FAST QUARTER SWEEP USING MODIFIED SUCCESSIVE OVER-RELAXATION ITERATIVE METHODS FOR SOLVING TWO DIMENSIONAL HELMHOLTZ EQUATION

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By

MOHD KAMALRULZAMAN BIN MD AKHIR

Thesis Submitted to the School of Graduate Studies,
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DEDICATIONS

To my Father, Md Akhir Ahmad, My Mother, Zaiton Jaafar, My Wife, Khairunnisa Rahim, To ALL my Sister and Brothers, My Father-in-law, Rahim Sarijau, My Mother-in-law, Zaharah Melan. Thank You for everything.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Master of Science

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October 2012

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New over-relaxation methods for the solution of two-dimensional Helmholtz partial differential equations (PDEs) are described. In the Helmholtz equation, when solving the resulting PDEs using a finite difference (FD) scheme, the computations involve large sparse systems of linear equations (SLEs). These require considerable computation time. Hence, to overcome this problem, the development of faster iterative techniques is desirable.

Point iterative methods, which are based on full-, half-, and quarter-sweep discretization, are commonly used to solve the Helmholtz equation. Due to the large scale of the resulting SLE, many studies have attempted to speed up the convergence rate of the solution. Hence, Young (1971) has already elaborated and discussed the concepts behind various iterative methods. In addition, block (or group) iterative methods, whereby the mesh points are grouped into blocks, have been shown to reduce the number of iterations and execution time, because the solution at the mesh points can be updated in groups instead of pointwise. Among these group iterative methods, Explicit Group (EG), Explicit Decoupled Group (EDG), and Modified Explicit Group (MEG) methods have been expansively researched, and have been shown to converge faster than their pointwise counterparts. Apart from this approach, in order to improve the rate of convergence of these techniques, conjoint accelerated methods, such as Successive Over-Relaxation (SOR), may be applied to reduce the number of iterations. Whereas the above methods have already been implemented with SOR, the quarter-sweep pointwise and MEG methods have never been implemented with Modified Successive Over-Relaxation (MSOR).

This thesis explains the construction and formulation of a quarter-sweep method combined with MSOR, namely QSMSOR. In addition, a computational complexity analysis is presented, and the method is compared with half-sweep MSOR (HSMSOR) and full-sweep MSOR (FSMSOR). Next, the derivation of a combined MEG and MSOR method for solving the two-dimensional Helmholtz equation iteratively is discussed in detail, and a computational complexity analysis of the proposed method is conducted. The numerical results illustrate the improvement of the MEGMSOR method over the combined EDGMSOR and EGMSOR methods in terms of number of iterations, execution timing and maximum absolute error. This is shown to be true for both nonhomogeneous and homogeneous problems in second-order schemes.

In conclusion, the newly developed method is a viable alternative for solving the Helmholtz equation iteratively.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Sarjana Sains

KAEDAH LELARAN PENGENDURAN BERLEBIHAN BERTURUT-TURUT TERUBAHSUAI MENGGUNAKAN SAPUAN SUKU PANTAS UNTUK MENYELESAIKAN PERSAMAAN HELMHOLTZ DUA DIMENSI

Oleh

MOHD KAMALRULZAMAN MD AKHIR

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Kaedah Pengenduran Berlebihan baru untuk penyelesaian sistem persamaan pembezaan separa (PPS) Helmholtz dua dimensi telah diterangkan. Dalam persamaan Helmholtz, apabila menyelesaikan PPS menggunakan skema kaedah beza terhingga (BT), pengiraan yang melibatkan sistem besar jarang persamaan linear (SPL) besar, jarang. Ini memerlukan memerlukan masa pengiraan yang tinggi. Oleh itu, membangunkan teknik lelaran yang lebih pantas adalah wajar untuk mengatasi masalah ini.

Kaedah lelaran titik yang berasaskan pendiskretan sapuan-penuh (SP), separuh-(SS) dan suku (SK) lazimnya digunakan untuk menyelesaikan persamaan Helmholtz. Oleh kerana skala sistem linear yang besar, banyak kajian telah dicadangkan untuk mempercepatkan kadar penumpuan dalam penyelesaian SPL. Oleh itu, Young (1971) telah menghuraikan dan membincangkan pelbagai konsep kaedah

lelaran. Di samping itu, kaedah lelaran blok (atau kumpulan), di mana titik-titik grid dikelompokkan ke dalam blok, didapati mengurangkan bilangan lelaran dan masa perlaksanaan yang diperlukan, kerana penyelesaian bagi titik-titik grid dikemaskinikan dalam blok atau kumpulan tetapi bukan titik demi titik. Antara kaedah-kaedah lelaran berkumpulan, kaedah kumpulan tak tersirat (KTT), kaedah kumpulan nyah pasangan tak tersirat (KNPTT) dan kaedah kumpulan tak tersirat terubahsuai (KTTT) telah banyak dikaji dan terbukti bahawa mempunyai penumpuan lebih cepat berbanding kaedah titik demi titik. Selain dari pendekatan ini, dalam usaha untuk mempercepatkan kadar penumpuan teknik-teknik ini, kombinasi kaedah pengenduran berlebihan berturut-turut (PBB), boleh digunakan untuk mengurangkan bilangan lelaran. Walaupun semua kaedah di atas telah dilaksanakan dengan kaedah PBB, kaedah titik demi titik sapuan suku dan kaedah KTTT belum pernah dilaksanakan dengan pengenduran berlebihan berturut-turut terubahsuai (PBBT).

Tesis ini menerangkan pembangunan dan penerbitan kaedah SK bersama dengan PBBBT, iaitu PBBTSK. Di samping itu, analisis kekompleksan pengiraan diperihalkan dan kaedah ini telah dibandingkan dengan kaedah PBBT sapuan separuh (PBBTSS) dan PBBT sapuan penuh (PBBTSP). Seterusnya terbitan kaedah PBBTKTTT dibincangkan secara terperinci untuk menyelesaikan persamaan Helmholtz dua dimensi secara lelaran dan analisis kekompleksan pengiraan kaedah yang dicadangkan dibincangkan. Keputusan berangka menunjukkan peningkatan ciri-ciri kaedah PBBTKTT berbanding dengan kaedah PBBTKNPTT dan PBBTKTT dari segi bilangan lelaran, masa perlaksanaan dan ralat mutlak maksimum. Dalam skim peringkat kedua ini dapat ditunjukan benar bagi kedua-dua masalah tak homogen dan homogen.

Kesimpulannya, kaedah baru yang dibangunkan adalah alternatif yang berdaya maju untuk menyelesaikan persamaan Helmholtz secara lelaran.

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I certify that a Thesis Examination Committee met on 19 October 2012 to conduct the final examination of Mohd Kamalrulzaman Bin Md Akhir on his thesis entitled "Fast Quarter-Sweep using Modified Successive Over-Relaxation Iterative Methods for Solving Two-Dimensional Helmholtz Equation" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

MOHD KAMALRULZAMAN MD AKHIR
Date: 19 October 2012

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LIST OF ABBREVIATIONS

AD	Adomian Decomposition
AGE	Alternating Group Explicit
AM	Arithmetic Mean
AMG	Algebraic Multigrid
AOR	Accelerated Over-Relaxation
BCs	Boundary Conditions
BE	Boundary Element
BVPs	Boundary Value Problems
CO	Consistently Ordered
EG	Explicit Group
EGAOR	Explicit Group Accelerated Successive Over-Relaxation
EGMSOR	Explicit Group Modified Successive Over-Relaxation
EDG	Explicit Decoupled Group
EDGAOR	Explicit Decoupled Group Accelerated Successive Over-Relaxation
EDGMSOR	Explicit Decoupled Group Modified Successive Over-Relaxation
FD	Finite Difference
FDBP	Finite Difference Beam Propagation
FE	Finite Element
FSAM	Full-Sweep Arithmetic Mean
FSGS	Full-Sweep Gauss–Seidel
FSAMG	Full-Sweep Algebraic Multigrid
FSMSOR	Full-Sweep Modified Successive Over-Relaxation

GS	Gauss–Seidel
GM	Geometric Mean
HSAM	Half-Sweep Arithmetic Mean
HSMSOR	Half-Sweep Modified Successive Over-Relaxation
IADE	Iterative Alternating Decomposition Explicit
ILU	Incomplete LU
ILU(0)	Zero Fill-in ILU
ILU(p)	Level of Fill-in and ILU
MEG	Modified Explicit Group
MEGMSOR	Modified Explicit Group Modified Successive Over-Relaxation
MEGSOR	Modified Explicit Group Successive Over-Relaxation
MILU	Modified Incomplete LU
PDEs	Partial Differential Equations
QSAM	Quarter-Sweep Arithmetic Mean
QSMSOR	Quarter-Sweep Modified Successive Over-Relaxation
RIADE	Reduced Iterative Alternating Decomposition Explicit
SE	Spectral Element
SLE	System of Linear Equations

CHAPTER 1

INTRODUCTION

1.1 Overview

In solving science and engineering problems via numerical methods, many discretization techniques can be taken into account such as FD, boundary element (BE), spectral element (SE) and finite element (FE) methods, which can be used to construct approximation equations for approximating the proposed problems. However, it is still very difficult to gain any solution in solving these problems, either analytically or numerically. As a matter of fact, FD method which is categories as mesh based methods has been used widely to obtain the numerical solution. This method can be used for solving the proposed problems. Next, these approximation equations will be used to generate the corresponding system linear of equation (SLE). Due to the large scale of SLE, the theory or numerical methods in solving such systems has become one of the most popular research areas in modern science.

The over-relaxation theory, which is established in early 1950, is based on solving SLE, which emerges in application of FD schemes to differential equations. This theory comprises different variants of SOR (Young, 1950) and MSOR (De Vogelaere, 1958) methods. On the other hand, the performance is improved through complexity reduction approaches and other approaches to speed up the convergence rate.

1.2 Problem Statement

Many problems in engineering and science involve Helmholtz equation, occur in real time application. On the other hand, the applications of Helmholtz equation are encountered in many fields such as time harmonic acoustic and electromagnetic fields, optical waveguide, acoustic wave scattering, noise reduction in silencer, water wave propagation, radar scattering and lightwave propagation problems (Nabavi et al., 2007; Kassim et al., 2006; Yokota and Sugio, 2002). For example, there is a high important in improving the performance of the methods for solving Helmholtz equation. Hence, the development of fast methods is essential in this research area.

Recent research discovered the formulations and implementations of the FD scheme in the based on the point EG, EDG and MEG iterative methods combined with full-, half- or quarter-sweep iteration concept implemented with SOR methods (Othman and Abdullah, 2004, 2000b,a, 1998; Abdullah, 1991)and to some extent, with Accelerated Over-Relaxation (AOR) method (Rakhimov, 2011; Rakhimov and Othman, 2009; Ali and Lee, 2007; Martins et al., 2002; Hadjidimos and Saridakis, 1992; Hajidimos and Yeyios, 1991). Among these iterative methods, the four-point MEGAOR and quarter-sweep AOR (QSAOR) iterative methods are shown to be the fastest and require fewer arithmetic operations. While all the above methods were implemented with SOR and AOR, the quarter-sweep point and block have never been implemented with MSOR before.

In addition, the most important characteristic of the MSOR methods is the performance can be improved significantly with a wider choice of the relaxation parameter. Moreover, the relaxation parameter can be calculated practically by consecutively choosing a value with some precision until the optimal value is obtained (Kincaid and Young, 1972; Young, 1971).

1.3 Objectives of Thesis

The primary objective in this thesis is to develop a new fast and efficient pointwise and block algorithms for solving two-dimensional Helmholtz equation by utilizing the quarter-sweep concept with MSOR approach. The objective is obtained through research on different problems for solving large sparse SLE. Characteristics associated with the new methods such as number of iterations, computational times and accuracy will be discussed. Besides developing the new algorithms, the opportunity to show on computational complexity for the proposed methods will be elaborated to clarify the effectiveness of the proposed algorithms.

1.4 Scopes and Limitations

This thesis is concentrates on the development of new algorithms for solving Helmholtz equation. More specifically, the research focused primarily on implementation of MSOR method in quarter-sweep point and MEG methods. Meanwhile, in the literature review, it is discovered that there has been a general theory on MEG iterative method for solving PDEs. However, there has been no work on computational complexity analysis on MEGMSOR methods for solving PDEs, mainly on two-dimensional Helmholtz equation. The detail of the proposed methods will be given of the relevant chapters namely Chapter 3 and 4.

1.5 Methodology

In order to develop the new algorithms for solving Helmholtz equation, there are several steps to follow, which are

1. Literature review on:

   - Basic mathematical concepts.
   - The Helmholtz equation and general theory of PDEs.

- Iterative methods and Over-Relaxation theory.

2. Development and implementation of the new algorithms for solutions of the Helmholtz equation, such as

   - Point iterative FSMSOR, HSMSOR and QSMSOR methods.
   - Block iterative EGMSOR, EDGMSOR and MEGMSOR methods.

3. Experiments to benchmark the new algorithms with point and block iterative algorithms studied in step two with different grid sizes.

4. Development of computational complexity analysis of the new proposed methods.

1.6 Outline of Thesis

In Chapter 1, provide an overview and introduction on the application of numerical methods used in the later chapters.

Chapter 2, will give a brief introduction of the numerical solution of PDEs. Basic theory on mathematical concepts like matrix algebra concepts and SLE are discussed. This chapter also focused on the fundamental concepts in direct and iterative methods in solving SLE. A brief explanation of the numerical solution of Helmholtz equation is given. The chapter end with a review of all related researchers on point and block methods.

Chapter 3, provided the detail for derivation of the FSMSOR, HSMSOR and QSMSOR methods by using FD approximate equations. The computational complexities were also discussed. Additionally, the numerical results based on the QSMSOR method and comparisons of their performance to the existing FSMSOR and HSMSOR methods.

Chapter 4, will covered the explanation on deriving MEGMSOR method using the FD approximation equations, for solving two-dimensional Helmholtz equation are in Chapter 4. The details of the computational complexity were discussed. Numerical results are presented and comparisons of their performance to the existing method are made in the final section of the paper.

Chapter 5, summarizes the most important aspects of research. A discussion and suggestion for future work pertaining to this research will be given in highlight the opportunities.

REFERENCES

Abdullah, A. R. 1991. The Four Point Explicit Decoupled Group (EDG) Method: A Fast Poisson Solver. *International Journal of Computer Mathematics* 38: 61–70.

Ali, N. and Lee, S. 2007. Group Accelerated Over Relaxation Methods on Rotated Grid. *Applied Mathematics and Computation* 191 (2): 533–542.

Ali, N. H. M. 1998. *The Design and Analysis of Some Parallel Algorihmns for The Iterative Solution of Partial Differential Equations*. Doctoral thesis, Universiti Kebangsaan Malaysia.

Bayliss, A., Goldstein, A. C. I. and Turkel, E. 1983. An iterative method for the Helmholtz equation. *Journal Computational Physics* 49: 443–457.

Brandt, A. 1986. Algebraic multigrid theory: The symmetric case. *Applied Mathematics and Computation* 19: 23–56.

Chang, Q., Wong, Y. S. and Fu, H. 1996. On the Algebraic Multigrid method. *Journal of Computational Physics* 125: 279–292.

De Vogelaere, R. 1958. Over-relaxations. *Notices American Mathematical Society* 5: 147–273.

Ehrlich, L. W. 1971. Solving the biharmonic equation as coupled finite difference equations. *Siam Journal on Numerical Analysis* 8: 278–287.

El-Sayed, S. M. and Kaya, D. 2004. Comparing numerical methods for Helmholtz equation model problem. *Applied Mathematics and Computation* 150: 763–773.

Evans, D. J. 1997. *Group Explicit Methods for The Numerical Solution of Partial Differential Equations*. Australia: Gordon and Breach Science Publishers.

Evans, D. J. and Abdullah, A. R. 1983. Group explicit methods for parabolic equations. *International Journal of Computer Mathematics* 14 (1): 73–105.

Evans, D. J. and Sahimi, M. S. 1989. The Alternating Group Explicit (AGE) iterative method for solving parabolic equations II: 3D problems. *International Journal of Computer Mathematics* 26: 117–142.

Galanis, S., Hadjidimos, A. and Noutsos, D. 1989. Optimum first and second order extrapolations of successive overrelaxation type methods for certain classes of matrices. *BIT* 29: 477–490.

Gerald, C. F. and Wheatly, P. O. 1999. *Applied Numerical Analysis*. 6th edn. United States of America: Addison-Wesley Longman.

Grusser, O. 1997. The Life of Hermann von Helmholtz. *Physiology* 12 (2): 90–94.

Hadjidimos, A. 2000. Successive overrelaxation (SOR) and related methods. *Journal of Computational and Applied Mathematics* 123: 177–199.

Hadjidimos, A. and Saridakis, Y. G. 1992. Modified successive overrelaxation (MSOR) and equivalent 2-step iterative methods for collocation matrices. *Journal of Computational and Applied Mathematics* 42: 375–393.

Hajidimos, A. and Yeyios, A. K. 1991. Some Recent Results on the Modified SOR Theory. *Linear Algebra and Its Applications* 151 (156): 5–21.

Harari, I. and Nogueria, C. L. 2002. Reducing dispersion of linear triangular elements for the helmholtz equation. *Journal of Engineering Mechanics* 128: 351–358.

Harari, I. and Turkel, E. 1995. Accurate finite difference methods for time-harmonic wave propagation. *Journal of Computational Physic* 119 (2): 252–270.

Ho, C. M., Toh, K. H. and Amran, A. 2004. *Linear Algebra*. Malaysia: Universiti Malaysia Sabah.

Houstis, E. N., Mitchel, W. F. and Papathheodorou, T. S. 1983. A collocation method for mildly nonlinear elliptic problem. *International Journal for Numerical Methods in Engineering* 19: 665–709.

Howard, C. E. and Dianne, P. O. 1998. Efficient iterative solution of the three-dimensional Helmholtz equation. *Journal of Computational Physics* 142: 163–181.

Howard, C. E. and Dianne, P. O. 1999. Eigen analysis of some preconditioned Helmholtz problems. *Numerische Mathematik* 83: 231–252.

Howard, C. E., Oliver, G. E. and P. O. Dianne, P. O. 2001. A multigrid method enhanced by Krylov subspace iteration for discrete Helmholtz equations. *SIAM Journal Scientific Computing* 23 (4): 1291–1315.

Ibrahim, A. and Abdullah, A. R. 1995. Solving the two-dimensional diffusion equation by the four point explicit decoupled group (EDG) iterative method. *International Journal of Computer Mathematics* 58: 253–256.

Ihlenburg, F. and Babuska, I. 1995. Finite-element solution to the Helmholtz equation with high wavenumber part I: the h-version of the FEM. *Computers and Mathematics with Applications* 30: 9–37.

Kassim, N. M., Mohammad, A. B. and Ibrahim, M. N. 2006. Optical Waveguide Modelling Based on Scalar Finite Difference Scheme. *Jurnal Teknologi* 45: 181–194.

Kincaid, D. R. and Young, D. M. 1972. The modified successive overrelaxation method with fixed parameters. *Mathematics and Computations* 119: 705–717.

Manteuffel, T. A. 1978. Adaptive procedures for estimating parameters for the nonsymmetric Tchebychev iteration. *Numerische Mathematik* 31: 183–208.

Martins, M. M., Yousif, W. S. and Evans, D. J. 2002. Explicit Group AOR Method for Solving Elliptic Partial Differential Equations. *International Journal of Computer Mathematics* 10 (4): 411–422.

Mehdizadeh, M. and Paraschivoiu, M. 2003. Investigation of a two-dimensional spectral method for Helmholtz's equation. *Journal of Computational Physics* 189: 111–129.

Moussavi, S. 1984. *Generalized Two-parametric SOR Method for Linear Sparse Systems*. Riverside,United States of America: University of California.

Muthuvalu, M. S. and Sulaiman, J. 2011. Half-Sweep Arithmetic Mean method with composite trapezoidal scheme for solving linear Fredholm integral equations. *Applied Mathematics and Computation* 217 (12): 5442–5448.

Nabavi, M., Sidiqui, M. H. K. and Daraghi, J. 2007. A new 9-point sixth-order accurate compact finite difference method for Helmholtz equation. *Journal of Sound and Vibration* 307: 972–982.

Oberai, A. and Pinsky, P. 2000. A numerical comparison of finite-element methods for the Helmholtz equation. *Journal of Computational Acoustics* 8: 211–221.

Othman, M. and Abdullah, A. R. 1998. A New Technique Point Iterative Method for Solving Poisson Equation on MIMD Computer System. *Sains Malaysiana* 27: 107–117.

Othman, M. and Abdullah, A. R. 2000a. An Efficient Parallel Quarter-Sweep Point Iterative Algorithm for Solving Poisson on SMP Parallel Computer. *Pertanika Journal of Science and Technology* 8 (2): 191–174.

Othman, M. and Abdullah, A. R. 2000b. An Efficient Four Points Modified Explicit Group Poisson Solver. *International Journal of Computer Mathematics* 76: 203–217.

Othman, M. and Abdullah, A. R. 2004. A Parallel Four Points Modified Explicit Group Algorithm on shared Memory Multiprocessors. *International Journal of Parallel, Emergent and Distributed Systems* 19 (1): 1–9.

Rakhimov, S. I. 2011. *New Quarter Sweep Based Accelerated Over-Relaxation Iterative Algorithms and Their Parallel Implementations in Solving 2D Poisson Equation*. Master of science thesis, Institute for Mathematical Research. Universiti Putra Malaysia.

Rakhimov, S. I. and Othman, M. 2009. An Accelerated Over-Relaxation Quarter Sweep Point Iterative Method for Two-Dimensional Poisson Equation. *Sains Malaysiana* 38 (5): 729–733.

Ruggiero, V. and Galligani, E. 2009. An iterative method for large sparse systems on a vector computer. *Computers and Mathematics with Applications* 20 (1): 25–28.

Saad, Y. and Vorst, H. A. V. D. 2000. Iterative Solution of linear systems in the 20th century. *Applied Mathematics and Computations* 123: 1–33.

Sahimi, M. S., A. A. and Bakar, A. A. 1993. The Iterative Alternating Decomposition Explicit (IADE) method to solve the heat conduction equation. *International Journal Computer Mathematics* 47 (1): 219–229.

Sahimi, M. S. and Khatim, M. 1993. The Reduced Iterative Alternating Decomposition Explicit (RIADE) Method for the Diffusion Equation. *Pertanika Journal of Science and Technology* 9 (1): 13–20.

Shaw, R. P. 1988. Integral equation methods in acoustics. *Boundary Elements* 4 (1): 221–244.

Singer, I. and Turkel, E. 1998. High-order finite difference method for the Helmholtz equation. *Computer Methods in Applied Mechanics and Engineering* 163: 343–358.

Sulaiman, J. and Abdullah, A. 2000. Perlaksanaan Beberapa kaedah lelaran Kumpulan Tak Tersirat Terubahsuai Bagi unsur terhingga dalam masalah resapan satu matra. *Sains Malaysiana* 16 (2): 87–94.

Sulaiman, J., Hasan, M., Othman, M. and Karim, S. 2010. MEGSOR iterative method for triangle element solution of 2D Poisson equation. *Procedia Computer Science* 1: 377–385.

Sulaiman, J., Hasan, M. K. and Othman, M. 2004b. The Half sweep Iterative Alternating Decomposition Explicit (HSIADE) method for diffusion equations. In *Lecture Note on Computer Science (LNCS 3314)*, 57–63. Berlin: Springer-Verlag.

Sulaiman, J., Hasan, M. K. and Othman, M. 2004c. A Fourth Compact Finite Difference Solver for One- Dimensional Poisson equation via the Arithmetic Mean Method. In *Proceedings of The Seminar on Science and Technology*, 57–63. Universiti Malaysia Sabah, Malaysia.

Sulaiman, J., Hasan, M. K. and Othman, M. 2005. Half sweep arithmetic mean (HSAM) algorithm to access water quality model. In *Proceedings of National Conference on Computer Science, Technology and Networking*, 145–148. Malaysia.

Sulaiman, J., Hasan, M. K. and Othman, M. 2008. Half-Sweep Algebraic Multigrid (HSAMG) method applied to diffusion equations. In *In H.G. Bock et al. (Eds). Modeling, Simulation and Optimization of Complex Processes*, 547–556. Berlin: Springer-Verlag.

Sulaiman, J., Othman, M. and Abdullah, A. R. 1998. Skema kaedah lelaran Kumpulan Tak Tersirat Terubahsuai Bagi persamaan resapan satu matra. *Borneo Science* 4 (1): 57–66.

Sulaiman, J., Othman, M. and Hasan, M. K. 2004a. Quarter-Sweep Iterative Alternating Decomposition Explicit algorithm applied to diffusion equations. *International Journal of Computer Mathematics* 81 (12): 1559–1565.

Sulaiman, J., Othman, M. and Hasan, M. K. 2009. A New Quarter Sweep Arithmetic Mean (QSAM) Method to Solve Diffusion Equations. *Chamcuri Journal of Mathematics* 2: 93–103.

Tavakoli, R. and Davami, P. 2010. New stable group explicit finite difference method for solution of diffusion equation. *Applied Mathematics and Computation* 181 (2): 1379–1386.

Taylor, P. J. 1969. A generalization of systematic relaxation methods for consistently ordered matrices. *Numerische Mathematik* 13: 379–395.

Varga, R. S. 1962. *Matrix Iterative Analysis*. New York: Prentice-Hall, Englewood Cliffs.

Yokota, M. and Sugio, N. 2002. Application of Multigrid Method to Lightwave Propagation in Two-Dimensional Optical Waveguide. *Journal of Electronics and Communication* 86 (7): 1168–1175.

Young, D. M. 1950. *Iterative methods for solving partial differential equations of elliptic type*. Doctoral thesis, Harvard University.

Young, D. M. 1954. Iterative Methods for solving Partial Difference Equations of Elliptic Type. *Transactions of the American Mathematical Society* 76 (7): 92–111.

Young, D. M. 1971. *Iterative Solutions of Large Linear Systems*. New York: Academic Press.

Young, D. M. 1989. A historical overview of iterative methods. *Computer Physic Communication* 53: 1–17.

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Mohd Kamalrulzaman Md Akhir was born on the 26th of October 1985 in Kuantan, Pahang. He went to Sekolah Rendah Kebangsaan Taman Selasih, Selangor Darul Ehsan for his primary education and then completed his secondary education from Sekolah Menengah Teknik Setapak, Kuala Lumpur, Wilayah Persekutuan. Later, he furthered his study at Negeri Sembilan Matriculation College in 2004.

A year later, he went to Universiti Malaysia Sabah, to pursue his first degree in April 2005. He receives his Bachelor of Science in Mathematics in October 2010. Later, he continued his study at Universiti Putra Malaysia, Selangor Darul Ehsan, at Institute for Mathematical Research to pursue the Master of Science.

LIST OF PUBLICATIONS

JOURNALS

1. **Akhir, M.K.M.,** Othman, M. Sulaiman, J. Majid, Z.A. and Suleiman, M. 2011. Numerical Solutions of Helmholtz Using New Four Point EGMSOR Iterative Method equations. *Applied Mathematical Science*. 80(5): 3991 – 4004.

2. **Akhir, M.K.M.,** Othman, M. Sulaiman, J. Majid, Z.A. and Suleiman, M. 2012. Half-Sweep Modified Successive Over-Relaxation for Solving Two-Dimensional Helmholtz Equations. *Australian Journal of Basic and Applied Sciences*. 5(12): 3033 – 3039.

3. **Akhir, M.K.M.,** Othman, M. Sulaiman, J. Majid, Z.A. and Suleiman, M. 2012. Half Sweep Iterative Method for Solving Two-Dimensional Helmholtz Equations. *International Journal of Applied Mathematics and Statistics*. 29(5): 101 – 109.

4. **Akhir, M.K.M.,** Othman, M. Sulaiman, J. Majid, Z.A. and Suleiman, M. 2012. Four Point Explicit Decoupled Group Iterative Method Applied to Two-Dimensional Helmholtz Equation.*International Journal of Mathematical Analysis*. 20(6): 963 – 974.

5. **Akhir, M.K.M.,** Othman, M. Sulaiman, J. Majid, Z.A. and Suleiman, M. 2012. Efficient Quarter-Sweep Modified Successive Over-Relaxation for Solving Two-Dimensional Helmholtz Equations.*International Journal of Pure and Applied Mathematics*. 80(5): 751 – 764.

CHAPTER IN BOOKS

1. **Akhir, M.K.M.,** Othman, M. Sulaiman, J. Majid, Z.A. and Suleiman, M. 2011. The Four Point-EDGMSOR Iterative Method for Solution of 2D

Helmholtz Equations. *Lecture Notes In Communications in Computer and Information Science, A. Abd Manaf. Berlin: Springer-Verlag.* 253: 218 – 227.

2. **Akhir, M.K.M.,** Othman, M. Sulaiman, J. Majid, Z.A. and Suleiman, M. 2013. The Solution of 2D Helmholtz Equations by Modified Explicit Group Iterative Method. *Lecture Notes In Computer and Information Science.* In Press.

PROCEEDINGS

1. **Akhir, M.K.M.,** Othman, M. Sulaiman, J. Majid, Z.A. and Suleiman, M. 2010. Solving Two-Dimensional Helmholtz Equations by Modified SOR iterative method. In *Proceedings of Fundamental Science Congress*, Universiti Putra Malaysia, MALAYSIA.

2. **Akhir, M.K.M.,** Othman, M. Sulaiman, J. Majid, Z.A. and Suleiman, M. 2010. A New Half-Sweep Modified SOR Iterative Method for Solving Two-Dimensional Helmholtz Equation. In *International Conference On Intelligent Network And Computing*, 26 – 28 November 2010, Kuala Lumpur, MALAYSIA, pp: 26 – 28.

LIST OF AWARDS

1. **Akhir, M.K.M.,** Othman, M. Sulaiman, J. Majid, Z.A. and Suleiman, M. 2012. Fast Helmholtz Solver - SILVER MEDAL. In *Exhibition of Invention, Research and Innovation (PRPI) 2012*, 17 – 19 July 2012, Universiti Putra Malaysia,MALAYSIA.