SOME ASPECTS OF THE SPATIAL UNILATERAL AUTOREGRESSIVE MOVING AVERAGE MODEL FOR REGULAR GRID DATA

By

NORHASHIDAH BINTI AWANG

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

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To

Mother and Family

And

In the memory of a loving father,
Awang bin Ahmad (1928-1983).
May Allah rest his soul.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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Spatial statistics has received much attention in the last three decades and has covered various disciplines. It involves methods which take into account the locational information for exploring and modelling the data. Many models have been considered for spatial processes and these include the Simultaneous Autoregressive model, the Conditional Autoregressive model and the Moving Average model. However, most researchers focused only on first-order models. In this thesis, a second-order spatial unilateral Autoregressive Moving Average (ARMA) model, denoted as ARMA(2,1;2,1) model, is introduced and some properties of this model are studied. This model is a special case of the spatial unilateral models which is believed to be useful in describing and modelling spatial correlations in the data. It is also important in the field of digital filtering and systems theory and for data whenever there is a natural ordering to the sites.

Some explicit stationarity conditions for this model are established and some numerical computer simulations are conducted to verify the results. The general explicit correlation structure for this model over the fourth quadrant is obtained which is then specialised to AR(2,1), MA(2,1) and the second-order separable models. The results from simulation studies show that the theoretical correlations are in good agreement with the empirical correlations. A procedure using the maximum likelihood (ML) method is provided to estimate the parameters of the AR(2,1) model. This procedure is then extended to the case of spatial AR model of any order. For the AR(2,1) model, in terms of the absolute bias and the RMSE value, the results from simulation studies show that this estimator outperforms the other estimators, namely the Yule-Walker estimator, the 'unbiased' Yule-Walker estimator and the conditional Least Squares estimator. The ML procedure is then demonstrated by fitting the AR(1,1) and AR(2,1) models to two sets of data. Since the AR(2,1) model has the second-order terms which are only in one direction, two types of data orientation are taken into consideration. The results show that there is a preferred orientation of these data sets and the AR(2,1) model gives better fit. Finally, some directions for further research are given.

In this research, inter alia, the field of spatial modelling has been advanced by establishing the explicit stationarity conditions for the ARMA(2,1;2,1) model, by deriving the explicit correlation structure over the fourth lag quadrant for ARMA(2,1;2,1) model and its special cases and by providing a modified practical procedure to estimate the parameters of the spatial unilateral AR model.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

BEBERAPA ASPEK TENTANG MODEL RERUANG SESISI AUTOREGRESI PURATA BERGERAK BAGI DATA GRID SEKATA

Oleh

NORHASHIDAH BINTI AWANG

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Statistik reruang mula mendapat lebih perhatian semenjak tiga dekad lalu dan ia mencakupi pelbagai disiplin ilmu. Statistik ini melibatkan kaedah-kaedah yang mengambilkira maklumat lokasi dalam menjelajah dan memodel data reruang. Banyak model yang telah dipertimbangkan bagi proses reruang termasuk model autoregresi serentak, model autoregresi bersyarat dan model purata bergerak. Namun demikian, hampir kesemua kajian ditumpukan pada model peringkat pertama. Dalam tesis ini, model reruang sesisi autoregresi purata bergerak (ARMA) peringkat kedua, ditulis sebagai ARMA(2,1;2,1) diperkenalkan dan sifat-sifatnya dikaji. Ia adalah kes istimewa model reruang sesisi yang bermanfaat dalam menerang dan memodel korelasi reruang yang wujud dalam data. Ia juga penting dalam ilmu penyaringan digital dan sistem teori dan bilamana terdapat penertiban semulajadi pada tapak data.

Beberapa syarat tak tersirat bagi kepegunan model ini diperolehi dan keputusan disahkan dengan ujian simulasi komputer berangka. Struktur korelasi tak tersirat

bagi model ini berserta kes-kes khasnya seperti model AR(2,1), model MA(2,1) dan model-model terpisahkan peringkat kedua diperolehi bagi sukuan keempat jeda. Keputusan ujian simulasi menunjukkan bahawa struktur korelasi yang diperolehi ini berpadanan dengan korelasi empirik. Prosedur penganggaran menggunakan kaedah kebolehjadian maksimum (ML) diperolehi bagi menganggar parameter model AR(2,1). Prosedur ini kemudiannya diperluaskan kepada model reruang sesisi autoregresi (AR) sebarang peringkat. Bagi model AR(2,1), kajian simulasi menunjukkan kaedah ML ini adalah lebih baik secara keseluruhannya berbanding kaedah-kaedah lain seperti Yule-Walker (YW), YW saksama dan kuasa dua terkecil bersyarat berdasarkan nilai pincang mutlak dan punca min ralat kuasa dua. Kaedah ML ini didemonstrasi dengan menyuai model AR(1,1) dan model AR(2,1) pada dua set data. Memandangkan model AR(2,1) mengandungi sebutan-sebutan peringkat kedua pada satu arah sahaja, dua jenis orientasi data dipertimbangkan. Kajian mendapati orientasi yang berbeza memberikan keputusan yang berbeza dan secara amnya model AR(2,1) adalah lebih baik bagi dua set data ini. Akhir sekali, beberapa arahtuju bagi penyelidikan lanjut dicadangkan.

Dalam penyelidikan ini, ilmu permodelan reruang dimajukan antaranya dengan menyediakan syarat-syarat kepegunan tak tersirat bagi model ARMA(2,1;2,1), dengan menerbitkan struktur korelasi tak tersirat bagi sukuan keempat jeda untuk model ini dan kes-kes khasnya, dan dengan menyediakan prosedur terubahsuai yang praktikal untuk menganggar parameter model reruang sesisi AR.

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Lastly, my thanks also go to my family. Without their love, encouragement and support I would not have been able to complete this work. This thesis is dedicated to them.

I certify that an Examination Committee met on 22nd February 2005 to conduct the final examination of Norhashidah Awang on her Doctor of Philosophy thesis entitled "Some Aspects of the Spatial Unilateral Autoregressive Moving Average Model for Regular Grid Data" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulation 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

NORHASHIDAH BINTI AWANG

Date:

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LIST OF ABBREVIATIONS

AR	Autoregressive
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
CAR	Conditional Autoregressive
LS	Conditional Least Squares
MA	Moving Average
ML	Maximum Likelihood
RMSE	Root Mean Square Error
SAR	Simultaneous Autoregressive
YW	Yule-Walker

CHAPTER 1

INTRODUCTION

Spatial statistics has received much attention in the last three decades and interest in this area is increasing rapidly. A large amount of research in modelling spatial processes has been conducted and they have covered various applications. Spatial statistics involves methods which take into account the locational information for exploring and modelling the data.

Many observed phenomena are spatial in nature. For examples, the spread of infectious diseases, rainfall, ore grade in mining blocks, tumour growth and plant yields in agricultural experiments or plantation. It is believed that data which are close together tend to be alike than those which are far apart. In contrast to the non-spatial models, the spatial models admit this spatial variation into the generating mechanism.

In this introductory chapter, some background on spatial processes, the statement of problems, list of the research objectives and the outline of the thesis structure are given.

1.1 Introduction to Spatial Processes

A formal definition of spatial series is a sequence of d-dimensional random variables $\{Y_X, X \in A\}$ on a probability space (Ω, F, P), where A is a denumerable subset of R^d (Tjøstheim, 1993). A process which generates such random variables is called a spatial process. Spatial processes have been analysed and studied in wide varieties of disciplines such as agriculture field trials (Kempton and Howes, 1981, Gleeson and Cullis, 1987, Cullis et. al, 1989, Martin, 1990 and Cullis and Gleeson, 1991), business microdata (Franconi and Stander, 2003), plant ecology (Besag, 1974), geography (Cliff and Ord, 1981 and Bronars and Jansen, 1986), geology (Cressie, 1993), biology, image processing, meteorology and so on.

Most studies on spatial processes are focused on two-dimensional cases although there has been some work for general d-dimensional processes (Tjøstheim, 1978 and 1983 and Guyon, 1982). In recent years, there has been an interest examining processes on higher dimensions, for example, a three-dimensional process considered by Martin (1997).

There are many types of spatial data and they are classified according to

i) whether the associated random variables are continuous or discrete,

ii) whether they are spatial aggregations or observations at points in space,

iii) whether their spatial locations or system of sites regular or irregular, and

iv) whether those locations are from a spatial continuum or a discrete set.

Besag (1974) discussed broadly and provided many examples of various kinds of spatial data. Generally, spatial data may be categorised into three main classes namely, geostatistical data, lattice data and point patterns (Cressie, 1993) as explained in the following paragraphs.

The data are called geostatistical data if they are indexed over continuous space, or by a formal definition, if A is a fixed subset of R^d and Y_X is a random variable at location $X \in A$. The word geostatistics is meant by a hybrid discipline of mining engineering, geology, mathematics and statistics. It recognises spatial variability for both large scale (trend) and small scale (correlation). Trend-surface methods deal with large scale variation and assume the errors are independent. Some examples of geostatistical data are the soil pH in water, rainfall and mining data, for instance, ore-reserve in a mining field which is important in analysing and predicting the ore grade in a mining block (i.e. kriging).

For the processes which are indexed over lattices in space, the data are called the lattice data. In this case, A is a fixed (regular or irregular) subset of Z^d, where Z is the set of integers, and Y_X is a random variable at location $X \in A$. Some examples include grid data obtained from remote sensing (Kiiveri and Campbell, 1989) and field trials (Modjeska and Rawlings, 1983 and Besag and Kempton, 1986). A spatial data on regular lattice is analogous to a time series observed at equally spaces time points.

When A is a point process in R^d or a subset of R^d and Y_X is a random variable at location $X \in A$, we obtain the point patterns. In this case, the important variable to be analysed is the location of 'events' and we examine whether the pattern is exhibiting complete spatial randomness, clustering or regularity. Examples include spread of infectious diseases and tumour growth.

In this thesis, spatial lattice processes on two-dimensional regular grid are considered.

1.2 Statement of Problems

Although a spatial series may be considered as a generalisation of time series, analyzing it is considerably more difficult (Tjøstheim, 1978), including estimating the parameters of the models. Unlike time series which is unidirectional following a natural distinction made between past and present, dependence in spatial series extends in all directions.

Spatial series encounter larger proportion of edge effects compared to time series and hence, analysing the data is not easy due to substantial mathematical and computational difficulties. This problem has been discussed in Whittle (1954), Besag (1972 and 1974), Ord (1975), Haining (1978a, b), Martin (1979 and 1990), Tjøstheim (1978 and 1983), Guyon (1982), Dahlhaus and Kunsch (1987) and Kiiveri and Campbell (1989). To overcome this problem, Haining (1978a) and Gleeson and McGilchrist (1980) considered the likelihood methods conditional on