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GENERALIZATIONS OF PARALINDELÖF PROPERTY IN BITOPOLOGICAL SPACES

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GENERALIZATIONS OF PARALINDELÖF PROPERTY IN BITOPOLOGICAL SPACES

By

HEND MOHAMED BOUSELIANA

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

April 2015

DEDICATIONS

To my husband, my mother, my father,
and
my kids

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy.

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Chair: Prof. Adem K l cman, Ph.D.
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The idea of bitopological spaces was initiated first by Kelly in 1969 which defined as two topologies defined on one set (Kelly, 1963). Furthermore, Kelly has extended several topological properties and their results to a bitopological space such as pairwise Hausdorff, pairwise regular, pairwise normal spaces. After that, many authors deal with the extensions to bitopology theory and also with its applications. The main goal of this thesis is to study and focus on the notions of pairwise paralindelöfness, pairwise para-m-lindelöf and generalizations of pairwise paralindelöfness in bitopological spaces derived by the well-known notions of paralindelöf, para-m-lindelöf and generalized paralindelf in topological settings.

During this work, we define five types of paralindelöf spaces in bitopological setting. Namely, paralindelöf, FHP-pairwise paralindelöf, RR-pairwise paralindelöf, p-paralindelöf and pairwise para-m-lindelöf spaces. In the spaces of paralindelöf and p-paralindelöf, they are depended on open and $_1$ $_2$-open covers respectively. Whereas the FHP-pairwise paralindelöf and RR-pairwise paralindelöf spaces are both depended on i-open cover. The difference between these spaces is on the definition of the refinement of their covers. For example, a bitopological space X is called p-paralindelöf if every $_1$ $_2$-open cover of X admits $_1$ $_2$-open refinement p-locally countable. The generalizations of pairwise paralindelöf space are pairwise nearly paralindelöf and pairwise almost paralindelöf spaces that are defined by open covers and pairwise regular open covers.

In this study we state some characterizations of pairwise paralindelöf space and its generalizations. Further, the relationships between these concepts are studied and supported by examples and even counterexamples. In addition, their subspaces and subsets are introduced, and investigated some characterizations and their behavior. Moreover, we present the idea of pairwise locally generalizations of paralindelöfness in bitopological spaces. For example, pairwise locally nearly paralindelöf and pairwise locally almost paralindelöf spaces. Further, we introduce the notion of one-point extension of pairwise almost paralindelöf spaces and study some properties of this notion.

This research is also concerned on mappings and generalized pairwise continuity and some relations among them were presented. The effect of some kinds of some generalized pairwise continuous and generalized pairwise open mappings on those generalized pairwise paralindelöfness are introduced and studied. For instance, pairwise perfect mappings with Lindelöf inverse point (denoted by pairwise L-perfect mappings) map RR-pairwise paralindelöf space to paralindelöf bitopological space. Moreover, pairwise L-perfect and pairwise almost closed mappings are preserved pairwise almost paralindelöf property. In contrast, we show that FHP-pairwise paralindelöf space is not preserved under closed mappings which was given as an example. Furthermore, we add some extra conditions on the maps and sometimes on the spaces to guarantee as some weak types of pairwise mappings preserve the characters of some these generalized classes of pairwise covering properties.

The product property of pairwise paralindelöfness and their generalizations are also investigated. We show that these covering properties are not preserved by product, even with the case of the finite product. To ensure the product is preserved, we restrict some conditions to these bitopological properties to prove that these properties are preserved by finite product under these conditions. For instance, in P-spaces, the product of two paralindelöf bitopological spaces is also paralindelöf bitopological space.

As the theory of multifunctions arise, many of their classes are extended to bitopological settings. We extend some of multifunction to bitopological spaces as pairwise almost continuous, pairwise super continuous, pairwise -continuous and pairwise -continuous multifunctions. Furthermore, we give some characterization of these concepts and study some their properties. In addition, we show the effectiveness of those multifunctions on the generalized pairwise paralindelöf properties.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

PENGITLAKAN SIFAT PARALINDELÖF DI RUANG BITOPOLOGI

Oleh

HEND MOHAMED BOUSELIANA

April 2015

Pengerusi: Prof. Adem K l cman, Ph.D.
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Idea ruang bitopologi telah dimulakan oleh Kelly pada tahun 1969 yang ditakrifkan sebagai dua topologi ditakrifkan pada satu set. Tambahan pula, Kelly telah melanjutkan beberapa ciri-ciri topologi dan keputusan mereka untuk ruang bitopologi seperti berpasangan Hausdorff , berpasangan tetap, berpasangan ruang normal. Semenjak itu, ramai penulis memberi huraian lanjutan kepada teori bitopologi dan juga aplikasinya. Matlamat utama tesis ini adalah untuk mengkaji dan memberi tumpuan kepada tanggapan paralindelöfness berpasangan, para-m-lindelöf berpasangan dan generalisasi daripada paralindelöf berpasangan di dalam ruang bitopologi yang diperolehi oleh tanggapan terkenal paralindelöf, para-m-lindelöf dan paralindelöf umum dalam tetapan topologi.

Dalam karya ini, kita menentukan lima jenis ruang paralindelöf dalam tanggapan bitopologi. Iaitu, ruang paralindelöf, paralindelöf FHP-berpasangan, paralindelöf RR-berpasangan, p-paralindelöf dan para-m-Lindelöf berpasangan.Untuk ruang paralindelöf dan p-paralindelöf, mereka bergantung kepada penutup terbuka dan $_1$ $_2$-terbuka masing-masing, manakala paralindelöfness, paralindelöfness FHP-berpasangan dan paralindelöfness RR-berpasangan kedua-duanya bergantung kepada perlindungan i-terbuka. Perbezaan di antara ruang ini ialah kepada takrif penghalusan penutup mereka. Sebagai contoh, ruang bitopologi X dipanggil p-paralindelöf jika setiap penutup $_1$ $_2$-terbuka X menerima $_1$ $_2$-terbuka p-tempatan bolehbilang. Pengitlakan ruang paralindelöf berpasangan adalah ruang hampir-hampir paralindelöf berpasan-

gan dan ruang hampir paralindelöf berpasangan yang ditakrifkan oleh penutup terbuka dan penutup terbuka biasa berpasangan .

Dalam kajian ini, kami menyatakan beberapa pencirian ruang paralindelöf berpasangan dan pengitlakannya. Seterusnya, hubungan antara konsep tesis dikaji dan disokong dengan contoh-contoh dan juga penyangkal. Di samping itu, subruang dan subset mereka diperkenalkan, beberapa pencirian dan tingkah laku mereka turut disiasat. Selain itu, kami membentangkan idea paralindelöfness berpasangan tempatan dalam ruang bitopologi dan pengitlakan mereka. Sebagai contoh, ruang tempatan hampir-hampir paralindelöf berpasangan dan tempatan hampir paralindelöf berpasangan. Seterusnya, kami memperkenalkan konsep lanjutan satu titik dari ruang hampir-hampir paralindelöf berpasangan dan mengkaji beberapa sifat-sifat tanggapan ini.

Kajian ini juga berkenaan pada pemetaan dan kesinambungan dari segi pasangan umum dan hubungan di antara mereka telah dibentangkan. Kesan beberapa jenis beberapa berpasangan umum pemetaan terbuka dari segi pasangan berterusan dan umum kepada umum paralindelöfness berpasangan diperkenalkan dan dikaji. Sebagai contoh, pemetaan dari segi pasangan sempurna dengan Lindelöf titik songsang ditandakan dengan berpasangan L-sempurna pemetaan paralindelöf RR-berpasangan ke ruang paralindelöf bitoplogi. Selain itu, pemetaan pasangan L-sempurna dan berpasangan hampir tertutup dipelihara dari segi pasangan hampir paralindelöf sifat. Sebaliknya, kita menunjukkan bahawa ruang paralindelöf FHP-berpasangan tidak dipelihara di bawah pemetaan tertutup yang diberi sebagai contoh. Tambahan pula, kami menambah beberapa syarat tambahan kepada peta dan kadang-kadang pada ruang untuk menjamin beberapa jenis pemetaan berpasangan lemah mengekalkan ciri-ciri beberapa kelas-kelas teritlak berpasangan meliputi properti.

Sifat hasildarab paralindelöf berpasangan dan pengitlakan mereka juga disiasat. Kita menunjukkan bahawa sifat peliputan tidak dipelihara oleh hasildarab, walaupun dengan kes hasildarab terhingga. Untuk memastikan hasildarab terpelihara, kita membatasi beberapa syarat terhadap sifat bitopologi untuk membuktikan bahawa sifat-sifat ini dipelihara oleh hasildarab terhingga di bawah syarat-syarat ini. Sebagai contoh, adalah dalam ruang-P, hasildarab dua ruang paralindelöf bitopologi juga ruang paralindelöf bitopologi.

Disebabkan kemunculan teori multifungsi, banyak kelas mereka diperluaskan ke tetapan bitopologi. Kami perluaskan beberapa multifungsi untuk ruang bitopologi seperti berpasangan hampir multifungsi berterusan, berpasangan super multifungsi berterusan, berpasangan multifungsi-berterusan dan berpasangan multifungsi-berterusan. Tambahan pula, kami memberikan beberapa pencirian konsep-konsep ini dan mengkaji beberapa sifat mereka. Di samping itu, kami menunjukkan keberkesanan multifungsi tersebut kepada sifat paralindelöf berpasangan teritlak.

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I certify that a Thesis Examination Committee has met on 30 April 2015 to conduct the final examination of Hend Mohamed Bouseliana on his thesis entitled "Generalizations of Paralindelöf Property in Bitopological Spaces" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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LIST OF ABBREVIATIONS AND SYMBOLS

$i\text{-}int(A)$ or $i\text{-}int_X(A)$	interior of A in X with respect to τ_i
$i\text{-}cl(A)$ or $i\text{-}cl_X(A)$	closure of A in X with respect to τ_i
$\aleph_0$	cardinality of $\mathbb{N}$
$\|A\|$	Cardinal of A
$X \setminus A$	complement of A in X
(X, τ)	topological space X
(X, τ_1, τ_2)	bitopological space X
$(X, \tau^s_{(1,2)}, \tau^s_{(2,1)})$	pairwise semiregularization of (X, τ_1, τ_2)
τ_A	The induced topology on A
$(A, \tau_1\|A, \tau_2\|A)$	Subspace of X whenever $A \subseteq X$
Ω	First uncountable ordinal
τ_{dis}	Discrete topology
τ_{ind}	Indiscrete topology
τ_u	Usual (Euclidean) topology
τ_{ni}	Nested interavel topology
τ_{coc}	Cocountable (countable complement) topology
τ_s	Sorgenfrey topology
$\tau_{l.r}$	Left ray topology
$\tau_{r.r}$	Right ray topology
τ_{pp}	Particular point topology
$\tau_{e.o}$	Either-or topology
FHP (resp. RR)	Fletcher, Hoyle and Patty (resp. Raghavan and Reilly)
DCCC	Discrete countable chain condition
CCC	Countable chain condition
cwH (resp. CwN)	collectionwise Hausdorff (resp. collectionwise normal)

CHAPTER 1

INTRODUCTION

Within theoretical and applied fields of Mathematics, we repeatedly deal with sets endowed with various structures. However, it may happen that the consideration of a set with a specific structure, say topological, algebraic, order, uniform, convex, et cetera is not sufficient to solve the problem posed and in that case it becomes necessary to introduce an additional structure on the set under consideration. To confirm this idea, some theories of topological structures do not complete without adding the theory of bitopological spaces.

The theory of bitopological spaces is a comparatively young area of topology. Its origin is usually associated with the paper of J. C. Kelly in 1969. In his fundamental paper, the concept of bitopological space $(X;\ {}_1;\ {}_2)$ is formulated as a set with two topological structures ${}_1$ and ${}_2$ given on it, generally not at all related to one another, and initial results are given which to a considerable extent determined the content of the future studies. Since then several papers devoted to bitopological spaces have been published at the present time. The overwhelming majority of them contain generalizations of various concepts and assertions of the theory of topological spaces to bitopological spaces in the sense of Kelly.

From the above-said it follows that due to the specific properties of the considered structures two topologies are frequently generated on the same set and can be either independent of each other though symmetric by construction or closely interconnected. Certainly, the investigation of a set with two topologies, interconnected by relations of bitopological character, makes it possible on some occasions to obtain a combined effect, that is, to get more information than we would quire if we considered the same set with each topology separately.

If we compare all the results available in the theory of bitopological spaces from the general point of view, we shall find that in different cases two topologies on a set are not, generally speaking, interconnected by some common law that takes place for all bitopological spaces. However if, when defining a bitopological notion, the closure and interior operators are successively applied in an arbitrary initial order to the same set, then, in general, these operators will interchange in topologies as well. we would like to note that we firmly believe that from the standpoint of applications the theory of bitopological spaces has no less promising prospects than the theory of topological spaces.

For convenience, neither separation axioms nor bitopological separation axioms are assumed unless otherwise stated and bitopological space $(X;\ _1;\ _2)$ is denoted by X .

1.1 Historical Remarks

A topological space $(X;\ )$ is said to be paralindelöf if every open cover of X has a locally countable open refinement. The subject of paralindelöf spaces is a wide open field, with very little is known about which implications hold between covering properties or separation axioms (regular or beyond), besides those that hold for topological spaces in general. Consider the following properties: regular, normal, collectionwise Hausdorff, Lindelöf, paracompact. It is not known whether paralindelöf together with any of these properties implies another property if it does not already do so for all spaces.

The concept of paralindelöf spaces was introduced by (Greever, 1968) "On Some Generalized Compactness Properties". Since then, many topologists worked on paralindelöf spaces, as (Blair, 1986) and (Fleissner and Reed, 1977a), mainly trying to solve the problem whether every regular paralindelöf space is paracompact. The solution, in the form of a counterexample, was given by (Navy, 1981) "Nonparacompactness in Para-Lindelöf Spaces". Her paper was important in the development of metrizability theory. It shows the properties of paralindelöf topological spaces.

The idea of paralindelöfness came from studying paracompact property. Since the relationship between paracompactness and paralindelöfness are very strong, where every paracompact space is paralindelöf but not the converse, many properties of paracompact spaces were generalized onto paralindelöf spaces. The concept of paracompactness was introduced by (Dieudonn, 1944). As time passes, more and more people are being attracted by it. A space is said to be paracompact if every open cover of the space admits a locally finite open refinement. Generalizations of paracompactness can be obtained by putting cardinality restrictions on the cover or by modifying the nature of the cover or by requiring a refinement to be a different type, or by combinations of these such as countable paracompact, pointwise paracompact spaces. Some authors call these spaces weakly paracompact, strongly paracompact spaces and nearly paracompact etc, see (Dowker, 1951), (Arens and Dugundji, 1950), (Singal and Arya, 1969b) and others. Mathematicians called these concepts covering properties. Also, they generalized these concepts again to other covering properties such as paralindelöf, nearly paralindelöf almost paralindelöf and other generalizations.

Moreover, since every Lindelöf space is paralindelöf space, some mathematicians generalized the notion of paralindelöf from Lindelöf property such as (Ewert and Ponomarev, 2000). In general topology, several generalizations of Lindelöf space and the relations among them have been studied by several authors, for example, (Balasubramaniam, 1982), (Cammaroto and Santoro, 1999), (Fawakhreh and Kılıçman, 2004) and (Kılıçman and Fawakreh, 2000) later in 2000s. Further, in bitopological settings, also many people work on the notion of Lindelöfness. They called these concepts pairwise Lindelöf spaces. For instance, (Fora and Hdeib, 1983), (Kılıçman and Salleh, 2007a), (Kılıçman and Salleh, 2009), (Kılıçman and Salleh, 2008b) and others.

Many generalizations of paralindelöf spaces have been introduced by some authors for different reasons and purposes. Some of them are depended on open covers which had been studied by several authors as (Burke, 1980), (Burke and Davis, 1982) and (Fleissner and Reed, 1977b). Other generalizations depended on regular open covers and open covers which are introduced by (Thanapalan, 1991b). He used the modification of nearly paracompact spaces studied by (Singal and Arya, 1969b). About the concept of almost paralindelöf, the authors in (Ewert and Ponomarev, 2000), (Thanapalan, 1991a), (Thanapalan, 1992a) and (Thanapalan, 1994) discussed this notion and several properties of it.

1.2 Research Objectives

The generalizations of covering properties have been done to bitopological setting which are called pairwise covering properties. The earlier generalizations to pairwise covering properties are pairwise compact, pairwise Lindelöf, pairwise paracompact spaces, see, for example, (Kılıçman and Salleh, 2009), (Kılıçman and Salleh, 2007a), (Kılıçman and Salleh, 2008b), (Konstadilaki-Savopoulou and Reilly, 1981), (Munshi and Bassan, 1982), (Fora and Hdeib, 1983), (Kelly, 1963), (Raghavan and Reilly, 1986) and others. The next generalization of pairwise covering properties is pairwise paralindelöfness analogous paracompactness and lindelöfness generalized to paralindelöfness in covering properties. Our objectives are:

1. To define several types of pairwise paralindelöf spaces in bitopological settings namely, paralindelöf, FHP-pairwise paralindelöf, RR-pairwise paralindelöf, p_1-paralindelöff (or p-paralindelöf), B_w-paralindelöf, B_ww-paralindelöf and pairwise para-m-lindelöf spaces for infinite cardinal m.
2. To study the classes of pairwise paralindelöf bitopological spaces which are depended on pairwise open and regular open covers pairwise nearly paralindelöf, B_1-nearly paralindelöf, B_r-nearly paralindelöf, pairwise almost paralindelöf and B_1-almost paralindelö. The relations between these generalizations, and their properties also shall be introduced.
3. To introduce the concept of collectionwise Hausdorff bitopological spaces and the relation of collectionwise Hausdorff with paralindelöf bitopological spaces.

4. To introduce and investigate locally pairwise nearly paralindelöfness, locally pairwise almost paralindelöfness and the one point extension of pairwise almost paralindelöf.

5. To show that pairwise paralindelöf space preserve under certain generalized pairwise mappings and do not under others.

6. To show the preservation of generalized pairwise paralindelöf spaces under product topology.

7. We extend some kinds of multifunctions to bitopology which are pairwise super continuous, pairwise almost continuous, pairwise -continuous, -continuous multifunctions.

8. To study the effect of those multifunctions on paralindelöf bitopological space and its classes.

1.3 Organization of the Thesis

This research is organized as eight chapters. The Chapter 1 describes all information about the theory of bitopology. Also, the objectives of the study are listed. Chapter 2 shows all background which we need in the rest of chapters as separation axioms and covering properties. The third chapter introduces and studies many types of pairwise paralindelöfness in bitopological spaces and studies its properties and relations with some bitopological concepts.

The Chapter 4 defines the classes of pairwise paralindelöf bitopological spaces: pairwise nearly paralindelöf, B_1-nearly paralindelöf, B_r-nearly paralindelöf, pairwise almost paralindelöf and B_1-almost paralindelö, the relations between them and their properties. The fifth chapter shows that the pairwise paralindelöf spaces preserve under certain generalized pairwise mappings and do not under others. The effect of classes of pairwise paralindelöfness under some mappings is investigated. The Chapter 6 introduces the preservation of generalized pairwise paralindelöf spaces under product topology.

The Chapter 7 discusses the idea of multifunction in bitopology. It studies the concepts of pairwise super continuous, pairwise almost continuous, pairwise -continuous, -continuous multifunctions. Lastly, Chapter 8 covers some questions and ideas for future studies.

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