DIRECT ONE-STEP BLOCK METHODS FOR SOLVING GENERAL SECOND ORDER NON-STIFF ORDINARY DIFFERENTIAL EQUATIONS

NUR ZAHIDAH MUKHTAR

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By

NUR ZAHIDAH MUKHTAR

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

December 2011

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DEDICATION

To

My Parents,

Mukhtar Che Salleh

Che Rodziah Ali

My Siblings,

Muhammad Ihsanuddin Mukhtar

Nur Syakirah Mukhtar

Muhammad Syafiq Mukhtar

Nur Najihah Mukhtar

Nur Nabilah Mukhtar

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

DIRECT ONE-STEP BLOCK METHODS FOR SOLVING GENERAL SECOND ORDER NON-STIFF ORDINARY DIFFERENTIAL EQUATIONS

By

NUR ZAHIDAH BINTI MUKHTAR

December 2011

Chairman : Zanariah Abdul Majid, PhD

Institute : Institute for Mathematical Research

In this thesis, one-step block methods are developed for solving Initial Value Problems (IVPs) of general second order Ordinary Differential Equations (ODEs). These methods is used to solve the general second order ODEs using variable step size. The proposed methods will obtain the approximation solutions at two, three, four and five points simultaneously in a block. These methods will also solve the general second order ODEs directly. This approach is more efficient than the common technique in reducing the problems to a system of first order equations.

These methods will be formulated in terms of multistep method but the implementation is equivalent to the one-step method i.e. Runge-Kutta method. Lagrange interpolation polynomial is applied in order to compute the coefficients of the developed block methods formulae by integrating the closest point in the interval

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to obtain the approximate solutions. The stability region of the proposed method has also been studied.

The numerical results showed that as the number of point increased in the block, the total number of steps is reduced. In addition, at smaller tolerances, the execution times of the proposed methods were faster in the tested problems as the number of points increased. In all cases, the accuracy of the proposed methods gave acceptable accuracy within the given tolerances. In conclusion, the proposed direct one-step block methods in this thesis are suitable for solving the general second order ODEs directly.

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Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

KAEDAH BLOK TERUS SATU LANGKAH BAGI MENYELESAIKAN PERSAMAAN PEMBEZAAN BIASA PERINGKAT KEDUA AM TIDAK KAKU

Oleh

NUR ZAHIDAH BINTI MUKHTAR

Disember 2011

Pengerusi : Zanariah Abdul Majid, PhD

Institut : Institut Penyelidikan Matematik

Dalam tesis ini, kaedah blok satu langkah telah dibangunkan bagi menyelesaikan Masalah Nilai Awal (MNA) untuk Persamaan Pembezaan Biasa (PPB) peringkat kedua am. Kaedah ini berupaya menyelesaikan PPB peringkat kedua am menggunakan panjang langkah berubah. Kaedah yang dicadangkan ini akan memperoleh nilai anggaran penyelesaian pada dua, tiga, empat dan lima titik secara serentak di dalam satu blok. Kaedah yang dicadangkan akan menyelesaikan PPB peringkat kedua am secara langsung. Pendekatan ini lebih efisien berbanding teknik biasa yang menurunkan permasalahan kepada sistem persamaan peringkat pertama.

Kaedah ini akan dirumuskan di dalam bentuk kaedah multilangkah tetapi pelaksanaannya setara dengan kaedah satu langkah iaitu kaedah Runge-Kutta. Interpolasi polinomial Lagrange diterapkan untuk mengira nilai pekali-pekali bagi formula-formula kaedah blok yang dibangunkan dengan mengamirkan titik yang

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terdekat di dalam selang untuk mendapatkan nilai anggaran penyelesaian. Rantau kestabilan untuk kaedah yang dicadangkan juga dikaji.

Keputusan berangka menunjukkan bahawa semakin bertambah titik di dalam blok, maka semakin berkurang jumlah langkah. Tambahan pula, pada toleran yang kecil, masa yang di ambil oleh kaedah-kaedah yang dicadangkan adalah lebih singkat di dalam permasalahan yang diuji seiring dengan bilangan titik yang meningkat. Di dalam semua kes, ketepatan untuk kaedah yang dicadangkan adalah boleh diterima dengan toleran yang diberikan. Kesimpulannya, kaedah blok terus satu langkah yang dicadangkan di dalam tesis ini adalah sesuai untuk menyelesaikan PPB peringkat kedua am secara langsung.

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The Most Beneficent, The Most Merciful

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In addition, I must thank everybody who is important to the success of this thesis and I express my apology in everything that I could not mention personally here.

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I certify that a Thesis Examination Committee has met on **9 December 2011** to conduct the final examination of Nur Zahidah Mukhtar on her thesis entitled "**Direct One-Step Block Methods for Solving General Second Order Non-Stiff Ordinary Differential Equations**" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently submitted for any other degree at Universiti Putra Malaysia or at any other institutions.

_____

NUR ZAHIDAH BINTI MUKHTAR

Date : 9 December 2011

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TABLE OF CONTENTS

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LIST OF TABLES

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LIST OF FIGURES

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LIST OF ABBREVIATIONS

BBDF	:	Block Backward Differentiation Formula
BDF	:	Backward Differentiation Formula
DI	:	Direct Integration
IVPs	:	Initial Value Problems
IVMs	:	Initial Value Methods
LMM	:	Linear Multistep Method
ODEs	:	Ordinary Differential Equations
ODE45	:	MATLAB built-in solver ode45 (Runge-Kutta method of order fourth and fifth).
RKF34	:	An embedded Runge-Kutta Fehlberg method of order third and fourth
RKF45	:	An embedded Runge-Kutta Fehlberg method of order fourth and fifth
2PD	:	Two Point One-Step Block Method for Solving Second Order ODEs
3PD	:	Three Point One-Step Block Method for Solving Second Order ODEs
4PD	:	Four Point One-Step Block Method for Solving Second Order ODEs
5PD	:	Five Point One-Step Block Method for Solving Second Order ODEs

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Numerical analysis is the branch of mathematics considered with the study of algorithms that use numerical approximation for solving mathematical problems. Mathematical problems arise in scientific applications especially in the fields of science and engineering such as the motion of rocket or satellite, fluid dynamics, electrical circuits and other areas of applications. These mathematical problems can be represented in terms of linear or non-linear differential equations. Some of these problems cannot be solved analytically, therefore the numerical methods are applied.

In general, the common numerical methods used to solve the problems can be categorized as one-step method and multistep method. A one-step method such as implicit Runge-Kutta method used information from only one previous point to obtain the solutions. A multistep method or a *k*-step method such as Adams type formula determines the solutions referred to more than one previous point. Hence, any appropriate method can be used to obtain the accurate approximate solution.

The basic concept of the block method is to obtain the approximate solutions at several points simultaneously in a block. As a sequence, a long computational can be avoided in order to save the computational works. Furthermore, this block method is

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also able to reduce the computational time which makes the method more competitive.

1.2 Objective of the Thesis

The main idea of the thesis is to propose a direct one-step block method for solving second order ODEs. The objectives of the study can be achieved by:

a) deriving the formulae of two point, three point, four point and five point one-step block methods for solving the general second order ODEs
b) determining the order of the proposed block methods for solving the general second order ODEs
c) performing the stability region of the proposed block methods for solving the general second order ODEs
d) developing a code for each proposed method by implementing block methods using variable step size and for solving the general second order ODEs.

1.3 Scope of Study

This study is intended to solve the general second order non-stiff ODEs directly without reducing to a system of first order ODEs. The proposed method involves the derivation of two point until five point one-step block methods based on the closest point in a block. Besides that, the order of each developed method is obtained. In order to show the proposed method is suitable and stable, the stability region is

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presented. The programmes for all the proposed methods are written in C language and implemented using variable step size technique.

1.4 Outline of the Thesis

In Chapter I, an introduction about the field of numerical analysis is reviewed. The objectives and the scope of the study are also discussed. Chapter II describes the literature review related to this study and concerns with the concept of numerical solution. The preliminary theory of numerical methods is presented and the review of previous works related to this study is also explained in Chapter II.

Chapter III deals with the derivation of two point and three point one-step block methods for solving the general second order non-stiff ODEs. Then, the necessary conditions to determine the order of the methods will be shown. Those methods are implemented using variable step size and the stability regions are also discussed and compared. In order to study the performance of the proposed methods, the numerical results are presented and comparison of each method is made.

In Chapter IV, the extensions of the work in Chapter III are discussed to derive the four point and five point one-step block methods. The order of the derived methods is also exposed. Next, the stability regions and the numerical results of both methods are also presented and compared based on the specified test problems.

Finally the summary of this study and suggestions for future research are presented in Chapter V.

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