are very useful for data that do not have any prior information available and give better result in many cases when compared to the classical estimation approach. Modified Jeffreys prior with covariate, which we are going to develop by introducing it in the power of a function and this gives better results than the classical method in many cases.

The Bayesian model with Jeffrey prior information for the Exponential distribution can be seen in Al-Kutubi and Ibrahim (2009). As far as the Bayesian model is concerned, the extension of Jeffreys prior information has not been used in the analysis of Bayesian Weibull distribution.

Sinha (1986) used Lindley's approximation technique to estimate the survival and hazard functions of Weibull distribution with Jeffreys prior information, and there were extensively large number of researchers using this technique such as

Nassar&Eissa (2005) andPerda&Constantinescu(2010). However, the model as found in Lindley's approximation technique forextension of Jeffreys prior information with unknown shape using right censored data,has not been used in the analysis of Bayesian Weibull distribution.

Singh *et al*. (2002) and Singh *et al.* (2005) obtained the Bayesian model with Jeffreys prior information by using Gauss quadrature formula to estimate the parameter with complete and Type-II censored data, wherein the Bayesian model, it have been seen no incorporate the covariate into the Jeffreys prior and modified Jeffreys priorto estimate the parameters of covariate, the shape parameter, the survival function and hazard function of Weibullregression model with known and unknown shape.

It is quite difficult to fit survival models with the Bayesian approach but with the use of techniques like MCMC, fitting complex survival models can be straightforward. Also, with the availability of software, it is easy to implement. Kundu&Howlader (2010) obtained Bayesian model using Markov Chain Monte Carlo for constructing the Bayesian estimation and credible intervals. None in the literature review so far has the Bayesian model to estimate the parameters and the survival and hazard functions of the Weibull regression distribution using right censored data with Jeffreys prior via Markov Chain Monte Carlo (MCMC).

1.3 Research Objectives

In view of the importance of the Bayesian model discussed in section 1.2 the following objectives will be addressed:

1. To extend the Bayesian model for Weibull distribution with known shape using right censored data obtained by extension of Jeffreys prior information.

2. To extend the Lindley's approximation technique for Weibull distribution with unknown shape using right censored data obtained by Bayesian using Jeffreys prior and extension of Jeffreys prior information.

3. To develop the Bayesian model for Weibull regression distribution with known and unknown shape using right censored data for Jeffreys prior and modified Jeffreys priors obtained by Gauss quadrature method.

4. To extend the Bayesian model for Weibull regression distribution using right censored data with Jeffreys prior and modified Jeffreys prior via Markov Chain Monte Carlo (MCMC).

5. To assess the performance of all developed models with its maximum likelihood counterparts through simulation study.

1.4 Outline of Thesis

In chapter 2, we present review of related literature to our work. The mathematical expressions and techniques for estimating parameters under maximum likelihood and Bayesian with right censored data are discussed. Other than that, discussion on different distributions that have made use of the above censoring scheme have also been considered especially those that employed maximum likelihood and Bayesian estimators with respect to Markov Chain Monte Carlo, Lindley and the Gauss quadrature rule.

Chapter 3 presents the maximum likelihood estimator that is used to estimate the scale parameter, the survival function and hazard function of Weibull distribution given shape. Also, the scale parameter, the survival function and hazard function of Weibull distribution given shape are estimated using Bayesian with Jeffreys prior and extension of Jeffreys. The Bayesian estimates are obtained by using Lindley's approximation and are compared to its maximum likelihood counterpart. The comparison criteria is the mean squared error (MSE) and absolute bias. The performance of these three estimators are assessed through simulation by considering various sample sizes, several specific values of Weibull parameters and several values of extension of Jeffreys prior.

Chapter 4 deals withthe Bayesian using Jeffreys prior and modified Jeffreys priors with covariate obtained under the Gauss quadrature numerical approximation method and that of the maximum likelihood estimator. The parameters of the covariate, the survival function and hazard function of the Weibull regression distribution given shape with right censored data are estimated. We have also considered a case where the shape parameter is unknown with covariates and made use of the same censoring scheme and numerical approximation as stated above. The comparison criteria is the mean squared error (MSE) and absolute bias. The performance of these three estimators are assessed with and without covariate by using simulation considering various sample sizes, several specific values of Weibull shape parameter. We have in this chapter analyzed real data set and have obtained standard errors and confidence/credible intervals for both the maximum likelihood estimator and that of the Bayesian for the purpose of comparison.

In chapter 5we consider the estimation of the scale and shape parameters, the survival function and hazard function of Weibull distribution with right censored data under Bayesian with Jeffreys and extension of Jeffreys prior by using Markov Chain Monte Carlo (MCMC) method. Here Gibbs sampling technique is used to estimate the scale parameter and Metropolis- Hastings algorithm for the shape parameter. Importance sampling techniqueis used to solve the covariate with Jeffreys prior and modified Jeffreys prior and compared with the maximum likelihood estimator.

Finally in chapter 6, conclusions of the research work are given and several considerations for further research are stipulated

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LIST OF PUBLICATIONS

Thesis related Journal Publications, Seminars and Exhibitions

Journal Publications:

1. Al Omari, M. A. and Ibrahim, N. K. (2010). Bayesian Survival Estimator for Weibull Distribution with Censored Data. *Journal of Applied Sciences,* 11: 393-396.

2. Al Omari, M. A. & Ibrahim, N. A. & Arasan, J. Adam, M. B. (2011). Extension of Jeffreys's Prior Estimate For Weibull Censored Data Using Lindley's Approximation. *Australian Journal of Basic and Applied Sciences*, 5(12): 884-889.

3. Al Omari, M. A. & Ibrahim, N. A. & Adam, M. B. and Arasan, J. (2012). Bayesian Survival and Hazard Estimate for Weibull Censored Time Distribution. *Journal of Applied Sciences*, 12: 1313-1317.

Proceedings:

1. Al Omari, M. A. and Ibrahim, N. K. (2010). Bayesian Estimator for Weibull Distribution with Censored Data using Extension of Jeffrey Prior Information. Proceeding of the International Conference on Mathematics Education Research 2010 (ICMER 2010). Procedia Social and Behavioral Sciences 8 :663–669.

Paper presented:

1. Al Omari, M. A. & Ibrahim, N. A. & Adam, M. B. and Arasan, J. Bayesian Estimate for Weibull Censored Data Using Lindley's Approximation. Paper presented at The regional Fundamental Science Congress 2011 (FSC2011). Universiti Putra Malaysia, 5th-6th July 2011.

2. Al Omari, M. A. & Ibrahim, N. A. & Adam, M. B. and Arasan, J. Jeffreys's prior for Weibull regression censored data. Paper presented at Seminar on Applications of Cutting-edge Statistical Methods in Research. Dewan Taklimat, Universiti Putra Malaysia, 4-5 January 2012.

3. Ibrahim, N. A. & Al Omari, M. A. & Adam, M. B. and Arasan, J. Extension of Jeffreys prior estimation for Weibull censored data. Poster presented at Exhibition of Invention, Research & Innovation (PRPI) 2012, UPM.

4. Al Omari, M. A. and Ibrahim, N. K. Bayesian Estimation of Hazard Rate for Weibull Distribution with Censored Data. Paper presented at 1st ISM International Statistical Conference. Persada Johor, 4-6 September 2012.

Award:

1. Ibrahim, N. A. & Al Omari, M. A. & Adam, M. B. and Arasan, J. Extension of Jeffreys prior estimation for Weibull censored data. Exhibition of Invention, Research & Innovation (PRPI) 2012, UPM. Bronze medal.