UNIVERSITI PUTRA MALAYSIA

MATHEMATICAL MODELLING OF AN UNMANNED AERIAL VEHICLE

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This thesis presents the results of a study on the development of a mathematical model of the flight dynamics of a small-scaled aircraft. The aircraft, due to its small size has the potential to be converted to an unmanned aerial vehicle (UAV). A mathematical model of an aircraft flight dynamics can be derived through wind tunnel testing or Computational Fluid Dynamics (CFD) analysis. But the work presented in this study develops an alternative method by means of flight testing. The method shows that by proper analysis of the data from sensors that measure the aircraft state variables such as pitch angle, altitude, speed, a profile of the aircraft flight dynamics can be deduced. It is expected that the results from this study will benefit projects which interest is to convert existing flyable aircraft to be an UAV, such as the Composites Technology Research Malaysia (CTRM) project in Eagle Airborne Reconnaissance Vehicle (ARV) System. In this study, a small-scaled remote-controlled plane was equipped with onboard sensors and a data logger. A software program has been written for this project which interfaced the data-logging system, with a computer aided engineering (CAE) software MATLAB and its toolbox SIMULINK. Using the program developed, a profile of the aircraft longitudinal motion was collected through a series of flight tests. It was found that the aircraft longitudinal dynamics developed can be represented as a simple second order transfer function equation. Using the correlation coefficient method, it was found that 99.5% of the variability in short period flight test result is explained by the simulation result while 87.5% of the variability in Phugoid flight test result is explained by the simulation result. The results and method proposed in this study helps to re-create the flight dynamic characteristics of the small-scaled aircraft through flight simulation, which will be important for such work as control law determination in the future.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
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PROSES MEMBINA MODAL MATEMATIK UNTUK SEBUAH PESAWAT TANPA PEMANDU

Oleh

CHONG CHIEW CHOY

November 2005

Pengerusi: **Zairil Azhar Zaludi, PhD**

Fakulti: **Kejuruteraan**

Tesis ini menunjukkan suatu hasil kajian dalam proses membina modal matematik untuk dinamik penerbangan bagi sebuah pesawat kecil. Dengan saiz yang kecil, pesawat tersebut berpotensi untuk diubahkan sebagai sebuah pesawat tanpa pemandu. Modal matematik untuk dinamik penerbangan pesawat dapat diperolehi daripada kajian terowong angina atau *Computational Fluid Dynamics* (CFD). Hasil kerja dalam kajian ini memproseskan satu cara yang berlain iaitu kajian penerbangan. Cara ini menunjukkan dengan pengajian yang berpatutan mengenai maklumat daripada pengesan-pengesan untuk mengukur pembolehubah pesawat umpamanys sudut *pitch*, ketinggian, kelajuan, satu profil untuk dinamik penerbangan pesawat dapat dicapai. Hasil kerja dalam kajian ini dijangkai dapat memberi faedah kepada projek untuk mengubah pesawat yang terwujud kepada sebuah UAV, umpamanya projek *Composites Technology Research Malaysia* (CTRM) dalam system *Eagle Airborne Reconnaissance Vehicle* (ARV). Dalam kajian ini, sebuah kawalan jauh pesawat kecil dilengkapi dengan pengesan-pengesan dan perakam maklumat. Satu program perisian telah dikarang untuk kajian ini membenarkan perakam maklumat berinteraksi dengan satu perisian kejuruteraan komputer MATLAB dan peralatannya SIMULINK. Melalui program ini, satu profil untuk

pergerakan longitud pesawat dapat dikumpul melalui satu siri pengajian penerbangan. Kajian ini menunjukkan dinamik longitude pesawat tersebut dapat digambarkan dengan satu persamaan *second order transfer function* yang ringkas. Dengan cara pekali untuk hubungan menyaling, sebanyak 99.5% daripada kualiti sebagai pembolehubah dalam penerbangan jangka pendek dapat dijelaskan oleh simulasi manakala 87.5% daripada kualiti sebagai pembolehubah dalam penerbangan *Phugoid* dapat dijelaskan oleh simulasi. Hasil kajian dan tindakcara tercadang dalam kajian ini membantu penghasilan semula ciri-ciri khas dinamik penerbangan untuk pesawat kecil melalui simulasi penerbangan, kegunaannya adalah penting dalam kerja-kerja membina peraturan kawalan dalam masa hadapan.

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NOMENCLATURE

A	state coefficient matrix
AGL	ultrasonic altimeter
A_0	oscillation peak
A_n	nth peak
B	driving matrix
CAD	computer aided design
CIC	computer-induce-control
cg	center of gravity
FCS	flight control system
g	gravitational acceleration
GCS	Ground Control System
GPS	Global Positioning System
GUI	graphical user interface
I	identity matrix
l.h.s.	left hand side
m	number of matrix column
MIMO	multi-input/multi-output
M_q	pitch rate dimensional stability derivative
M_u	speed dimensional stability derivative
M_w	dimensional stability derivative
$M_{\delta E}$	dimensional control derivative
n	number of matrix row
PIC	pilot-induce-control
q	perturbation in angular velocity about the y stability axis

r.h.s.	right hand side
SISO	single-input/single-output
t_0	time of maximum peak A_0
t_1	time of maximum peak A_1
T	undamped period of an oscillation
T_d	damped natural period of oscillation
tf	transfer function
u	control vector
u	linearized perturbation in x axis velocity of aircraft cg
UAV	unmanned aerial vehicle
U_o	x axis velocity component of aircraft cg at trim conditions
w	linearized perturbation in z axis velocity of aircraft cg
x	state vector
$\dot{x}$	state-space equation
X_u	speed dimensional stability derivative
X_w	incidence dimensional stability derivative
$X_{\delta E}$	dimensional control derivative
Z_w	incidence dimensional stability derivative
Z_u	speed dimensional stability derivative
$Z_{\delta E}$	dimensional control derivative
d	logarithmic decrement
d_E	elevator deflection angle
λ	eigenvalue
θ	pitch angle
θ	perturbation in inertia pitch angle

ω undamped natural frequency

ω_d damped natural frequency

ω_{ph} Phugoid natural frequency

ω_{sp} short period natural frequency

ζ damping ratio

ζ_{ph} Phugoid damping ratio

ζ_{sp} short period damping ratio

CHAPTER 1

INTRODUCTION

1.1 Background

During the development of aviation many configurations and designs were constructed and modified; computer technology becomes more important in aircraft flight control system. Aircraft are initially fully controlled by human pilot and gradually partial controlled by computer, with minimal or without any human intervention, in which Unmanned Aerial Vehicle (UAV) is dealing with the frontier of the aviation history. Before discussing with the main subject of the thesis, it was considered important to define what is meant by UAV. For nearly 75 years, the American Institute of Aeronautics and Astronautics (AIAA) has been the principal voice and technical society devoted to continuing contributions and global leadership in the aerospace community. A definition of UAV is found in the AIAA Committee of Standards [1], "Lexicon of UAV/ Remotely Operated Aircraft (ROA) Terminology". It defines an UAV to be "An aircraft which is designed or modified, not to carry a human pilot and is operated through electronic input initiated by the flight controller or by an onboard autonomous flight management control system that does not require flight controller intervention."

Autopilot could perform a high level of autonomy in the UAV mission; military aircraft control system or routine flight of commercial aircraft. An autopilot system for an UAV has several capabilities; one important capability is to provide stability assistance. This part of the autopilot system is also known as Stability Augmentation System (SAS) [2]. Proportional, Integral and Derivatives (PID) control law [3] is sometimes used in SAS to stabilize the airframe of the aircraft. Another capability of an autopilot system is to provide autonomous guidance and navigation to lead the aircraft to the preprogrammed destinations. However, the PID controller setting in autopilot system is required to be fine tuned before it can provide a reliable SAS operation.

One of the main obstacles toward a deployment of UAVs is given by the fact that most autopilot manufacturers do not provide a specifically tuned autopilot system for their customers. For instance, to tune the PID controllers, the Micropilot autopilot manufacturer [4] suggests either by 'trial and error' method or simulation. The former involves adjusting the gains of Proportional (P), Integral (I) and Derivatives (D). This method is faster perhaps, but clearly is not the safer since the only way to test the effectiveness of the gains is by risky flying the aircraft and engaging the autopilot system. By simulation, however, 'the best possible' PID gains is deduced before commencing the flight testing.

To simulate the flight dynamics behaviour of an aircraft, sufficient flight dynamics characteristics of the aircraft are required. For instance, if the dynamic oscillation of the aircraft longitudinal and lateral motions is a typical second order system, then the values of the flight dynamic parameters such as natural frequency (ω_n), and damping ratio (ζ) are required. The source of these values could be obtained through the analysis of the flight data. Substituting these parameters into the theoretical mathematical equation of motion [5, 6, 7, 8] will result in a mathematical model, which describes the flight dynamics behaviour of the aircraft. The scope of the work in this thesis is to develop a longitudinal dynamics mathematical model of a small-scaled fixed wing aircraft which has the potential to be converted to become an UAV.

To build a fixed wing UAV, one would need a suitable airframe. This airframe is known as a 'platform' in this work. The platform must be flyable and unless it is a glider UAV, it must have a suitable propulsion system. Manufacturers of UAV have two options to select appropriate airframe. The first choice is to design a completely new flyable airframe, and the second choice is to use an existing flyable airframe. If manufacturers choose to design a new airframe, it is likely (though not always) that the process includes some form of aerodynamic analysis which can be wind tunnel testing [9], and/or Computational Fluid Dynamics (CFD) analysis [10]. From the aerodynamic analysis, a conclusion can be deduced of the state of the airframe stability during flight. If the manufacturer chooses to use an existing, flyable airframe, such as in the case of Composites Technology Research Malaysia (CTRM) when their EAGLE Lightweight Composite Aircraft [11] is converted to be an UAV, it may be physically challenging to place the actual airframe of the aircraft into a wind tunnel to deduce the parameters of the aircraft stability. In this case, a scaled model of the aircraft is required to be placed in the wind tunnel. The method proposed in this thesis provides an alternative method to conventional ways to obtain mathematical model of an aircraft. The method need not the knowledge of the aerodynamic coefficients but instead of aircraft dynamics parameters such as ω_n and ζ. These parameters can be obtained from the flight data analysis through flight testing. Such work done by Luetzen [12] and Rafter [13], Moes and Iliff [14] are good examples of such attempts. For Luetzen, the C-172 Cessna aircraft was tested in which ω_n and ζ were obtained from the flight testing. For Rafter, a transfer function is obtained for a Boeing commercial airline where PID controller can be used to stabilize the large aircraft and for Iliff, the SR-71 airplane has completed a series of flight tests at the NASA Dryden Flight Research Center (Edwards, California). Instead of using wind tunnel testing or CFD analysis, the series of flight tests was performed to determine stability and control characteristics. The flight testing method has been used on larger aircraft but for a small-scaled aircraft much more suitable for UAV operations, not much is known if it is possible.

The mathematical model developed in this work would be an important tool to study the aircraft stability. The results of this work provide an understanding of the aircraft dynamic behaviour which can be used to assist in the design of SAS.

The technique proposed in this work is useful for future SAS design and recommendation for the research which involves conversion from an existing aircraft to an UAV.

1.2 Objectives

The main objective of this study is to develop the longitudinal motion mathematical model of a small-scaled fixed wing aircraft by means of flight data analysis. To achieve this, it was found necessary to allow the onboard data logging system to transmit the sensor signal data from the aircraft to the on-ground computer using a computer aided engineering (CAE) software, MATLAB and its toolbox, SIMULINK

[15]. It will be an advantage for researchers to obtain ω_n and ζ if the sensor signal data could be processed in real time. Hence it is also an objective of this research to write a computer program which allows the data logging software be interfaced with MATLAB and SIMULINK in real time. The main objective will be achieved if the developed mathematical equation could reproduce a short period and Phugoid motion in simulation, in which the oscillation of the motions is as accurate as the flight testing motion.

1.3 Contribution of This Study

The following are the main contributions made by this thesis to the area of mathematical modelling for small-scaled aircraft with a potential to be converted to an UAV: the longitudinal motion mathematical model of a small-scaled fixed wing aircraft can be developed through extraction of flight dynamics parameters [16] obtained from a series of flight tests. A specially written software code allows the flight data from the data logger to be interfaced in real time with MATLAB and SIMULINK for instant data analysis. The results from the longitudinal motion mathematical model can be used in the future for control law determination through flight simulation.