CLAIM DEPENDENCE IN HIERARCHICAL CREDIBILITY MODELS AND ESTIMATION OF STRUCTURAL PARAMETERS

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By

MAHDI EBRAHIMZADEH

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

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DEDICATION

To

My wife and my son

Zahra and Shahriar

For their great patience

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

CLAIM DEPENDENCE IN HIERARCHICAL CREDIBILITY MODELS AND ESTIMATION OF STRUCTURAL PARAMETERS

By

MAHDI EBRAHIMZADEH

February 2012

Chair: Prof. Noor Akma bt Ibrahim, PhD

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One of the most important techniques used in general insurance pricing is the credibility ratemaking. In general we can say, credibility theory is a quantitative tool that allows an insurer to combine the past experience of a policyholder to the premium in a risk class or group of risk classes.

In the usual credibility model, observations are made of a risk or group of risks selected from a population and claims are assumed to be independent between different risks. However, there are some problems in practical applications and it may be violated in some situations. Some credibility models typically allow for one source of claim

dependence only that is across time for an individual insured risk or a group of homogeneous insured risks. There exist some other credibility models in the literature which have been developed on two-level common effects model that allows for two possible sources of dependence: across time for the same individual risk and that between individual risks.

In this thesis, we established the notion of modeling claim dependence in credibility models with dependence induced by three-level common effects that allows for three possible sources of dependence: the dependence among portfolio risks, dependence of the individual risks and the dependence of experience for a particular individual risk over time. Using conditional expectation, the credibility premium formulas in which the common effects random variables have a normal distribution are calculated and we present some obvious asymptotic properties of the credibility premium formula. We further give illustrative example to demonstrate the ideas. We also obtain the corresponding credibility formulas for the general (distribution-free) hierarchical structure credibility premiums in the model with three-level of common effects by using the projection method. Then we derive the general hierarchical structure or multi-level credibility premiums for the models with h-level of common effects.

We also estimate the structural parameters of credibility models with dependence induced by common effects. The main advantage of our estimators is their simplicity in calculation and application. We derive unbiased estimators of structural parameters for

two- and three-level common-effect models for portfolios with the Bühlmann model's structure. The results are extended to the Bühlmann-Straub model. We conjecture the *h*-level model formulas when all assumptions are maintained. To illustrate numerically the three-level common effects model, claims data are generated. The result showed that the differences between the true values and our unbiased estimators are generally rather small.

Lastly we illustrate the application of our model using real data.The result showed that the three-level model is better than two- and one-level common effects models. Furthermore, three-level model has the advantage of determining the influence of common effects at each level.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

TUNTUTAN KEBERSANDARAN DALAM MODEL KREDIBILITI BERHIRAKI DAN ANGGARAN BAGI PARAMETER BERSTRUKTUR

Oleh

MAHDI EBRAHIMZADEH

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Pengerusi: Prof. Noor Akma bt Ibrahim, PhD

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Salah satu teknik yang paling penting digunakan dalam penentuan harga insurans adalah penentuan kredibiliti yuran. Secara umum, kita boleh katakan teori kredibiliti adalah alat kuantitatif yang membolehkan pemegang insurans menggabungkan pengalaman yang lepas pemegang polisi kepada polisi premium tulen dalam kelas risiko atau kumpulan kelas risiko.

Dalam model kredibiliti biasa, pemerhatian dibuat bagi suatu risiko atau kumpulan risiko yang terpilih daripada satu populasi dan tuntutan dianggap bebas diantara risiko yang berbeza. Walau bagaimanapun, terdapat beberapa masalah dalam aplikasi secara

praktik dan ia boleh dicabuli dalam sesetengah keadaan. Sesetengah model kredibiliti biasanya membenarkan satu sumber tuntutan kebersandaran sahaja iaitu yang merentasi masa untuk risiko individu berinsuran atau kumpulan risiko homogen berinsuran. Terdapat beberapa model kredibiliti lain dalam literatur yang telah dibangunkan pada model kesan biasa dua-peringkat yang membolehkan dua sumber kebersanderan: merentas masa bagi risiko individu yang sama dan diantara risiko individu.

Dalam tesis ini, kami telah mencetuskan ide untuk memodelkan tuntutan kebersandaran dalam model kredibiliti dengan kebersandaran didorong oleh kesan biasa tiga-peringkat yang membolehkan tiga punca kebersandaran yang mungkin: kebersandaran dikalangan risiko portfolio, kebersandaran bagi risiko individu dan kebersandaran pengalaman bagi risiko individu tertentu dari masa ke masa. Dengan menggunakan jangkaan bersyarat, rumus premium kredibiliti yang mana pembolehubah rawak kesan biasa bertaburan normal dihitung dan kami membentangkan beberapa sifat asimptotik yang jelas bagi rumus premium kredibiliti ini. Kami juga memperolehi premium kredibiliti setara secara berhiraki dengan menggunakan kaedah unjuran. Selepas itu kami terbitkan struktur hiraki umum atau premium kredibiliti pelbagai peringkat bagi model dengan kesan biasa *h*-peringkat.

Kami juga menganggar parameter struktur model kredibiliti dengan kebersandaran didorong oleh kesan biasa. Kelebihan utama penganggar kami adalah pengiraan dan aplikasi yang mudah. Kami terbitkan penganggar saksama parameter berstruktur bagi model kesan biasa dua- dan tiga-peringkat untuk portfolio dengan struktur model

Bühlmann. Keputusan diperluaskan kepada model Bühlmann-Straub. Kami konjektur rumusan bagi model *h*-peringkat apabila semua andaian dikekalkan. Untuk gambaran secara bernumerik model kesan biasa tiga-peringkat, data tuntutan dijana. Keputusan menunjukkan perbezaan diantara nilai sebenar dan pengaggar saksama kami secara umumnya adalah kecil. Akhir sekali kami ilustrasi aplikasi model kami menggunakan data sebenar. Keputusan menunjukkan model tiga-peringkat adalah lebih baik daripada model kesan biasa dua- dan satu-peringkat. Tambahan pula, model tiga-peringkat mempunyai kelebihan untuk mengenalpasti pengaruh kesan biasa pada setiap peringkat.

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Last but not least, I also would like to thank my father in law, my mother in law and my brother in law, Reza, for their encouragement and financial support over the last three years.

I certify that a Thesis Examination Committee has met on 24 February 2012 to conduct the final examination of Mahdi Ebrahimzadeh on his thesis entitled "Claim Dependence in Hierarchical Credibility Models and Estimation of Structural Parameters" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

Mahdi Ebrahimzadeh

Date: 24 February 2012

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CHAPTER 1

INTRODUCTION

1.1 Background

Generally, the insurance industry is faced with three different operational problems: evaluation of the premium, simulation studies of the future behavior of the risk portfolio and the use of principles of statistical control to reduce unnecessary expenditures.

In many countries for most lines of business, the insurance market has strong competition. This competition makes the insurers seek to determine the appropriate premium to charge for the risks of their portfolios. Thus, one of the basic challenges of developing insurance policies is determining their premiums. The insurance cost is determined after the contract is passed; thus, insurance premiums should be determined based on past experiences, statistical information and probability calculation. The fundamental problem is to predict future claims of a risk class, given past claims of that and related risk classes. Therefore, for the evaluation of risk, there should exist an accurate and dynamic statistical system.

If we have observations of past claims for a set of contracts, it might be possible to calculate an appropriate premium for a future period. These premiums must strongly reflect the features of the expected insurance risks. A common practice in calculating

premiums is to group individual risks to ensure homogeneity and achieve a fair and equitable premium across individuals. Under this approach, the risks within each group are as homogeneous as possible in terms of certain observable risk characteristics. However, not all risks in the group are truly homogeneous. Some unobservable factors will always affect the degree of heterogeneity among the individuals.

To determine the premium of next period in which both individual and collective experiences are to be considered, two extreme possibilities can be considered. One is to charge the same premium to everyone, estimated by the overall mean $\bar{X}$ of the data. This approach is reasonable if the portfolio is homogeneous, which means that all risk cells have identical mean claims. But if this is not the case, the 'good' risk will take their business elsewhere, leaving the insurer with only 'bad' risks. The other extreme is that for each group, to charge its own average claims as a premium. Such premiums are justified if the portfolio is heterogeneous, but the premiums are reasonable when the experiences of each group are large enough. Since the beginning of the twentieth century, often an intermediate between the two is considered, the weighted average

$$Z\times(experience)+(1-Z)\times(collective\ experience), \tag{1.1}$$

where $Z \in [0,1]$, is called the "credibility factor", expresses how 'credible' the individual experience.

Credibility theory is an old branch of risk theory and nonlife insurance mathematics. First results go back to Mowbray (1914). Using introductory statistical method, he identified what sample size is sufficient to consider past observations credible. Whitney (1918) suggested using a weighted average between the individual and the collective experience. It is the art and science of using both kinds of experience to adjust the insurance premiums and to improve their accuracy. A premium such as (1.1) is called a credibility premium. Adoption of this premium based on both individual experience and group experience is justified, because the portfolio is in general neither completely homogeneous, nor completely heterogeneous. This idea was formalized in a modern framework in this field by Bühlmann (1967). The risks in group *j* have characteristics in common with the risks in other groups, but they also possess unique group properties. Intuitively, Z_j will be close to one if sufficient past experience is in group *j*, variance within group is small or variance between groups is large. Bühlmann and Straub (1970) extended the Bühlmann model, signified by permitting heterogeneity in the variance of each observation as well as differences in the number of observations recorded for each risk entity.

Credibility models are actuarial tools to distribute premiums fairly among a heterogeneous group of policyholders. More generally, they can be seen as prediction methods applicable in any setting where repeated measures are made for subjects with different risk levels. Thus, credibility covers more broadly linear estimation and prediction in latent variable models.

In the classical credibility models introduced by Bühlmann (1967) and Bülmann and Straub (1970), a common assumption is that observable claims are independent across individuals. Also, for each individual, observable claims are conditionally independent given risk characteristic. While such independence assumptions may be appropriate in some practical situations, everyone agrees these may seem unrealistic.

In fact, it has been recognized that there exist many important insurance scenarios where these classical assumptions are certainly violated. Firstly, certain conditional dependence over time has been recognized as more appropriate to fit the practice in some circumstances and thus considerable attention has been drawn to the credibility models with time dependence structures. For example, after a car accident, a driver may suffer from accident proneness and this affects his/her next claim. The papers by Gerber and Jones (1975), Frees, *et al*. (1999, 2001), Purcaru and Denuit (2002, 2003), Bolancé *et al*. (2003) are examples of credibility models with time dependence of claims.

Secondly, it has been recognized that there exist many important insurance applications where the dependence over risks are common. For one, individual risks may display some traits of dependence with one another and the claims of one insured individual can directly impact those of other insured individuals. Therefore, the observations of each risk may contain useful information about other risks. Examples include house insurance for which geographic proximity of the insureds may result in exposures to common catastrophes and motor vehicle insurance where one accident may involve several insureds. There have been many remarkable efforts in the existing actuarial literature to

study the impacts of dependent risks in various aspects; see e.g., Dhaene and Goovaerts (1996), Dhaene *et al*. (2002a,b), Lu and Zhang (2004), Müller (1997), Wang (1998), Wang *et al.* (1997), Wu and Zhou (2006), and the references therein.

The importance of dependencies among risks is well recognized in actuarial theory and practice. Typical cases arise for policies covering natural disasters (hurricane, tornado, flood, etc.) or groups of individuals (household, staff of a large company, etc.). Recently, the models for which both sources of dependence of expression in the above are allowed, have attracted considerable research interest. These models are known as claim dependence with two-level common effects in credibility models. See e.g., Yeo and Valdez (2006), Wen *et al* (2009b), Zhang and Wen (2010) and Wen and Wu (2011).

More detailed discussions about the literature will be covered in chapter 2.

1.2 Motivation and Objectives

With increasing sources of dependence, one can generalize the models on levels higher than two in a hierarchical way. For example, a database containing a pooled experience of several portfolios thereby helps to produce a more fair, reliable and equitable premium structure for all risks concerned. Research and analysis of such multi-level insurance experience data is lacking in both the actuarial and statistical literature. The benefits of multi-level models go beyond the insurer; reinsurers (i.e. insurers of insurers)

together with regulators also benefit from statistical models of this type of data because they typically deal with analyzing the experience of a collection of insurers.

Hierarchical probability models are widely used for data classified in a tree-like structure and in Bayesian inference. The main characteristic of such models is to have the probability law at some level in the classification structure be conditional on the outcome in previous levels. Hierarchical models arise naturally in insurance applications. For example, they may be used to describe the probability structure of a portfolio of policies or as a means to incorporate collateral data from other cohorts, lines of business, or even companies in predictions.

In this thesis, we examined claim dependence induced by common effects for hierarchical credibility models. The main objectives of the research are as follows:

1. To introduce credibility models with dependence induced by three-level of common effects and to obtain credibility premium formula using conditional expectation and to consider asymptotic properties of credibility factors.

2. To develop credibility premium formula for model with h-level normal common effects using conditional expectation.

3. To establish the model with three-level of claim dependence in general case (distribution-free) and to derive credibility premium formula using the projection method.

4. To develop credibility premium formula for model with *h*-level common effects in general case (distribution-free) using the projection method.

5. To obtain unbiased estimation of structural parameters for the models with two- and three-level of common effects and to extend the estimators formulas to the model with *h*-level common effects in structures of Bühlmann and Bühlmann-Straub models.

1.3 Contribution

The main contribution of this thesis to the existing literature regarding claim dependence in credibility theory is the construction of a credibility model that allows for three possible sources of dependence: across time for an individual insured risk, between these insured risks and that between portfolio risks. Existing credibility models have mostly allowed for one or two source of dependence that is across time for the same insured risk and that between insured risks. There are some numerous circumstances demonstrating that this may be inadequate and insufficient. The development of the three-level common effects model and its extension to *h*-level aims to fill part of this gap in existing literature. Moreover, we present simple estimators of structural parameters of credibility

models with dependence induced by common effects in hierarchical way. We believe this work could be a worthwhile contribution to the literature of statistics and actuarial science.

1.4 Outline of Thesis

The outline of the thesis is as follows: In chapter 2, we introduce credibility theory, basic models of this theory and hierarchical credibility models. Then, we describe dependent claims in the credibility models and provide the motivation that leads to the proposal of the new model.

In chapter 3, we first introduce the model with two- and three-level of claim dependence. Then, using conditional expectation, we will calculate the three-level credibility premium formulas in which the common effects random variables have a normal distribution. To address the three sources of claim dependence mentioned above, we introduce the notion of a common effect affecting all portfolios and for each portfolio we introduce the notion of a common effect affecting all individuals and another common effect affecting a fixed individual over time. The dependence among portfolio risks is described by a common effect random variable Γ. For a fixed portfolio k, the dependence among individual risks is described by a common effect random variable Λ_k. Finally, for a fixed portfolio k and a fixed individual i, the dependence of claims across time is described by another common effect random variable denoted by $\Theta_{k,i}$.

In chapter 4, by means of the projection method, the model with three-level of claim dependence in general case (distribution- free) will be discussed and the model will be developed to *h*-level of claim dependence.

In chapter 5, unbiased estimation of structural parameters in credibility models with dependence induced by common effects will be discussed. We will obtain unbiased estimation of structural parameters for the models with two- and three-level of common effects. Subsequently we derive the formulas to the model with *h*-level common effects.

In chapter 6, we illustrate the application of hierarchical credibility model using real data and discuss model applications in some other areas of science.

Finally, in chapter 7, we conclude and offer suggestions for further research.

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LIST OF PUBLICATIONS

The following papers are extracted from the current thesis:

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Awards:

Bronze Medal for research entitled "*Credibility models with dependence induced by common effects and their applications in bank branch performance prediction"* in the PRPI 2011: Research and Innovation Exhibition, UPM, Malaysia, 19-21 July, 2011.