Computing two dimensional Poincaré maps for hyperchaotic dynamics

ABSTRACT

Poincaré map (PM) is one of the felicitous discrete approximation of the continuous dynamics. To compute PM, the discrete relation(s) between the successive point of interactions of the trajectories on the suitable Poincaré section (PS) are found out. These discrete relations act as an amanuensis of the nature of the continuous dynamics. In this article, we propose a computational scheme to find a hyperchaotic PM (HPM) from an equivalent three dimensional (3D) subsystem of a 4D (or higher) hyperchaotic model. For the experimental purpose, a standard four dimensional (4D) hyperchaotic Lorenz-Stenflo system (HLSS) and a five dimensional (5D) hyperchaotic laser model (HLM) is considered. Equivalent 3D subsystem is obtained by comparing the movements of the trajectories of the original hyperchaotic systems with all of their 3D subsystems. The quantitative measurement of this comparison is made promising by recurrence quantification analysis (RQA). Various two dimensional (2D) Poincaré mas are computed for several suitable Poincaré sections for both the systems. But, only some of them are hyperchaotic in nature. The hyperchaotic behavior is verified by positive values of both one dimensional (1D) Lyapunov Exponent (LE-I) and 2D Lyapunov Exponent (LE-II). At the end, similarity of the dynamics between the hyperchaotic systems and their 2D hyperchaotic Poincaré maps (HPM) has been established through mean recurrence time (MRT) statistics for both of 4D HLSS and 5D HLM and the best approximated discrete dynamics for both the hyperchaotic systems are found out.

Keyword: Poincaré map; Hyperchaos; Recurrence quantification; LE-I & LE-II; Mean recurrence time statistics