



UNIVERSITI PUTRA MALAYSIA

**EFFECTS OF THE USE OF CALCULATORS, THE POLYA
HEURISTIC AND WORKED EXAMPLES ON PERFORMANCE IN
LEARNING MATHEMATICS FROM A COGNITIVE LOAD
PERSPECTIVE**

LAWRENCE ALOYSIUS AERIA.

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By

LAWRENCE ALOYSIUS AERIA

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Faculty: Educational Studies

Cognitive load theory holds that if an instructional format reduces extraneous cognitive load and/or increases germane cognitive load during learning, as compared to another instructional format, then it will be more efficient in promoting learning, provided that the total cognitive load does not exceed the total mental resources. Based on this premise, a series of four experiments was conducted to investigate the use of calculators, the Polya heuristic and worked examples in an attempt to generate an alternative instructional format that is more instructionally efficient than the conventional one to teach Percentage to Form 1 students. It was hypothesized that the use of calculators and the use of worked examples can reduce cognitive load during learning and lead to better learning performance while the use of the Polya heuristic increases cognitive load and reduces learning performance. The results from the experiments



indicated that the use of calculators and worked examples reduced cognitive load and led to better learning performance but the use of the Polya heuristic produced zero effects. Based on the results from the experiments, an alternative instructional format, called the calcworked instructional format, that incorporated the use of calculators and the use of partial completion worked examples was generated. The calcworked instructional format was compared with the conventional instructional format and was found to be more instructionally efficient.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**KESAN PENGGUNAAN KALKULATOR, HEURISTIK POLYA
DAN CONTOH PENYELESAIAN TERHADAP PRESTASI
DALAM PEMBELAJARAN MATEMATIK DARIPADA
SATU PERSPEKTIF BEBANAN KOGNITIF**

Oleh

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Teori bebanan kognitif menegaskan bahawa jika sesuatu format pengajaran mengurangkan bebanan kognitif ekstraneous dan/atau menambahkan bebanan kognitif *germane* semasa pembelajaran, berbanding dengan sesuatu format pengajaran yang lain, maka format pengajaran tersebut akan lebih cekap dalam menggalakkan pembelajaran selagi jumlah bebanan kognitif tidak melebihi jumlah sumber mental. Berdasarkan pernyataan ini, satu siri yang mengandungi empat eksperimen telah dijalankan untuk mengkaji penggunaan kalkulator, heuristik Polya dan contoh penyelesaian supaya dapat diwujudkan satu format pengajaran alternatif yang lebih cekap berbanding format pengajaran konvensional, untuk mengajar tajuk Peratusan kepada pelajar Tingkatan 1. Hipotesis telah dibuat bahawa penggunaan kalkulator dan contoh penyelesaian dapat mengurangkan bebanan kognitif semasa



pembelajaran dan meningkatkan prestasi pembelajaran manakala penggunaan heuristik Polya akan menambahkan bebanan kognitif dan mengurangkan prestasi pembelajaran. Keputusan eksperimen menunjukkan bahawa penggunaan kalkulator dan contoh penyelesaian dapat mengurangkan bebanan kognitif dan meningkatkan prestasi pembelajaran tetapi penggunaan heuristik Polya menghasilkan kesan sifar. Berdasarkan keputusan eksperimen-eksperimen tersebut, satu format pengajaran alternatif, dinamakan format pengajaran *calcworked*, yang menggabungkan penggunaan kalkulator dan contoh penyelesaian separa lengkap telah diwujudkan. Jika dibandingkan dengan format pengajaran konvensional, format pengajaran *calcworked* didapati lebih cekap.

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TABLE OF CONTENTS

	Page
ABSTRACT	ii
ABSTRAK	iv
ACKNOWLEDGEMENT	vi
APPROVAL	x
DECLARATION	xii
LIST OF APPENDICES	xix
LIST OF TABLES	xx
LIST OF FIGURES	xxiii
CHAPTER	
1 INTRODUCTION	
Background of the Study	1
Introduction	1
Research in Mathematical Problem Solving	3
The Information Processing Theory	6
Cognitive Load Theory	8
Performance of Malaysian Lower Secondary School Students in Mathematical Problem Solving	12
The Use of Calculators	18
The Use of the Polya Heuristic	21
The Use of Worked Examples	24
Statement of the Problem	26
Purpose of the Study	29
Research Hypotheses	30
Significance of the Study	43
Limitations of the Study	46
Definition of Terms	48
2 REVIEW OF RELATED LITERATURE	
Introduction	55
Problem and Problem Solving in Mathematics	55
Meaning of Problem in Mathematics	55
Meaning of Problem Solving in Mathematics	59
The Information Processing Theory	62
The Modal Model of Memory	63
Sensory Buffer	64
Short Term Memory (STM)/ Working Memory	65
Long Term Memory	70
Encoding, Storage and Retrieval	72
Cognitive Load Theory	73
Overview of Cognitive Load Theory	73



Key Concepts in Cognitive Load Theory	74
Meaning of Cognitive Load	89
The Cognitive Load Construct	92
Measurement of Cognitive Load	97
Learning from the Perspective of Cognitive Load Theory	103
Expert-Novice Differences from the Perspective of Cognitive Load Theory	108
Principles of Cognitive Load Theory	112
Application of Cognitive Load Theory to Instructional Design	114
Conventional Problems and Conventional Problem Solving (Means-Ends Analysis)	118
The Instructional Techniques (Effects) Generated by Cognitive Load Theory	125
Methodology of Cognitive Load Studies	151
Cognitive Load Studies in Instructional Fields of Research	154
Recent Developments in Cognitive Load Theory Research	156
Calculators in Mathematics	164
Overview	164
Arguments For and Against the Use of Calculators	166
Research Findings On Calculators in Mathematics Education	168
Heuristics in Mathematics	173
What are Heuristics?	173
The Polya Heuristic	176
Research Findings on Heuristics	179
The Integrated Curriculum for Secondary Schools	183
Mathematics syllabus	184
Conceptual Framework of the Study	187
Summary	189
3	
METHODOLOGY	
Overview of Chapter	192
Introduction	192
Design of the Study	195
The Experimental Design	195
Overview of the Staggered Experimental Design	196
Justification for the Staggered Experimental Design	198
Design of Each Experiment	201
The Variables of the Study	206



Sample	210
Composition/Criteria	210
Sample Criteria	210
Sampling Procedure	211
Sample Size	212
Instrumentation	214
Pre-Experiment Preparation	216
Overview of Procedures for Experiments	217
Pilot Study	222
Preliminary Study (Experiment 1)	223
Development of Instrument 1 (The Selection Criteria Test)	224
Reliability of The Selection Criteria Test	227
Development of Instrument 3 (The General Guideline Lesson Plan)	229
Development of Instrument 4 (The Percentage Problems)	232
Reliability of Instrument 4 (The Percentage Problems)	235
Item Difficulty Index and Item Discrimination Index of Instrument 4	236
Development of Instrument 5 (The Worked Examples)	239
Development of Instrument 6 (The Paas Mental Effort Rating Scale)	240
Reliability of Instrument 6 (The Paas Mental Effort Rating Scale)	241
Details of the Procedures for the Experiments	243
Experiment 1	243
Experiment 2	253
Experiment 3	257
Experiment 4	260
Summary	267
4 RESULTS	
Introduction	269
Experiment 1	269
Effects of the Use of Calculators, the Polya Heuristic and Worked Examples on Performance and Mental Effort in the Acquisition Phase	271
Effects of the Use of Calculators, the Polya Heuristic and Worked Examples on Performance and Mental Effort in the Test Phase	277
Instructional Efficiency of the Conventional, Calculator, Heuristic and Worked Example Instructional Formats	281

	Correlation Between Cognitive Load and the Performance Variables	285
	Research Hypotheses	286
	Post-hoc Power Analyses	303
Experiment 2		304
	Effects of the Use of Calculators, the Polya Heuristic and Worked Examples on Performance and Mental Effort in the Acquisition Phase	305
	Effects of the Use of Calculators, the Polya Heuristic and Worked Examples on Performance and Mental Effort in the Test Phase	311
	Instructional Efficiency of the Conventional, Calculator, Heuristic and Worked Example Instructional Formats	320
	Correlation Between Cognitive Load and the Performance Variables	322
	Research Hypotheses	324
	Post-hoc Power Analyses	342
Experiment 3		342
	Effects of the Use of the Alternative (Calcworked) Instructional Format on Performance and Mental Effort in the Acquisition Phase	343
	Effects of the Use of the Alternative (Calcworked) Instructional Format on Performance and Mental Effort in the Test Phase	345
	Instructional Efficiency of the Conventional and Calcworked Instructional Formats	349
	Correlation Between Cognitive Load and the Performance Variables	350
	Research Hypotheses	352
	Post-hoc Power Analyses	367
Experiment 4		367
	Effects of the Use of the Alternative (Calcworked) Instructional Format on Performance and Mental Effort in the Acquisition Phase	368
	Effects of the Use of the Alternative (Calcworked) Instructional Format on Performance and Mental Effort in the Test Phase	370
	Instructional Efficiency of the Conventional and Calcworked Instructional Formats	374
	Correlation Between Cognitive Load and the Performance Variables	375



	Research Hypotheses	377
	Post-hoc Power Analyses	386
	Summary	387
5	DISCUSSION	
	Introduction	388
	Experiment 1	388
	Effects of the Use of Calculators on Performance in Learning	389
	Effects of the Use of the Polya Heuristic on Performance in Learning	390
	Effects of the Use of Worked Examples on Performance in Learning	391
	Summary of Research Findings of Experiment 1	392
	Experiment 2	393
	Effects of the Use of Calculators and Worked Examples on Performance in Learning	395
	Effects of the Use of the Polya Heuristic on Performance in Learning	396
	Summary of Research Findings of Experiment 2	398
	Experiment 3	399
	Effects of the Calworked Instructional Format Summary of Research Findings of Experiment 3	401
	Experiment 4	402
	Effects of the Calworked Instructional Format under Classroom Conditions Summary of Research Findings of Experiment 4	403
	Summary	404
	Summary	406
6	SUMMARY, CONCLUSIONS, IMPLICATIONS AND RECOMMENDATIONS	
	Introduction	409
	Overview of the Study	409
	Summary of the Purpose of the Study	409
	Summary of Research Methodology	410
	Summary of Research Findings	412
	Research Question No. 1	412
	Effects of the Use of Calculators	412
	Effects of the Use of the Polya Heuristic	417
	Effects of the Use of Worked Examples	418
	Research Question No. 2	422
	Effects of the Use of the Alternative Instructional Formats	422
	Conclusions of the Study	424



Implications of the Study	425
Theoretical Implications	425
Practical Implications	428
Recommendations	430
The Use of Calculators	430
The Use of the Polya Heuristic	432
The Use of Worked Examples	433
Recommendations for Further Research	434
BIBLIOGRAPHY	436
APPENDICES	466
BIODATA OF THE AUTHOR	535



LIST OF APPENDICES

Appendix		Page
A	Ujian Pemilihan Kriteria (Instrumen 1) Selection Criteria Test	466
B	Record Keeping And Observational Form (Instrumen 2)	473
C1	Rancangan Mengajar untuk Eksperimen 1,2 & 3 (Instrumen 3) General Guideline Lesson Plan For Experiments 1, 2 & 3	475
C2	Rancangan Mengajar untuk Eksperimen 4 (Instrumen 3) General Guideline Lesson Plan For Experiments 4	485
C3	Rumus-Rumus	491
D1	Instrumen 4 bagi Eksperimen 1 (Soalan-Soalan Peratus) (The Percentage Questions)	492
D2	Instrumen 4 bagi Eksperimen 2 & 3 (Soalan-Soalan Peratus) (The Percentage Questions)	496
D3	Instrumen 4 bagi Eksperimen 4 (Soalan-Soalan Peratus) (The Percentage Questions)	500
E1	Instrumen 5 bagi Eksperimen 1 (Worked Examples)	501
E2	Instrumen 5 bagi Eksperimen 2 (Worked Examples)	505
E3	Instrumen 5 bagi Eksperimen 3 (Worked Examples)	511
E4	Instrumen 5 bagi Eksperimen 4 (Worked Examples)	517
F	Instrumen 6 (Skala Kadar Daya Mental) The Paas Mental Effort Rating Scale	525
G	Panel of Content Validators And Translators	527
H	Letters of Permission	531



LIST OF TABLES

Table		Page
1.1	Differences between conventional instructional practices and cognitive load generated instructional practices	11
1.2	Performance of Malaysian students for PMR Mathematics Paper 1 (1993-1996)	12
1.3	Analysis of students' performance on percentage problems for PMR Mathematics Paper 1 (1993-1996)	14
1.4	The four-stage Polya heuristic	22
3.1	Rotation of control and treatment groups for Experiment 3 according to time allocation	205
3.2	The independent variable (instructional format) for Experiment 1 and Experiment 2	207
3.3	The independent variable (instructional format) for Experiment 3 and Experiment 4	208
3.4	Dependent variables (Measures of performance)	209
3.5	The recommended alpha range (DeVelli's Scale)	229
3.6	Difficulty and Discriminant indices of the acquisition and test items of instrument 4 in Experiment 1	237
3.7	Reliability estimates of the Paas Mental Effort Rating scale	243
3.8	Summary of Experiment 1	244
3.9	Summary of Experiment 2	255
3.10	Summary of Experiment 3	258
3.11	Summary of Experiment 4	261
4.1	Means and standard deviations of the dependent variables for the control and treatment groups for the acquisition phase of Experiment 1	273



4.2	One-way analysis of variance on the dependent variables for the acquisition phase of Experiment 1	274
4.3	Tukey post-hoc comparisons on the dependent variables for the acquisition phase of Experiment 1	275
4.4	Means and standard deviations of the dependent variables for the control and treatment groups for the test phase of Experiment 1	278
4.5	One-way analysis of variance on the dependent variables for the test phase of Experiment 1	279
4.6	Tukey post-hoc comparisons on time per test problem for the test phase of Experiment 1	280
4.7	Relative condition efficiency index of the control and treatment groups for Experiment 1	284
4.8	One-way analysis of variance on the relative condition efficiency index for Experiment 1	285
4.9	Pearson product moment correlation between mental effort and the performance variables of Experiment 1	286
4.10	Means and standard deviations of the dependent variables for the control and treatment groups for the acquisition phase of Experiment 2	306
4.11	One-way analysis of variance on the dependent variables for the acquisition phase of Experiment 2	307
4.12	Tukey post-hoc comparisons on the dependent variables for the acquisition phase of Experiment 2	308
4.13	Means and standard deviations of the dependent variables for the control and treatment groups for the test phase of Experiment 2	312
4.14	One-way analysis of variance on the dependent variables for the test phase of Experiment 2	313
4.15	Tukey post-hoc comparisons on the dependent variables for the test phase of Experiment 2	314
4.16	Relative condition efficiency index of the control and treatment groups for Experiment 2	321



4.17	One-way analysis of variance on the relative condition efficiency index for Experiment 2	321
4.18	Tukey post-hoc comparisons on the relative condition efficiency index for Experiment 2	321
4.19	Pearson product moment correlation between mental effort and the performance variables of Experiment 2	323
4.20	Means and standard deviations of the dependent variables for the conventional and calcworked groups and results of the <i>t</i> -test analyses for the acquisition phase of Experiment 3	344
4.21	Means and standard deviations of the dependent variables for the conventional and calcworked groups and results of the <i>t</i> -test analyses for the test phase of Experiment 3	346
4.22	Relative condition efficiency index of the conventional and calcworked groups and results of the <i>t</i> -test analyses for Experiment 3	350
4.23	Pearson product moment correlation between mental effort and the performance variables of Experiment 3	351
4.24	Means and standard deviations of the dependent variables for the conventional and calcworked groups and results of the <i>t</i> -test analyses for the acquisition phase of Experiment 4	369
4.25	Means and standard deviations of the dependent variables for the conventional and calcworked groups and results of the <i>t</i> -test analyses for the test phase of Experiment 4	371
4.26	Relative condition efficiency index of the conventional and calcworked groups and results of the <i>t</i> -test analyses for Experiment 4	374
4.27	Pearson product moment correlation between mental effort and the performance variables of Experiment 4	376



LIST OF FIGURES

Figure		Page
2.1	The structure of memory	64
2.2	Schema of a car	76
2.3	Total cognitive load	92
2.4	Learning facilitated (mental resources exceed total cognitive load)	93
2.5	Learning fails (total cognitive load exceeds total mental resources)	93
2.6	Learning facilitated (total cognitive load made less than total mental resources by reducing extraneous cognitive load)	93
2.7	Cognitive load construct	95
2.8	Diagram showing the redundancy effect	147
2.9	Conceptual Framework of the Study	188
3.1	A progressive series of four experiments	196
3.2	Post-test only control group design for Experiments 1 and 2	202
3.3	Post-test only control group design for Experiments 3 and 4	202
3.4	Relative condition efficiency as a function of instructional format type	251



CHAPTER 1

INTRODUCTION

Background of the Study

Introduction

In the last few decades, the mathematical community has struggled to define what mathematics is and what mathematical thinking means. One possible reason for this is the growing role of mathematics, science and technology in modern life (Organization of Economic Co-operation and Development, 2004). Consequently, there have been numerous changes made to the mathematics curriculum used in schools and institutions of higher learning.

The events in the United States typify the situation. According to Schoenfeld (1992), in 1957 when the Russians launched their space satellite *Sputnik*, it galvanized American educators, scientists and mathematicians to restructure the curriculum in line with their notion of what mathematics and mathematical thinking should be. Thus evolved *The New Maths*. When it was discovered that this new curriculum was not working, the pendulum swung to the other end and the *back-to-basics* movement grew in popularity. This too was eventually deemed to be inadequate as students were found wanting on measures of thinking. The dismal performance of the *back-to-basics* movement led

