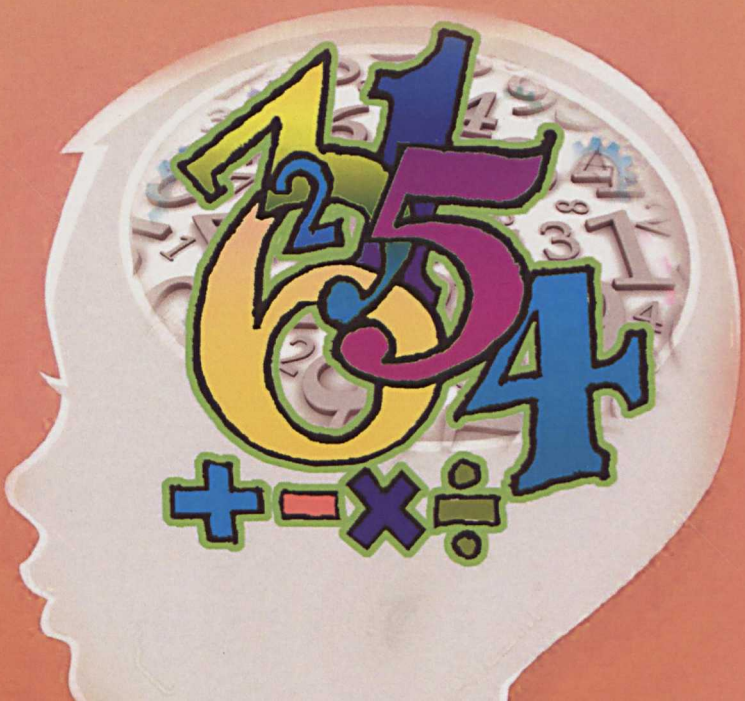


# INAUGURAL LECTURE series

Prof. Dr. Aida Suraya  
Md. Yunus



## Developing Students' Mathematical Thinking: How Far Have We Come?





**Developing Students'  
Mathematical Thinking:  
How Far Have  
We Come?**

PROFESSOR OF MATHEMATICS, UNIVERSITY OF TEXAS AT AUSTIN



**PROFESSOR DR. AIDA SURAYA MD. YUNUS**

# Developing Students' Mathematical Thinking: How Far Have We Come?

**Prof. Dr. Aida Suraya Md. Yunus**

BSc. (Mathematics) Ohio University, M.Sc (Mathematic) West Virginia University,  
Ph.D (Mathematics Education) University Malaya

**5 JUNE 2015**

Auditorium  
Faculty of Educational Studies  
Universiti Putra Malaysia



**Universiti Putra Malaysia Press**

Serdang • 2015

<http://www.penerbit.upm.edu.my>

© **Universiti Putra Malaysia Press**

First Print 2015

All rights reserved. No part of this book may be reproduced in any form without permission in writing from the publisher, except by a reviewer who wishes to quote brief passages in a review written for inclusion in a magazine or newspaper.

UPM Press is a member of the Malaysian Book Publishers Association  
(MABOPA)

Membership No.: 9802

Typesetting : Sahariah Abdol Rahim @ Ibrahim

Cover Design : Md Fairus Ahmad

*Design, layout and printed by*  
Penerbit Universiti Putra Malaysia  
43400 UPM Serdang  
Selangor Darul Ehsan  
Tel: 03-8946 8855 / 8854  
Fax: 03-8941 6172  
<http://www.penerbit.upm.edu.my>

## Contents

Abstract	1
Introduction	3
Direction of Malaysian School Mathematics	4
Mathematical Thinking	7
Conceptual and Procedural Knowledge	11
Finding on Students' Mathematical Thinking	14
Role of Technology in Facilitating Mathematical Thinking	39
Current Status of Malaysia School Mathematics	42
How Successful Have We Been in Facilitating the Development of Mathematical Thinking?	51
The Mathematical Curriculum	52
The Mathematics Classroom Instruction	53
The Mathematics Assessment	57
Conclusion	58
References	59
Biography	67
Acknowledgements	71
List of Inaugural Lectures	75

## ABSTRACT

Malaysian students' poor performance in the newly implemented Form Three Assessment (Pentaksiran Tingkatan 3, PT3), TIMSS and PISA has spurred many debates and criticism on the quality of our students' learning of mathematics and science. Since the inception of the Integrated Curriculum for Secondary Schools in 1989, the aim of secondary mathematics has been steadfastly on "developing individuals who are able to think mathematically and who can apply mathematical knowledge effectively and responsibly in solving problems and making decisions". So how far have we come? Are the classroom activities, assessment tasks and assessment questions geared towards cultivating mathematical thinking and decision making? Evidence show that there has been vast misalignment between the intended curriculum (the one prescribed by policy makers), the implemented curriculum (the one that is actually carried out by teachers in their classrooms) and the attained curriculum (the one learnt by students or on what was examined). Thus can the large number of students getting As in mathematics for Lower Secondary School Evaluation (PMR) and Malaysian School Certificate (SPM) be used as indicators of the success in developing students' thinking?

This paper discusses what is meant by mathematical thinking. There has not been a conclusive definition on mathematical thinking because some view it as a process and others as an outcome. Mathematical thinking is the foundation to do reasoning and problem solving and to develop conceptual knowledge, as opposed to procedural knowledge. Several findings from studies that focused on students' ability to provide reasoning and give meanings to concepts and algorithms are highlighted. Students' development in geometric thinking based van Hiele's levels of geometric thinking in learning, shapes and spaces is also discussed. van Hiele's

levels of geometric thinking includes higher order thinking and decision making skills and acquisition of mathematical concepts to enable learners to operate at higher levels in van Hiele's theory. The role of metacognition in facilitating mathematical thinking is also deliberated. Metacognition refers to higher order thinking which involves active control over the cognitive processes engaged in learning. The paper also touches on the role of technology in facilitating mathematics thinking.

This paper highlights that generally, Malaysian students are not meeting the international benchmark for mathematics performance. This may be partially attributed to students' inability to think mathematically and thus their inability to translate the contexts given and to use mathematics to provide solutions. Factors that may contribute to this phenomenon are further examined from the perspectives of the curriculum, the instruction and the assessment. The present curriculum may not be able to support the initiative to focus on higher order thinking skills (HOTS), as can be concluded from the expected outcomes stated in the curriculum guides. In terms of instruction, although it has been suggested since 20 years back, constructivist teaching has not been a preference of Malaysian teachers. Nevertheless, the shift to school based assessment, the HOTS initiative and the change in examination format to include more challenging questions, such those given in PT3, TIMSS and PISA, may well elicit a classroom atmosphere that cultivates HOTS as well as critical and creative thinking. To provide support for learning school textbooks need to be greatly improved as the current contents, contexts and examples do not stimulate students' thinking. As for assessment, as long as the 'teaching for examination' culture is strong and the right concept of school-based assessment is not implemented, not much change will happen in classroom instruction and the aspiration to get our students to think mathematically will remain far from reality.



## INTRODUCTION

In December 2014, the newly implemented Form Three Assessment (Pentaksiran Tingkatan 3, PT3) caused a lot of disappointment and commotion among students, parents, teachers and other interested stakeholders. The Ministry of Education (MOE) and the Examination Board in particular were bombasted with criticisms concerning the entire implementation of PT3, including the examination format, last minute release of information, security of examination papers, grading and scoring rubrics (Ahmad Nurulazam, Aida Suraya, et al., 2015). The public was comparing the performance of students in the Lower Secondary Examination (Penilaian Menengah Rendah, PMR) with that of the PT3. As reported by Ng (2014), 30988 out of 422506 (7.33%) students who sat for the PMR scored straight As in 2013, which was an increase of 0.41% from 2012. He elaborated further that in comparison only 80 students (0.02%) got straight As out of the 450,000 candidates who sat for the PT3 in 2014. The MOE was thus accused of inflating grades and “manipulating results” over all these years to ensure achievement of a string of As.

The assessment standards used in PT3 focuses more on higher order thinking (HOTS) skills. In the study on PT3 (Ahmad Nurulazam, Aida Suraya, et al., 2015), some teachers and administrators were of the opinion that the PT3 results reflect students’ actual performance much better than the PMR results. So which of the two assessments better portrays Malaysian students’ authentic performance? Is the problem related to the type of assessment used or on what has been learnt and emphasized in mathematics instruction over all these years? Are students able to think beyond the exemplars and stereotyped problems given during instruction and in textbooks? In trying to answer these questions it is worth revisiting the directions in school mathematics.

## **DIRECTION OF MALAYSIAN SCHOOL MATHEMATICS**

The National Philosophy of Education (NPE) introduced in 1988, provides essential principles and guidelines with respect to teaching and learning. It boldly emphasises on the need to produce Malaysian citizens who are knowledgeable and competent.

Education in Malaysia is an ongoing effort towards further developing the potential of individuals in a holistic and integrated manner so as to produce individuals who are intellectually, spiritually, emotionally and physically balanced and harmonious, based on a firm belief in and devotion to God. Such an effort is designed to produce Malaysian citizens who are knowledgeable and competent, who possess high moral standards, and who are responsible and capable of achieving a high level of personal well-being as well as being able to contribute to the betterment of the family, the society and the nation at large (Ministry of Education (2006a), p. v).

The philosophy was well translated in the aims for school mathematics performance. The aims for school mathematics were first formulated and released in 1989 and later revised in 2002. However, the statement on the need to “develop individuals who are able to think mathematically and who can apply mathematical knowledge effectively and responsibly in solving problems and making decisions” remains.

The Malaysian mathematics curriculum for the secondary school level aims to develop individuals who are able to think mathematically and who can apply mathematical knowledge effectively and responsibly in solving problems and making decisions. This will enable the individual to face the challenges in everyday life that arise due to the advancement of science and technology (MOE Malaysia, 2006a).

The curriculum and the text books have been well aligned to ensure that the aims are met. The curriculum guides and text books are meant to be guides to help teachers implement and promote mathematical thinking and application within and outside the context of mathematics. The curriculum emphasizes on problem solving, communication, reasoning, mathematical connections and application of technology (Malaysian MOE, 2006a). These are partly based on the National Council of Teachers of Mathematics (NCTM) process standards which include problem solving, reasoning and proof, communication, connections, and representations (NCTM, 2000).

The emphasis on reasoning is clearly stated in Malaysian Mathematics Curriculum as follows:

- i. Logical reasoning or thinking is the basis for understanding and solving mathematical problems.
- ii. Students are encouraged to estimate, predict and make intelligent guesses (conjectures) in the process of seeking solutions.
- iii. Students are to be trained to investigate their predictions or guesses by using concrete material, calculators, computers, mathematical representation etc.

- iv. Logical reasoning has to be absorbed in the teaching of mathematics so that students can recognize, construct and evaluate predictions and mathematical arguments. (Malaysian MOE, 2006a)

Back to the issue of students' poor performance in PT3, was there then a mismatch between what was intended and what was attained or examined? Howson and Wilson (1986) explained that the intended curriculum is the one prescribed by policy makers while the implemented curriculum is the one that is actually carried out by teachers in their classrooms, and the attained curriculum is the one learnt by students or on what was examined. As stated in the Malaysian Education Blueprint 2013-2025, preliminary report September 2012, p. 4-1, the Malaysian curriculum was examined from the following three aspects

- i. What is written in the curricula, or the "Written Curriculum": the knowledge, skills and values that form the content, outlining what is to be taught by teachers;
- ii. What is taught in the classroom, or the "Taught curriculum": the knowledge acquired, skills developed, and values inculcated in students; and
- iii. What is examined, or the "Examined curriculum": students' knowledge, skills, and values that are tested, either in summative national examinations such as the UPSR, PMR, and SPM, or through formative and/or summative school based assessments that guide teaching.

The written curriculum clearly dictates the need to develop individuals who are able think mathematically and who can apply mathematical knowledge effectively and responsibly in solving problems and making decisions. Are teachers focusing on these

aims? Are the classroom activities, assessment tasks and questions given in the classroom cultivating mathematical thinking and decision making? Are the assessment items measuring students' mathematical thinking? Can the large number of those getting As in mathematics for PMR and Malaysian School Certificate (SPM) examinations be used as indicators of the success in developing students' thinking skills? Or is the achievement of few As in PT3 reflect more accurately whether schools have been successful in developing mathematical thinking?

## MATHEMATICAL THINKING

An earlier definition on mathematical thinking as provided by Polya (1968) revolves around the process of understanding phenomena through exploration, making conjectures, developing and testing hypothesis, data collection and analysis. To Polya (1968), mathematical thinking is achieved through two processes, namely: (i) identifying patterns and principles which are consistent, through the process of induction; and (ii) by building chains of inferences to support an argument. Polya's construction of knowledge is based on logical reasoning, which involves inductive and deductive thinking. According to Polya (1965):

Mathematical thinking is not purely 'formal'; it is not concerned only with axioms, definitions and strict proofs, but many other things belong to it: generalizing from observed cases, inductive arguments, arguments from analogy, recognizing a mathematical concept in, or extracting it from a concrete situation. The mathematics teacher has an excellent opportunity to acquaint his students with these highly informal thought processes . . . stated incompletely but concisely: let us teach proving by all means, but let us also teach guessing. (p.100)

On the other hand, Schoenfeld (1992) is of the opinion that individuals must value the processes of mathematization, abstraction and having the predilection to make abstraction, symbolic representation and symbolic manipulation to understand mathematical structures. He affirmed that learning to think mathematically involves developing a mathematical point of view and developing competence with the tools of the trade, and using those tools in understanding structure and sense-making in mathematics.

Devlin (2012) provided a more simple explanation of mathematical thinking. According to him, mathematical thinking is not the same as doing mathematics, at least not as mathematics is typically presented in our school system. He highlighted that school mathematics usually requires students to solve highly stereotyped problems. Devlin (2012) elaborated that the focus of mathematics learning is normally on applying various procedures to solve math problems, whereas it should be more on “what” and “why” so that students can get the big picture.

To develop students' mathematical thinking and hence improve their ability in answering higher order thinking questions (HOTS) such as those given in TIMSS, PISA or PT3, the taught and examined curriculum must be aligned with the written curricula, as discussed earlier. Students' difficulties in solving problems that require application and reasoning are evident from examples of TIMSS questions that will be discussed later.

Consider the following simple question on equivalent fractions:

$$\frac{2}{8} = \frac{\quad}{16}$$

Many students in Year 5 in primary school may be able to provide the answer quite easily. However, if the question is

rephrased as shown below, where the answer required is the same, the complexity increases greatly.

*Zharif ate two pieces of a regular size pizza that is divided into eight equal parts. If you have a pizza of the same size but divided into 16 equal parts, how many should you eat so that you get the same amount as Zharif?*

This question requires understanding of what the question requires and translation of the scenario given into mathematical language. It may also require the student to understand some unfamiliar English words. Lastly, the student must be able to relate the context given to an equivalent fraction. In such instances some students may not even be motivated to read the lengthy question. As compared to the question that requires students to provide an equivalent of a given fraction, this wordy problem requires students to make interpretations, make connections, reason, and do the required algorithm. In short, they are required to think.

In contrast, if the student is able to provide the right answer to the direct question on equivalent fractions as given earlier, their ability cannot be used to imply that they have developed their thinking on the concept of equivalent fractions. It may just be a mechanical process to the students, providing the answer by following a certain procedure taught to them. Thus, the teacher may not have enough evidence on the student's ability to think mathematically. Kieran and Pirie (1991) characterized mathematical thinking in terms of the learner being able to develop strong understandings in mathematical situations.

Solving highly stereotyped problems may only require knowing the procedures of the algorithm, as opposed to what professional mathematicians do, which includes thinking of ways to solve real

problems. These problems may emerge from within and outside of mathematics. Mathematics can be related to Islamic Studies or any other subjects. For example, calculating the distribution of the estate of a deceased person among his heirs based on Faraid is an application of fractions. To Devlin (2012), “doing math” usually involves the application of formulas, procedures and symbolic manipulations, whilst mathematical thinking is a powerful way of thinking about things in the world logically, analytically, quantitatively and with precision.

Hwa and Stephens (2011) asserted that mathematical thinking involves mental operations that are facilitated by mathematical knowledge and productive disposition toward mathematical problem solving. Mason, Burton and Stacy (1982) view mathematical thinking as a process to improve mathematical problem solving performance by giving heuristic strategies as well as monitoring and controlling their outcomes in a meta-cognitive way. They also identified four fundamental processes, in two pairs, and showed how thinking mathematically very often proceeds by alternating between them: (i) specialising – trying special cases, looking at examples; (ii) generalising - looking for patterns and relationships; (iii) conjecturing – predicting relationships and results; and (iv) convincing – finding and communicating reasons why something is true. Harel and Sowder (2005) explained that understanding involves the particular meaning students give to a term, sentence or text, the solution they provide to a problem, or the justification they use to validate or refute an assertion, and the underlying actions are ways of thinking.

The Programme for International Student Assessment (PISA) popularised the term ‘mathematical literacy’. Mathematical literacy is the ability to use mathematics for everyday living, work and further study. In line with this definition, PISA assessments



present students with application problems, gauging students' ability to connect mathematics in realistic contexts. The framework used by PISA (2006) shows that mathematical literacy involves many components of mathematical thinking, including reasoning, modelling and making connections between ideas.

Scusa (2008) provided the five processes involved in mathematical thinking: (i) problem solving; (ii) making connections; (iii) representation; (iv) reasoning and proof; and (v) communication. These are the exact process standards for mathematics as drawn by NCTM. Mathematical thinking is often used synonymously with problem solving. However, I concur with the views of Mason, Burton and Stacy (1982), Hwa and Stephens (2011) and Devlin (2012), that mathematical thinking facilitates and helps improve mathematical problem solving. Mathematical thinking also develops conceptual knowledge, which is also a prerequisite to become good problem solvers. With the emphasis on thinking, Singapore adopted the vision statement for its MOE as *Thinking Schools, Learning Nation* (TSLN), way back in 1997.

## CONCEPTUAL AND PROCEDURAL KNOWLEDGE

Mathematical connection is one of the emphases in Malaysian school mathematics. The elaborated description is "opportunities for connection must be created so that students can link conceptual to procedural knowledge and relate topics within mathematics and other learning areas in mathematics" (MOE, 2006a). Hence, there is a need to clarify the difference between conceptual and procedural knowledge.

According to Hiebert and Lefevre (1986), conceptual knowledge is knowledge that is rich in relationships and meanings or underlying structure, thus a unit of conceptual knowledge cannot

be an isolated piece of information. According to Byrnes and Wasik (1991), it is assumed that conceptual knowledge is stored in some form of relational representation such as schemas, semantic network or hierarchies. For instance, if one has a conceptual understanding of fractions, they would be able to link fractions to decimal numbers, have an understanding of the concept of part-to-whole, understand proportion, understand what the numerator and denominator implies, and can represent fractions using discrete objects or continuous representations. One who has conceptual understanding of fractions can spontaneously give the answer for '0.25 of 8' or '0.25 x 88' because they can immediately 'see' 0.25 as  $\frac{1}{4}$ . Many others will depend on the procedure of long multiplication. This is part of number sense. Gersten and Chard (1999) explained that number sense refers to a student's fluidity and flexibility with numbers which include sense of what numbers mean, understanding of their relationships to one another, ability to perform mental mathematics, understanding symbolic representations and the ability to use those numbers in real world situations.

On the other hand, Hiebert and Lefevre (1986) defined procedural knowledge as formal language or symbolic representations of mathematics and the algorithm or rules in completing mathematical tasks. Algorithmic knowledge refers to step by step instructions that define precisely how to complete mathematical tasks in a predetermined linear sequence. Thus, conceptual knowledge will facilitate the acquisition of procedural knowledge but, having procedural knowledge alone may not eventually develop conceptual knowledge naturally. Simply stated, procedural knowledge or algorithmic knowledge can help students solve many kinds of problems but it cannot ensure that they have the conceptual knowledge to solve non-routine or novel problems (Steen, 1999; Sternberg, 1999).

One of the learning principles in NCTM's (2000) *Principles and Standards for School Mathematics* depicts the importance of helping students to develop conceptual understanding.

*Students must learn mathematics with understanding, actively building new knowledge from experience and previous knowledge. Research has solidly established the important role of conceptual understanding in the learning of mathematics. By aligning factual knowledge and procedural proficiency with conceptual knowledge, students can become effective learners. They will be able to recognize the importance of reflecting on their thinking and learning from their mistakes. Students become competent and confident in their ability to tackle difficult problems and willing to persevere when tasks are challenging.*

According to Harel and Sowder (2005), understanding involves three types of mathematical activities: (i) the particular meaning or interpretation a person gives to a concept, relationships between concepts, assertions or problems; (ii) the particular solution a person provides to a problem; and (iii) the particular evidence a person offers to establish or refute a mathematical assertion. In helping students to develop conceptual understanding and construct meaning or interpretation of a concept, relationships between concepts, assertions, or problems, the teacher must provide learning opportunities for students to explore meanings. NCTM (2000) asserted that "students must learn mathematics with understanding, actively building new knowledge by relating their previous experience and prior knowledge to the new situations" (p. 11). The NCTM's learning principle also emphasizes on the importance of the development of conceptual understanding. Students often rely on memorization of factors or procedures without understanding

them well. Conceptual understanding allows students to develop further by making them learn to solve the problems, explore mathematical ideas and eventually getting them to be confident and autonomous learners.

## **FINDINGS ON STUDENTS' MATHEMATICAL THINKING**

In understanding students' mathematical thinking, it is crucial to look into students' abilities and strategies in solving problems and their reasoning, rather than looking into the solutions alone.

Mathematical thinking needs to focus more on the ways that students conceptualize a problem and develop appropriate solution strategies rather than on whether or not they can carry out a formal algorithm to reach a solution. Furthermore, it is important to examine cognitive aspects of problem solving, such as the students' solution strategies, their mathematical misconceptions/errors, mathematical justifications and representations (Cai & Cifarelli, 2004, p. 73).

Cai and Cifarelli (2004) further elaborated that the examination of solution justifications and representations provides insights on how students process problems and how they communicate their ideas and thinking processes. This assertion may be built upon Sternberg's (1991) earlier statement that the examination of solution strategies provides qualitative information on students' mathematical thinking and reasoning.

In assisting students to develop their mathematical thinking, the basis for teaching must be on attaching meaning to what they learn. As an example, in learning about conversion of a number from base 10 to base 2, students may lose out totally on the meaning of

base-2 numbers if the focus is simply on the mechanical procedure of determining the binary number through repeated division by 2, followed by listing the remainder from the bottom up. Students may be able to get the answer correctly but there will be no meaning attached to the procedure.

$2 \overline{)156}$	Remainder:
$2 \overline{)78}$	0
$2 \overline{)39}$	0
$2 \overline{)19}$	1
$2 \overline{)9}$	1
$2 \overline{)4}$	1
$2 \overline{)2}$	0
$2 \overline{)1}$	0
	1

Therefore  $156_{10} = 10011100_2$

Likewise, instructions such as the following can be detrimental to the development of conceptual understanding of numbers in different bases.

Steps in converting numbers in base 10 to base 2, 8 and 5 are as follow:

1. Perform repeated division with the base that you need until the quotient is 0.
2. Write the number in the new base by referring to the remainders from bottom to the top.

In converting numbers in base 10 to base 2, students may need to be provided with reasons on why it needs to be divided repeatedly with 2 and why we take the remainders from the bottom up. To provide meaning to the procedure, the instructions must include the connection between place values of numbers in the decimal system, i.e.  $\dots 10^5, 10^4, 10^3, 10^2, 10^1$ , to place values of the new base, eg.  $\dots 2^5, 2^4, 2^3, 2^2, 2^1$ , and the equivalent decimal values,  $\dots 16, 8, 4, 2, 1$ . Students must also be made aware of the digits

used in the decimal system and be able to connect it to digits to be used in a binary system, base 5 system, etc.

Binary Evaluate	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$		Value	Decimal Number
Decimal Value	16	8	4	2	1			
					0	>	0	0
					1	>	1	1
			1	1	0	>	$4+2+0$	6
		1	0	1	0	>	$8+0+2+0$	10
	1	0	1	1	0	>	$16+0+4+2+0$	22
	1	1	0	0	1	>	$16+8+0+0+1$	25
	1	1	1	1	1	>	$16+8+4+2+1$	31

Thus, 3456 means  $3 \times 10^4 + 4 \times 10^3 + 5 \times 10^2 + 6 \times 10^1$  and similarly,  $1010_2$  means  $1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1$ .

### Ability to Reason and Give Meaning

Meaning must be orchestrated by the teacher in her instruction. Aida Suraya (2001a, 2003) explored the meanings attached by final year undergraduate students in mathematics education to common algorithms and concepts that they will have to teach in schools. The respondents were given time to solve the questions in a very relaxed situation and they were allowed to refer to the textbook provided. Once they had completed the tasks, they were asked to explain on how to convince school students of the concepts/algorithms given. Data comprises the interview transcripts, the respondents' written responses or solutions to the problem and the interviewer's notes on non-verbal behavior. Some of the tasks given were as follows:

<b>Justification for the selection of tasks</b>	
<b>Task</b>	
<p>Sample Task 1</p> <p>Explain why <math>\log_{10} XY = \log_{10} X + \log_{10} Y</math></p>	<p>This question was given because many school students make the common mistake that <math>\log_{10} XY = \log_{10} X \times \log_{10} Y</math>. Teachers normally ask students to memorize these rules.</p>
<p>Sample Task 2</p> <p>Solve <math>\frac{4}{6} + \frac{1}{3}</math></p> <p>Explain the algorithm.</p>	<p>There is no illustration or model that can be used to show division with a fraction. Further, prior to learning this content, students may have noticed or made generalization that “by dividing with a number, the dividend will always be smaller”. Thus, this is an algorithm that a teacher may simply tell the students “This is how to divide a fraction with a fraction” with an implied message, “Believe me, this procedure is valid, although it may not make much sense”. The change of operation from division to multiplication and reversal of the fraction, from <math>\frac{1}{3}</math> to <math>\frac{3}{1}</math>, will never be challenged by most students as to why one can “break all the rules”.</p>

Sample  
Task 3

Compare the means obtained from a set of 30 data items, when it is: (i) ungrouped; and (ii) grouped according to a certain class interval.

School students are taught to identify the means for ungrouped data at form 2 level and the means for grouped data at form 4 level. No focus is given to how the presentation of the data would affect the mean.

What happens to the mean of grouped data when the class interval is made larger? Why?

Sample  
Task 4

Provide a convincing argument to show that  $-(-a) = a$

This is usually left unexplained by most teachers. It is one of those "it is like that".



The categorization of the levels of mathematical reasoning used in this study were as follows: (a) Level 1: Unable to produce any reasoning; (b) Level 2: Have awareness of the models, known facts, properties and relationships to be used but cannot produce any arguments; (c) Level 3: Able to produce some reasoning although the arguments are weak; and (d) Level 4: Able to produce strong arguments to support their reasoning. In Aida Suraya's (2003) study, respondents were found to be operating at Level 1 for 46% of the tasks, Level 2 for 22% of the tasks, Level 3 for 15% of the tasks and Level 4 for 15% of the tasks. The respondents could not produce any arguments for quite a number of the tasks given. They were given scores on their ability to reason. The respondents found the most difficult task was to provide reasoning for  $-(-a) = a$ . In general, the ability of 50% of the respondents to provide reasoning for the tasks was low, 30% moderate and 20% high. This is of concern since the respondents were final year students who would soon become mathematics teachers.

For sample tasks 1 and 4, many of the respondents could not provide any explanation. The common response was: "The formula is like that. My teacher showed me like that". One tried to explain  $-(-a) = a$  by associating it with magnetic charge, "negative and negative becomes positive". She later added that "Someone borrows \$5, that means -5. Another person borrows \$5. How much do they borrow? This means 10" while writing  $(-5)(-5) = 10$ . In working out sample task 2, one respondent divided 4 by 6 and got 0.6. She did not even associate this with  $\frac{4}{6}$  and  $\frac{2}{3}$  to immediately get 0.67. She wanted to see if the answer would turn out to be the same.

There were some good reasoning provided by the respondents. For example, using the definition of log, one respondent followed it through and provided a convincing explanation for Sample Task

1. For  $-(-a) = a$ , only one respondent could provide an acceptable explanation such as the following:

$$a + (-a) = 0$$

$$-a + a = 0$$

$$-(-a + a) = 0$$

$$-(-a) - a = 0$$

$$-(-a) = a$$

For Sample Task 3, most of the respondents could not reason out whether the mean for the ungrouped data would be the same as the mean for the grouped data. One of the respondents anticipated that the means would be different but somehow by mistake, she arrived at the same answer. She reasoned this out by saying that the data that does not involve decimal numbers or data count that is even might result in the same mean. Another respondent was convinced that the two means would be the same because the data is the same. However, she later changed her mind and said that it will be different because the formula used in calculating the mean is different. One gave a good explanation as follows:

*The mean from ungrouped data will be more accurate. For grouped data, we use midpoints. For a small number of data, the difference is not much or the two means would be the same. But for large data sets and especially for intervals that are big, accuracy would be reduced. The means we get from the frequency distribution (ie. grouped data) are just estimates and might not be accurate.*

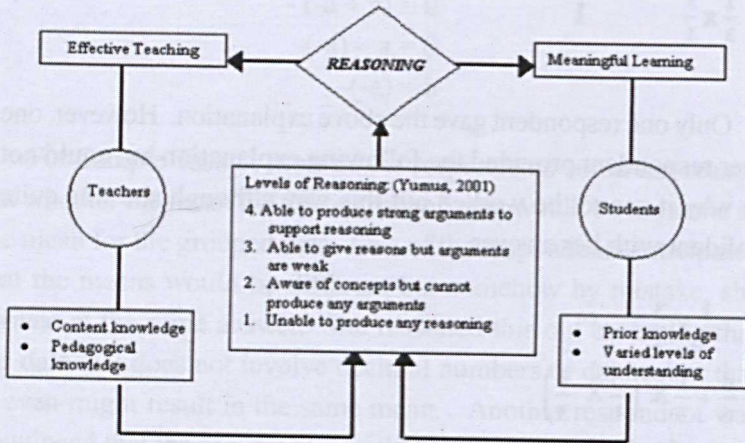
For  $\frac{4}{6} \div \frac{1}{3}$ , I had wanted to see the respondents viewing it as

$$\frac{\frac{4}{6} \times \frac{3}{1}}{\frac{1}{3} \times \frac{3}{1}} = \frac{\frac{4}{6} \times \frac{3}{1}}{1} = \frac{4}{6} \times \frac{3}{1} = \frac{12}{6} = 2$$

Only one respondent gave the above explanation. However, one other respondent provided the following explanation but could not see why it cannot be worked out this way although she was quite confident with her answer.

$$\begin{aligned} & \frac{4}{6} \div \frac{1}{3} \times \frac{3}{3} \\ & - \frac{4}{6} \div \frac{1}{3} \times \left( \frac{3}{1} \times \frac{1}{3} \right) \\ & - \frac{4}{6} \div \left( \frac{1}{3} \times \frac{3}{1} \right) \times \frac{1}{3} \\ & - \frac{4}{6} \div 1 \times \frac{1}{3} = \frac{4}{6} \times \frac{1}{3} \end{aligned}$$

The classification used by Aida Suraya (2003) in determining the levels of students' reasoning was later adapted by de Castro (2004). It relates to how effective teaching will bring about reasoning and meaningful learning (Figure 1).



**Figure 1** Mathematical Reasoning Imperative Factor in the Teaching-Learning Process

Source: de Castro (2004, p. 160)

In another study on students' algorithms of multiplication involving decimals (Aida Suraya, 2001b), seven algorithms used by students were identified. The algorithms that students construct or generalize on their own are part of their own thinking. Identification of algorithms from students' perspectives forms the basis in planning teaching strategies. The standard algorithm taught is to stack the numbers vertically, do the multiplication as is normally done for any two whole numbers and with the decimal point being subsequently inserted into the product. The number of decimal points is determined based on the sum of the number of decimal points of the multiplicand and the multiplier.

Algorithms are part of procedural knowledge unless the teacher provides the “what” and “why” so that students can see the big picture and the interrelatedness between concepts, thus allowing students to develop conceptual knowledge. This is evident from the study of decimals (Aida Suraya, 2001b). When asked, “If  $7 \times 34 = 238$ , what is  $0.7 \times 3.4$ ?”, all the respondents who were in Form 1 carried out the vertical process of multiplying and conducted the standard algorithm without using the information provided. This is a common problem posed in the Lower Secondary Examination (PMR). However, the respondents, which comprised of high, moderate and low achievers, were not able to make the connection on their own.

As in the case of  $0.55 \times 0.25$ , many of the respondents were able to provide the correct answer. However, they could not provide any explanation as to why the sum of the number of decimal points of the multiplicand and the multiplier is used to determine the decimal point of the product. It is only in later years that some students may have the insight that  $0.55 \times 0.25$  is the same  $\frac{55}{100} \times \frac{25}{100} = \frac{1375}{1000} = 0.1375$ . Most students will just see it as the procedure to be followed. In the case of  $0.55 \times 0.25$ , it is quite difficult to get students to make an estimate of the product before they carry out the algorithm. However, in cases such as with the question  $2.34 \times 5.2$ , students should be taught to make estimations, expecting the answer to be around 11 so that they can gauge the range of acceptable answers for the product.

Based on a series of observations and interviews, the most common algorithm used by students in Aida Suraya’s (2001b) study was the standard algorithm, as discussed above. In some cases the students used the “convert to fraction first” algorithm. Here, some students were able to convert the decimal given to a fraction first,

as in the case of computing  $0.25 \times 16$ . A few of the students could see 0.25 as  $\frac{1}{4}$  thus they could automatically provide the answer for  $0.25 \times 16$  as 4. Students who can see a decimal number as a fraction and vice versa and are able to use the equivalent in a different form are considered as having developed a good number sense.

In multiplying a decimal number with multiples of 10, some of the students used the algorithm 'shifting the decimal point to the right based on the number of zeros'. Thus in solving  $0.6 \times 10$  or  $0.6 \times 200$ , some of the students did not use the standard algorithm but determined how many decimal points to shift based on the number of zeros. However, when asked why the answer for  $0.6 \times 10$  is 6 and not 60, the students replied "I am not sure how the number of zeros is related to the decimal number".

It is apparent from this study (Aida Suraya, 2001b) that teachers need to emphasize on the relationship between whole numbers, fractions and decimals. To help students develop a good number sense, students need to be guided to look at problems from different perspectives and to decide on the easiest and most efficient algorithm to use. Students should be guided to see the relationships between multiplications, such as  $5 \times 62$ ,  $0.5 \times 0.62$ ,  $0.05 \times 0.062$  and  $\frac{5}{10} \times \frac{62}{100}$  (Aida Suraya, 2001b). Students' lack of development of number sense is also apparent in an earlier study by Aida Suraya, Sharifah and Habsah (1994) where only 21.2% of 151 students in Year 5, of varied mathematical competence, could solve the problem:  $20 + \frac{6}{100}$ . Surprisingly, 38.4% gave the answer  $\frac{26}{100}$ , while other answers given include  $\frac{206}{100}$ , 20.6 and 26.

## GEOMETRIC THINKING

Apart from studying mathematical thinking, there have been many researches that focus on thinking in specific areas of mathematics. Tan, Rohani, Aida Suraya and Ahmad Fauzi (2015) explored the understanding of primary school students' on van Hiele's (1986) levels of geometric thinking in learning shapes and spaces. The levels of geometric thinking proposed by van Hiele describes the progression that a student undergoes, independent of age or grade level. Table 1 shows the levels, description of the levels and what students need to illustrate at these levels. Students must progress from one level to another sequentially and without skipping any one level.

In the *Trends in International Mathematics and Science Study* (TIMSS), Malaysian students showed a gradual decline in performance in geometry with means scores of 497 in 1999, 478 in 2003, 477 in 2007 and 432 in 2011 (Tan et al., 2015). According to the National Centre for Education Statistics (NCES) in their highlights for TIMSS 2011 (NCES, 2013), Malaysian Form Two students were ranked in the "intermediate" category, which means that they can only apply basic mathematical knowledge, simple algebraic forms and two dimensional drawings.

**Table 1** The van Hiele Model of Geometric Thinking

Level	Name	Description	Example	Teacher Activity
0	Visualization	See geometric shapes as a whole; do not focus on their particular attributes.	A student would identify a square but would be unable to articulate that it has four congruent sides with right angles.	Reinforce this level by encouraging students to group shapes according to their similarities.
1	Analysis	Recognize that each shape has different properties; identify the shape by that property.	A student is able to identify that a parallelogram has two pairs of parallel sides and that if a quadrilateral has two pairs of parallel sides it is identified as a parallelogram.	Play the game "guess my rule", in which shapes that "fit" the rule are placed inside the circle and those that do not are placed outside the circle.
2	Informal deduction	See the interrelationships between figures.	Given the definition of a rectangle as a quadrilateral with right angles, a student can identify a square as a rectangle.	Create hierarchies (i.e. organizational charts of the relationships) or Venn diagrams of quadrilaterals to show how the attributes of one shape imply or are related to the attributes of others.



3	<p>Formal deduction</p> <p>Construct proofs rather than just memorize them; see the possibility of developing a proof in more than one way.</p>	<p>Given three properties about a quadrilateral, a student can logically deduce which statement implies what about a quadrilateral.</p>	<p>Provide situations in which students could use a variety of different angles depending on what is given (e.g. alternate interior or corresponding angles being congruent, or same-side interior angles being supplementary).</p>
4	<p>Rigor</p> <p>Learn that geometry needs to be understood in the abstract; see the “construction” of geometric systems.</p>	<p>Students should understand that other geometries exist and that what is important is the structure of axioms, postulates and theorems.</p>	<p>Study on non-euclidean geometries.</p>

Source: Breyfogle, M. L. & Courtney, M. L. (2010), p. 234

## Developing Students' Mathematical Thinking: How Far Have We Come?

In comparing what is covered at Year 6 level (Figure 2) and the van Hiele's levels shown in Table 1, the expected outcomes for Year 6 is only on finding the area, perimeter, surface area and volumes of cubes and cuboids. In Forms 1 and 2, students progress to other three dimensional shapes, including cones, pyramids, prisms, and spheres. However, the focus is not on the development of conceptual knowledge where students should be operating at least at van Hiele Level 2, which requires them to see the interrelationships between figures. Hence, learning tends to be compartmentalized, dealing with one object at a time and not seeing the interconnections, in terms of similarities and differences, to identify the important attributes that signify a shape.

### Topic 10: SHAPE AND SPACE

#### Learning Area: TWO-DIMENSIONAL SHAPES


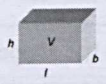
## Year 6

LEARNING OBJECTIVES <i>Pupils will be taught to</i>	SUGGESTED TEACHING AND LEARNING ACTIVITIES	LEARNING OUTCOMES <i>Pupils will be able to</i>	POINTS TO NOTE	VOCABULARY
	<ul style="list-style-type: none"> <li>Pupils construct two-dimensional composite shapes on the geo-board or graph paper. Pupils then find the area of the shapes.</li> <li>Teacher provides a two-dimensional composite shape with given dimensions. Pupils calculate the area of the shape.</li> <li>Pose problems of finding perimeters and areas of 2-D shapes in numerical form, simple sentences, tables or pictures.</li> <li>Teacher guides pupils to solve problems following Polya's four-step model of               <ol style="list-style-type: none"> <li>Understanding the problem</li> <li>Devising a plan</li> <li>Implementing the plan</li> <li>Looking back.</li> </ol> </li> </ul>	<p>(ii) Find the area of a two-dimensional composite shape of two or more quadrilaterals and triangles.</p> <p>Area <math>A</math>, of a square with sides <math>a</math> in length. <math>A = a \times a</math></p> <p>Area <math>A</math>, of a rectangle with length <math>l</math> and breadth <math>b</math>. <math>A = l \times b</math></p> <p>Area <math>A</math>, of a triangle with base length <math>b</math> and height <math>h</math>. <math>A = \frac{1}{2}(b \times h)</math></p> <p>(iii) Solve problems in real contexts involving calculation of perimeter and area of two-dimensional shapes.</p>	<p>To calculate area of 2-D shapes, use the following formulae...</p> <p>Area <math>A</math>, of a square with sides <math>a</math> in length. <math>A = a \times a</math></p> <p>Area <math>A</math>, of a rectangle with length <math>l</math> and breadth <math>b</math>. <math>A = l \times b</math></p> <p>Area <math>A</math>, of a triangle with base length <math>b</math> and height <math>h</math>. <math>A = \frac{1}{2}(b \times h)</math></p>	quadrilateral triangle grid geo-board

**Topic 10: SHAPE AND SPACE**

**Learning Area: THREE-DIMENSIONAL SHAPES**

**Year 6**

LEARNING OBJECTIVES <i>Pupils will be taught to</i>	SUGGESTED TEACHING AND LEARNING ACTIVITIES	LEARNING OUTCOMES <i>Pupils will be able to</i>	POINTS TO NOTE	VOCABULARY
<p><b>1</b> Find the surface area and volume of composite three-dimensional shapes.</p>	<ul style="list-style-type: none"> <li>Pupils draw net according to the given measurements, cut out the shape and fold to make a three-dimensional shape. Next, unfold the shape and use the graph paper to find the area. Verify that the area is the surface area of the 3-D shape.</li> <li>Teacher provides a three-dimensional composite shape with given dimensions. Pupils calculate the surface area of the shape.</li> <li>Pupils construct three-dimensional composite shapes using the Dienes's blocks. The volume in units of the block is determined by mere counting the number of blocks.</li> <li>Teacher provides a three-dimensional composite shape with given dimensions. Pupils calculate the volume of the shape.</li> </ul>	<p>(i) Find the surface area of a three-dimensional composite shape of two or more cubes and cuboids.</p> <p>(ii) Find volume of a three-dimensional composite shape of two or more cubes and cuboids.</p>	<p>Use only cubes and cuboids to form composite 3-D shapes. Examples of these shapes are as below...</p>  <p>For a cuboid with length <math>l</math>, breadth <math>b</math> and height <math>h</math>, the volume <math>V</math> of the cuboid is...</p> $V = l \times b \times h$ 	<p>cube cuboid three-dimensional volume length breadth height</p>

**Figure 2:** Curriculum Specifications for Shapes and Spaces, Year 6 Primary School

Source: Curriculum Specifications: Integrated Curriculum for Primary Schools, Year 6. (Ministry of Education, Malaysia 2006b). p. 21 – 22

In the study by Tan et al. (2015), respondents were asked to rank statements as “Understand very well” (+4) to “Least Understand” (-4). The 34 statements provided consisted of van Hiele’s Level 0 (visualization), 1 (analysis), and 2 (informal deduction). The Q-methodology showed the results of similarities, differences, correlation, factor analysis and examination of factor analysis scores by which the statements were sorted as factor 1 and factor 2. For statements in factor 1, the respondents showed very good understanding of statements of levels 0 and 1 (Table below) where 60% of the respondents achieved level 1 (analysis) which progressed from the basic level 0 (visualization). These groups of respondents could not however achieve level 2, as they declared ‘least understanding’ for the statements which were ranked -3 and -4.

**Table 2** Statements of Factor 1 by van Hiele's Levels and Given Ranks

Statement	van Hiele's Level	Rank
If four sides of a quadrilateral are equal in length, the figure is a parallelogram.	1	+4
A rectangle is a quadrilateral which has two longer sides and two shorter sides.	1	+4
There are many ways to cut a square into two exact halves.	1	+3
Rectangle has the shape of a long box.	0	+3
A encyclopedia looks like a cuboid.	0	+3
An equilateral triangle has an angle that is larger than 60 degrees.	2	-3
If a quadrilateral has four equal sides and three of its corners are 90 degrees, therefore it must be a rectangle.	2	-3
A rectangular container without its cover is still known as a cuboid.	0	-3
A square is not a parallelogram because parallelograms are slanted.	2	-4
If a corner of an isosceles triangle is 60 degrees, then three sides of this triangle cannot be unequal.	2	-4

Forty percent of the respondents only achieved level 0 (visualization) and the ranks given are as shown in Table 3. These groups of respondents chose 'least understand' in other levels and 'understanding very well' for statements which were 'incorrect'. Therefore they only achieved level 0.

**Table 3** Statements of Factor 2 by van Hiele's Levels and Given Ranks

Statement	van Hiele Level	Rank
Roof of a house is normally in the shape of a triangle.	0	+4
An encyclopedia book looks like a cuboid.	0	+4
The shape of a square looks like a perfect box.	0	+3
There are many ways to cut a square into two exact halves	1	+3
A rectangle is a type of square.	2	+3
If a quadrilateral has four equal sides, and three of its corners are 90 degrees each. Therefore, it must be a rectangle.	2	-3
A square is a subset of a cube.	2	-3
If a corner of an isosceles triangle is 60 degrees, then three sides of this triangle cannot be unequal.	2	-3
The sum of internal angles of a square is 360 degrees.	1	-4
Most rectangles are drawn horizontally or vertically and its length is two or three times the width.	2	-4

This result supports the van Hiele Theory (Van Hiele, 1986; Mason, 1998; Usiskin, 1982) as shown in the determination of the groups using Q-methodology. Students at Level 1 progress from visualization level to analytical level and it relates closely to Level 0 to proceed to the second level. The students may have perceived that

they have the ability to reason about a geometrical shape in terms of its properties, but they do not yet understand the relationships between the properties and between different figures.

The results also showed that most of the students seemed to be operating at the lower levels (L0 and L1) in the learning of Shapes and Spaces. This finding coupled with those found by Noraini (2007) suggest that a substantially large proportion of Malaysian primary school children are operating at the lower levels of van Hiele's levels of geometric thinking, a common phenomenon faced by other nations, including United States, China, Taiwan and United Kingdom. In Malaysia, several factors have been identified to explain why learning geometry is difficult, namely geometry language, visualization abilities and ineffective instructions (Noraini, 2007).

This various facets of learning difficulties in geometry have been made evident in numerous research findings. In United States, it was revealed that geometry poses learning difficulties consistently at all grades (Schäfer, 2003; Schwartz, 2008). Other studies conducted on the development of geometry proof competency, including in Taiwan and China, have provided empirical evidences showing that a large number of students had great difficulties in generating proofs, even for simple geometry problems (Wu & Ma, 2005; Ding & Jones, 2006).

This phenomenon gives rise to several interesting points. As pointed out by Tan et al. (2015), first, the deficiency of van Hiele's levels of geometric thinking appears to be global in nature, crossing boundaries of educational practices and curriculum. This deficiency might have been the major contributing factor in the learning difficulties encountered by learners around the globe. Secondly, the deficiency may hold the key that explains why Malaysian lower secondary school students showed lower performance in TIMSS and PISA. It should be noted that the van Hiele's levels

of geometric thinking includes higher order thinking and decision making skills, and acquisition of mathematical concepts to enable learners to operate at the higher levels in van Hiele's theory. Thirdly, this finding may raise concerns that many of the students do not seem to have acquired the targeted learning outcomes emphasised for Year 6 KBSR Mathematics, where they are expected to at least reach the level of L2 (Informal Deduction) at the end of the learning session. If this concern is true, then we can expect that this group of primary students will encounter more serious learning difficulties in geometry at secondary school level as the learning outcomes of KBSM Mathematics place emphasis on outcomes that reach higher levels (L3 and L4).

## ROLE OF METACOGNITION IN DEVELOPING THINKING

Metacognition is simply defined as "thinking about thinking." According to Livingston (2003), "metacognition refers to higher order thinking which involves active control over the cognitive processes engaged in learning" (p. 2). Sternberg (1984, 1986) associated metacognition with intelligence and posited that it enables people to be successful learners. Earlier works on metacognition can be traced back to Flavell (1976). According to Flavell (1976),

*Metacognition refers to one's knowledge concerning one's own cognitive processes or anything related to them, e.g. the learning of relevant properties of information or data.... Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of those processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete (problem solving) goal or objective (p. 232).*

The essence of the above definition is the notion of thinking about one's own thoughts. It is the ability to understand and monitor one's own thoughts, the assumptions and the implications of one's activity. According to Flavell (1976), if we have awareness of and control over our cognitive processes, then we are being metacognitive.

Aida Suraya and Wan Zah (2008) conducted a study which, among others, was to ascertain the relationship between the levels of metacognition and mathematics achievement and overall academic achievement. Data was collected from 195 final year students majoring in mathematics education from four Malaysian universities.

Metacognition is viewed by Schraw and Dennison (1994) as the ability of individuals to reflect, understand and control their own learning. Thus, they identified eight dimensions of metacognitive processes and these were incorporated in the Metacognitive Awareness Inventory (MAI). Sanchez-Alonso and Vovides (2006) verified that MAI, which comprises of 52 items, is a reliable measure of cognition and regulation of cognition. MAI measures eight specific dimensions of metacognition, namely, comprehension monitoring, procedural knowledge, declarative knowledge, conditional knowledge, evaluation, debugging strategies, information management strategies and planning. In the original inventory, respondents were asked to respond to the items as true or false. However, in the inventory that Aida Suraya and Wan Zah (2006) conducted, the scale used was a continuum from (1) Never to (5) Very Often. The survey was translated using the back to back translation technique. The reliability for the translated version of MAI was found to be very high ( $r = .930$ ). The definitions of each dimension of metacognition, as suggested by Schraw and Dennison (1994), are as follow:



- i. Comprehension Monitoring
  - Assessment of one's learning or strategy use.
- ii. Procedural Knowledge
  - The application of knowledge for the purpose of completing a procedure or process.
  - Knowledge about how to implement learning procedures (e.g. strategies).
  - Requires students to know the process as well as when to apply the process in various situations.
  - Students can obtain knowledge through discovery, cooperative learning and problem solving.
- iii. Declarative Knowledge
  - The factual knowledge that the learner needs before being able to process or use critical thinking related to the topic.
  - Knowing about, what or that.
  - Knowledge of one's skills, intellectual resources and abilities as a learner.
  - Students can obtain knowledge through presentations, demonstration, discussions.
- iv. Conditional Knowledge
  - The determination of under what circumstances specific processes or skills should be transferred.
  - Knowledge about when and why to use learning procedures.
  - Application of declarative and procedural knowledge with certain conditions presented.
  - Students can obtain knowledge through simulation.
- v. Evaluation
  - Analysis of performance and strategy effectiveness after a learning episode.

- vi. Debugging Strategies
  - Strategies used to correct comprehension and performance errors.
- vii. Information of Management Strategies
  - Skills and strategy sequences used to process information more efficiently (e.g. organizing, elaborating, summarizing, selective focusing).
- viii. Planning
  - Planning, goal setting and allocation of resources prior to learning.

Overall, the level of metacognition of the respondents was found to be moderate. The results indicated that the majority of the respondents' metacognitive awareness was high for three dimensions, debugging strategies (73.8%), information management strategies (54.3%) and conditional knowledge (63.4%), and moderate for the rest of the dimensions. Female respondents showed a higher level of metacognition as compared to the males and the females also showed consistently higher scores for all eight components.

Students' average grades in mathematics courses taken at the university level were used to indicate their mathematics achievements and their current CGPAs were used as a reflection of the respondents' overall academic achievements. The findings implied that there is a stronger correlation between mathematics achievement with levels of metacognition as opposed to overall academic achievement. This indicates that those with higher levels of metacognition do better in their mathematics courses. The study also showed that students with higher CGPA reported better use of metacognitive strategies. Three dimensions of MAI, procedural, declarative and conditional knowledge, were found to be significantly correlated with students' performance in university

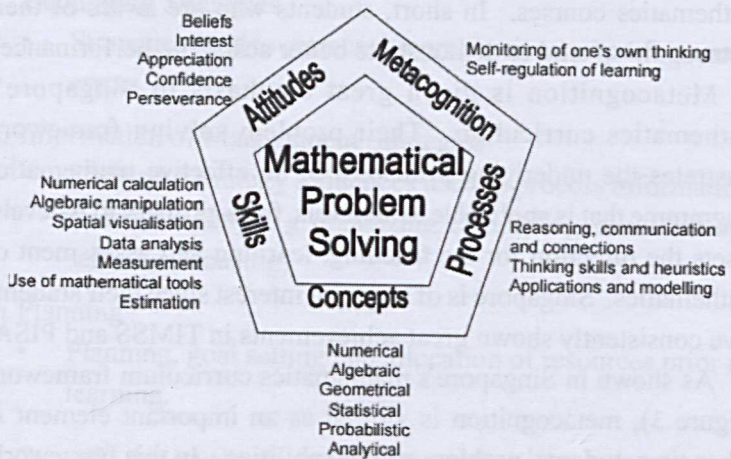
mathematics courses. In short, students who are aware of their metacognition tend to demonstrate better academic performance.

Metacognition is given great emphasis in Singapore's mathematics curriculum. Their problem solving framework illustrates the underlying principles of an effective mathematics programme that is applicable to all levels, from primary to A-levels. It sets the direction for the teaching, learning and assessment of mathematics. Singapore is of a special interest since their students have consistently shown great achievements in TIMSS and PISA.

As shown in Singapore's mathematics curriculum framework (Figure 3), metacognition is viewed as an important element in enhancing students' problem solving abilities. In this framework, problem solving is regarded as central to mathematics. Apart from metacognition, the framework illustrates the need to focus on mathematical reasoning, which is defined as the ability to analyze mathematical situations and construct logical arguments. Students are also exposed to the use of various thinking skills and heuristics to help them solve mathematical problems. Thinking skills are skills that can be used in a thinking process, such as classifying, comparing, sequencing, analysing parts and wholes, identifying patterns and relationships, induction, deduction and spatial visualization (Ministry of Education Singapore, 2006). Due to the high performance of Singaporean students in TIMSS, their framework and their textbooks have been adopted by many countries, including United States, Canada, Israel and United Kingdom (Prystay, 2004; Wong & Lee, 2009).

In Singapore, the primary focus of the mathematics curriculum is on mathematical problem solving. The mathematical problem solving framework incorporates processes which include reasoning, communication, connection, thinking skills, heuristics, application and modelling (Singapore MOE, 2006).

## Developing Students' Mathematical Thinking: How Far Have We Come?



**Figure 3** Singapore's Mathematics Framework. Ministry of Education, Singapore (2006).

Similar to Singapore, problem solving has always been a core in the Malaysian mathematics curriculum, since the introduction of the Integrated Curriculum for Secondary Schools in 1989. In fact, at the Form 4 level, a chapter was dedicated to guide students on problem solving, which includes the heuristics and the strategies. However, the topic was removed from the curriculum during a revision of the mathematics curriculum in year 2000 because the feedback received by the Curriculum Development Centre was that teachers were not teaching the topic because it was not directly tested in examinations (Aida Suraya, 2008). This is evidence of the misalignment between the intended, the taught and the examined curriculum.

## ROLE OF TECHNOLOGY IN FACILITATING MATHEMATICS THINKING

The use of calculators at an early age may hinder the ability to enhance mental computational skills, acquisition of number facts, development of number sense and mathematical thinking. Students used to be able to provide the values for equations such as the following, spontaneously, without depending on any gadgets:

$$\begin{aligned} \sin(60^\circ) &= \frac{1}{2}, & \sin(30^\circ) &= \frac{1}{2} \\ \sin(60^\circ) &= \frac{\sqrt{3}}{2}, & \cos(30^\circ) &= \frac{\sqrt{3}}{2} \\ \tan(60^\circ) &= \sqrt{3}, & \tan(45^\circ) &= 1 \end{aligned}$$

However the calculator-dependent students of today find the answers using calculators or computers, which I have often observed among my student teachers. Allowing the use of calculators or other technologies can be allowed but it is very much dependent on the tasks that have to be carried out. These tools can eliminate tedious computations, thus allowing students to focus more on the strategy, making and testing conjectures, observing patterns that emerge, making generalizations and engaging in more challenging tasks. As an example, if students are to conduct an activity to discover the value of pi and to suggest the formula to calculate the circumference of a circle, they may have to complete tasks such as the following:

### Task 1

Circular objects	Circumference (C)	Diameter (D)	$\frac{C}{D}$
Object 1			
Object 2			
Object 3			
Object 4			

**Task 2**

Determine the sign for sine, cosine and tangent in all four quadrants. What can be generalized?

**Task 3**

Determine relationship between  $a$ ,  $\sin A$ ,  $b$ ,  $\sin B$ ,  $c$ ,  $\sin C$  for a few triangles.

In such task the students should be relieved of such tedious computations and be allowed instead to focus on the emerging patterns. This is because the outcome sought is for students to make and test conjectures and observe patterns. The use of calculators in such instances will help reduce the mental efforts involved in completing the task.

In a study by Kamariah, Rohani, Ahmad Fauzi and Aida Suraya (2009) on the effect of utilizing Geometer's Sketchpad (GSP) on the performance and mathematical thinking of secondary learners, the findings indicated that the use of GSP induced higher mathematical thinking process and performance among the learners. Software such as GSP, Autograph and Mathlab are classified as mind tools to facilitate learning. Unlike a calculator which provides answers, a mind tool allows students to explore and learn. GSP allows students to manipulate dynamic models of fractions, number lines and geometric patterns, particularly primary school students. At higher levels, GSP can build students' readiness for algebra by exploring ratio and proportion, rate of change and functional relationships through numeric, tabular and graphical representations, construct and transform geometric shapes and functions, thus promoting deeper understanding.

The results of the independent t-test showed that there was no significant difference in post test scores between the control and GSP groups. Neither were there differences in the conceptual skills

and procedural skills of the individuals. It can be concluded that the 6-hour session may not have provided enough exposure and time for the students to get familiarized with the use of the technology, which may be why it showed less promising results on the use of technology.

In another study on the integration of the dynamic mathematical software, Autograph, in the learning of Quadratic Functions, the findings also showed that the tool did not help improve mathematical performance nor did it reduce the extraneous cognitive load as compared to the conventional strategy (Rohani, Aida Suraya, Ahmad Fauzi & Kamariah, 2009). Cognitive load refers to the total amount of mental effort being used in the working memory. This finding is in contrast to Burrill et al.'s (2002) extensive review of the use of the handheld graphing calculator, which provided evidences that secondary school students showed improvement in their conceptual understanding. They stressed further that it is not just due to the presence of the graphing calculators but more due to how it was used in the teaching of mathematics. The "how" in the use of the technology must be explored because the positive impact can only be seen when the tool is used efficiently.

While there are inconclusive findings on the impact of the utilization of technology in promoting learning and mathematical thinking, Rohani, Ahmad Fauzi, Kamariah and Aida Suraya (2008) emphasized on the need to address issues of instructional efficiency.

*In utilizing any technological tools, comprehensive measures addressing issues of instructional efficiency is crucial, especially when involving large scale and formal implementation of technology integration in teaching and learning ... dynamic software, particularly, graphing calculators provide positive impact upon learners thus becoming potential tools in teaching mathematics ..."*  
(p. 191).

It is not just the tool that is selected for use, but on how it is used, what it is used for and when it is used during instruction. A lot more work needs to be done to propose how best mind tools can be integrated in classroom instruction.

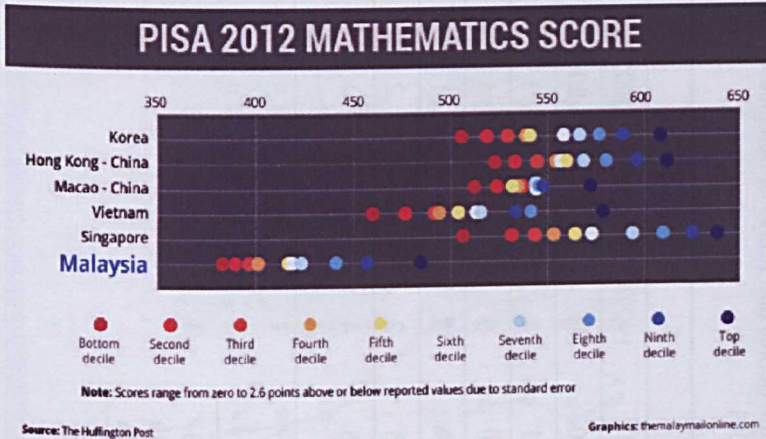
## CURRENT STATUS OF MALAYSIAN SCHOOL MATHEMATICS

Apart from PT3, other indicators of Malaysian students' performance are evident in international studies, namely in the *Trends in International Mathematics and Science Study* (TIMSS) and *Programme for International Student Assessment* (PISA). TIMSS is conducted by the International Associations for the Evaluation of Educational Achievement (IEA), every four years. TIMSS is a massive study and in 2011, the study managed to get 57 countries involved in the administration of tests for Grade 4 students and 56 countries for Grade 8 students. One hundred and eighty Malaysian schools participated in TIMSS 2011 with a total of 5,733 students. PISA, on the other hand, is administered by the Organisation for Economic Co-operation and Development (OECD) every three years on 15-year-olds in both OECD and non-OECD countries, on mathematics, science and reading skills, as well as critical problem-solving as opposed to memorization. As reported by Kang (2013), in PISA 2012, Malaysia scored 421 in Mathematics, 398 in Reading and 420 in Science, respectively, all of which are below average scores, with a ranking at 52 out of 65 countries. The global average score was 494 in Mathematics, 496 in Reading and 501 in Science. Shanghai, Singapore, Hong Kong, Taiwan and Korea were the top five countries, with even Vietnam being ranked higher at 17<sup>th</sup>.

The Huffington Post of the United States reported that even the best students in Malaysia performed far below the average score in PISA 2012, as shown in Figure 4. It was stated that the average



scores for the top 10 percent (decile) of Malaysia's elite 15-year-olds were far lower than the average scores of students in a number of its Asian neighbours, including Singapore and Vietnam.



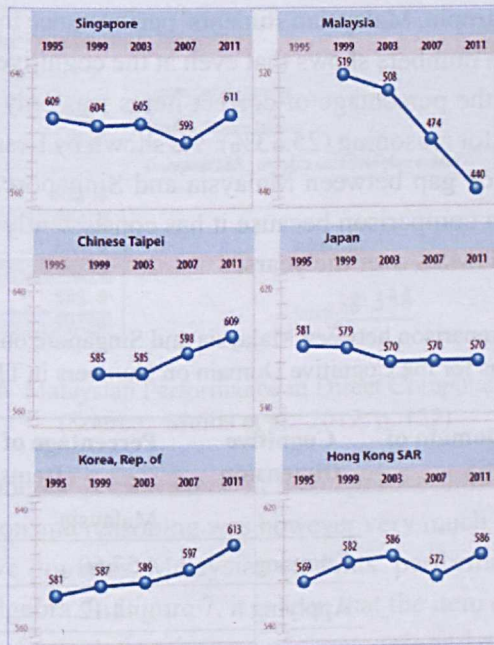
**Figure 4** Malaysian Students' Performance in PISA 2012  
*Source: Malay Mail Online, Jan 25, 2014.*

As shown in Table 4 and Figure 5, Malaysian students' performances have declined tremendously over the years and it was ranked at 26<sup>th</sup> place out of the 42 countries participating in TIMSS 2011. Many have contributed the results to the change in the language of instruction from Bahasa Melayu to English.

**Table 4** TIMSS Results for 8<sup>th</sup> Grade Students Based on Scores and Ranks of Selected Countries in 1999 to 2011

Country	Score					Rank				
	1999	2003	2007	2011	2011	1999	2003	2007	2011	2011
Malaysia	519	508	474	440	440	16	10	20	26	
Singapore	604	605	593	611	611	1	1	3	2	
Japan	579	570	570	570	570	5	5	5	5	
Chinese Taipei	585	585	598	609	609	3	4	1	3	
South Korea	587	589	597	613	613	2	2	2	1	
Hong Kong SAR	582	586	572	586	586	4	3	4	4	
International Average	487	466	500	467	467					
Highest Score	604	605	598	613	613					
	Singapore	Singapore	Chinese Taipei	South Korea						
Number of countries participating	38	45	48	42	42					

Source: Mullis et al. (2004), Mullis et al. (2008) and Mullis et al. (2012))



**Figure 5** Trends in Average Mathematics Achievement for 8<sup>th</sup> Grade students in TIMSS 1995 to 2011.

Source: Mullis et al. (2012), pp. 60-64

The TIMSS mathematics assessment which is based on a comprehensive framework developed collaboratively with the participating countries, is organized around two dimensions:

- i. Content dimension specifying the domains or subject matter to be assessed within mathematics, including Number (30%), Algebra (30%), Geometry (20%) and Data & Chance (20%);
- ii. Cognitive dimension specifying the domains or thinking processes expected of students as they engage with the mathematics content, including Knowing (35%), Applying (40%) and Reasoning (25%).

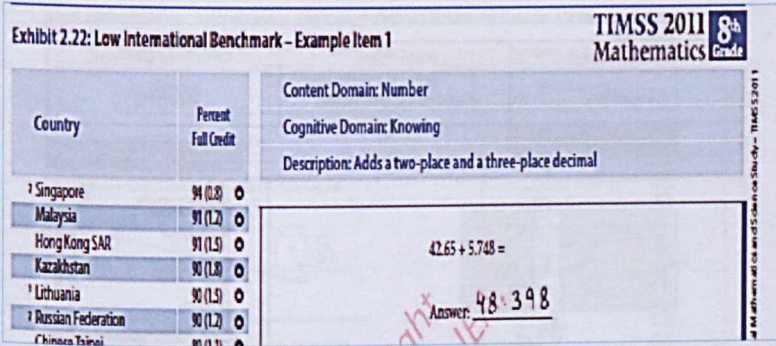
Developing Students' Mathematical Thinking: How Far Have We Come?

As an example, Malaysian students' performance in the content dimension on numbers shows that even at the cognitive dimension of knowing, the percentage of correct items was only 57.6% and the least was for reasoning (25.33%). As shown by Lessani (2015), there is a wide gap between Malaysia and Singapore. Singapore was used as a comparison because it has consistently been in the top three in TIMSS over the years.

**Table 5** Comparison between Malaysia and Singapore on Percent of Correct Items for the Cognitive Domain on Numbers in TIMSS 2011

Cognitive Domain of TIMSS	Cognitive Dimension	Percentage of Correct Items	
		Malaysia	Singapore
Number	Knowing	57.60	86.90
	Applying	44.12	77.62
	Reasoning	25.33	60.00
	Total	47.85	79.52

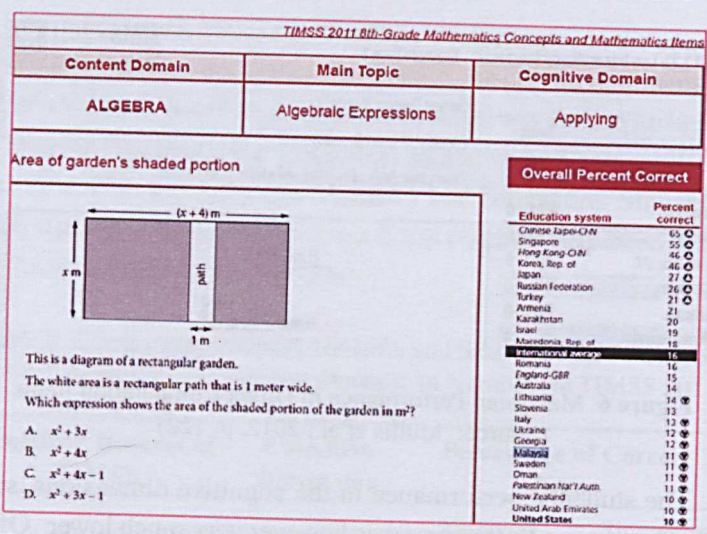
Malaysian students are only good at solving direct computational items, such as,  $42.65 + 5.748$ . Malaysian students managed to get 91% correct for this item, although they still lost out to Singapore who had the highest ranking for this item, as shown in Figure 6. This implies that Malaysian students are given adequate exposure to solving direct computations such as this.



**Figure 6** Malaysian Performance in Direct Computation Items  
(Source: Mullis et al., 2012, p. 122)

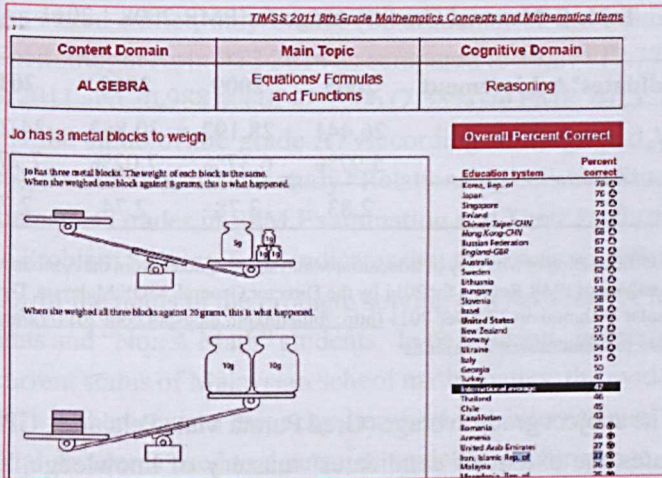
The students' performance in the cognitive dimensions, such as application and reasoning was however very much lower. Of all the cognitive domains, Malaysian students' performance was the worst for Algebra. In Figure 7, it is seen that the item only requires knowledge of calculating the area of a rectangle and multiplication of simple algebraic expressions. However, the Malaysian students' performance was below average for this item.

Developing Students' Mathematical Thinking: How Far Have We Come?



**Figure 7** Malaysian students' Performance in Applying Algebraic Concepts (Source: Mullis et al., 2012, p. 65)

Another example, shown in Figure 8, shows an item that requires reasoning in algebra. Only 36% of the Malaysian participants managed to get the correct answer. This shows that students cannot translate the diagram into inequalities such as:  $x < 8$  and  $3x > 20$ . If the item is rephrased as "Given  $x < 8$  and  $3x > 20$ , find possible values for  $x$ ", the students may have been able to perform much better.



**Figure 8** Malaysian participants' Performance in Providing Reasoning in Algebra (Source: Mullis et al., 2012, p. 30)

While the TIMSS and PISA results show a declining trend in students' achievements, the national benchmark used seems to indicate otherwise. The national grade average (Gred Purata Nasional, GPN) for PMR examinations have been showing steady increase in students' performance over the years. As shown in the following table, the performance of students in PMR 2011 in the 17 subjects tested showed the best results in four years with GPN of 2.71.

Developing Students' Mathematical Thinking: How Far Have We Come?

**Table 7** National Grade Average for PMR 2008 – 2011

Candidates' Achievement	2008	2009	2010	2011
All As	26,441 5.97%	28,192 6.37%	30,863 7.02%	34,271 7.77%
GPN	2.83	2.78	2.74	2.71

*Source:* Official Blog of Ministry of Education Malaysia. Extracted from the Announcement of the Analysis of PMR Results for 2011 by the Director General, MOE Malaysia, Dato' Sri Abd Ghafar Mahmud on 22<sup>nd</sup> Dec 2011 (<http://buletinkpm.blogspot.com/2011/12/analisis-keputusan-penilaian-menengah.html>)

The subject grade average (Gred Purata Mata Pelajaran, GPMP) indicates the extent of candidates' mastery of knowledge, skills and values of the subject. The table below shows the GPMP for mathematics in 2010 and 2011. There is shown to be an increase in the percentage of candidates getting As, of 1.3%. The figures show that the GPMP for mathematics showed better performance among candidates, with a decrease of 0.04 points. The GPMP for the 2013 PMR results was also reported to be better than the previous year.

**Table 8** Performance Analysis and GPMP for mathematics  
2010 – 2011

Year	Grade					No of Candidates	GPMP
	A	B	C	D	E		
2010	28.9	12.5	15.5	34.7	8.3	438,284	2.81
2011	30.2	12.5	15.4	34.3	7.6	439,938	2.77

*Source:* Official Blog of Ministry of Education Malaysia. Extracted from the Announcement of the Analysis of PMR Results for 2011, by the Director General, MOE Malaysia, Dato' Sri Abd Ghafar Mahmud on 22<sup>nd</sup> Dec 2011 (<http://buletinkpm.blogspot.com/2011/12/analisis-keputusan-penilaian-menengah.html>)



As stated earlier, only 0.02% (80 students) of the candidates obtained straight As in PT3 2014 as compared to 34,271 (7.77%) in PMR 2011 and 30,988 out of 42,2506 (7.33%) in PMR 2013. What then is the value of the grade A? According to Singh and White (2006), the outcome of their study “Relationship between Students’ Mathematics Grades in SPM Examination and Their Performance in the Problem Solving Test” indicates that there was no difference in 50% of the items in the problem solving test between “A Math” students and “Non A Math” students. In conclusion, in relation to the current status of Malaysian school mathematics, the evidences show that there exists disparity between the international and national standards used to determine students’ performance.

## **HOW SUCCESSFUL HAVE WE BEEN IN FACILITATING THE DEVELOPMENT OF MATHEMATICAL THINKING?**

This article has highlighted that generally, Malaysian students are not meeting the international benchmark for mathematics performance. This could be partially attributed to the students’ inability to think mathematically, and thus not being able to translate the contexts given and to use mathematics to provide solutions. Many have raised issues on the validity and reliability of the instruments used in TIMSS, PISA and even PT3. However, the fact is that these international standardized tests are being used in many countries to determine their targeted standards, as reflected in their policies and education reforms. Apart from the findings of these international comparative studies, the studies discussed earlier in this paper have also highlighted the lack of thinking skills amongst Malaysian school students and even university students. Thus, the factors that may contribute to this phenomenon need to

be examined from the perspectives of the curriculum, instruction and the assessment methodology.

## THE MATHEMATICS CURRICULUM

Since it was introduced in 1989, the Integrated Curriculum for Secondary School (KBSM) Mathematics has not changed much. The emphases on problem solving, communication, reasoning, making connections and application of technology are still relevant today. However, the expected outcomes, as stated in the curriculum guides, may not be able to support the initiative to focus on higher order thinking skills (HOTS). As shown below, typical learning outcomes for form 5 topic, on gradient and area under a graph, only expect the students to 'find, determine and solve'. This is not in line with HOTS initiatives that require cognitive operations of analyzing, evaluating and synthesizing or creating.

- i. Find the area under a graph.
- ii. Determine the distance by finding the area under the following types of speed-time graphs:
  - a.  $v = k$  (uniform speed),
  - b.  $v = kt$ ,
  - c.  $v = kt + h$ ,
  - d. a combination of the above.
- iii. Solve problems involving gradient and area under a graph.

The only learning outcomes that may require some investigation at the form 5 level is on determining whether the combined transformation AB is equivalent to the combined transformation BA. Yet, this can be easily verified and does not appear to be very challenging.

At the form 2 level, there are a few learning outcomes that require students to derive the formula for circumference, area of circle and area of a sector. It is noted in the curriculum guide that students should explore the relationship between the area of a sector and the angle at the centre of the circle using dynamic geometry software. However, how is this actually implemented in the classroom? Are students given the opportunity to experiment, explore, and test their conjectures?

## THE MATHEMATICS CLASSROOM INSTRUCTION

The curriculum guide (MOE Malaysia, 2006a) explicitly states that approaches such as contextual learning and enquiry-discovery should be practiced in mathematics instruction. These approaches are deeply rooted in the constructivist principles. The constructivist-based approaches are learner-centered approaches that emphasize on the importance of providing opportunities for learners to actively construct their knowledge with guidance from the teacher. In the constructivist view, teachers should not attempt to simply pour information into children's minds. Rather, children ought to be given confidence to discover their world, find out knowledge, consider and think critically, with the vigilant supervision and significant guidance of the teacher (Eby, Herrel & Jordan, 2005).

Hanley (1994) provided more guidelines on implementing a constructivist classroom, among which are as follows:

1. Seek out and use students' questions and ideas to guide lessons and whole instructional units;
2. Accept and encourage student initiation of ideas;
3. Promote student leadership, collaboration, location of information and their taking actions as a result of the learning process;

4. Use students' thinking, experiences and interests to drive lessons;
5. Encourage the use of alternative sources of information, both from written materials and experts;
6. Encourage students to suggest causes for events and situations and encourage them to predict consequences;
7. Seek out students' ideas before presenting teachers' ideas or before studying ideas from the textbooks or other sources;
8. Encourage students to challenge each other's conceptualizations and ideas;
9. Encourage adequate time for reflection and analysis; respect and use all ideas that students generate;
10. Encourage self-analysis, collection of real evidence to support ideas and reformulation of ideas in light of new knowledge;
11. Use students' identification of problems with local interests and play the role as the organizers for the course;
12. Use local resources (human and material) as original sources of information that can be used in problem resolution;
13. Involve students in seeking information that can be applied in solving real-life problems; and
14. Extend learning beyond the class period, classroom and the school.

Teachers have been encouraged to use constructivist teaching methods since the early years of the KBSM implementation (MOE Malaysia, 1989). However, research by Klieme and Vieluf (2009) provided evidences that teachers in Malaysia, South America and southern Europe showed lesser preference for a constructivist view as compared to teachers in Australia, Korea, northwestern Europe and Scandinavia. Thus after more than 20 years since it was first

suggested in curriculum guides, constructivist teaching has not been a preference of Malaysian teachers. Nevertheless, with the shift to school based assessment, the initiative on HOTS and the change in examination format to include more challenging questions such as PT3, this may well elicit a classroom atmosphere that cultivates HOTS, as well as critical thinking and creative thinking.

To provide support for learning, school textbooks need to be greatly improved. The existing contents, contexts and examples do not stimulate students' thinking. For example, in a form 1 topic on percentage, the learning outcomes only require students to be able to do basic tasks such as the following:

1. Express percentages as the number of parts in every 100.
2. Change fractions and decimals to percentages and vice-versa.
3. Find the percentage of a quantity.
4. Find the percentage one number is of another.
5. Find a number given the percentage.
6. Find the percentage of increase or decrease. Solve problems involving percentages.

In contrast, the Singapore textbook meant for secondary 2 students provides contexts and tasks that challenge students and expose them to issues such as water rates, money exchange, income tax, goods and service tax (GST) and the Central Provident Fund, all in one chapter.

# Developing Students' Mathematical Thinking: How Far Have We Come?

## 1 Ratio, Proportion, Percentages, Simple Financial Transactions and Standard Form

Scale drawings, models, public utility rates and fares are some important applications of rate, proportion and percentages to our daily lives. Maps and plans of buildings are examples of scale drawings. Developers of housing projects make models for interested buyers to see what the finished project would be like.

In Singapore, we have a Goods and Services Tax (GST) of 8% (1997). In the United Kingdom, they have a Value Added Tax (VAT), similar to our GST, of 16% (1997).

Not all taxes are collected for the purpose of revenue. Some are introduced as a control over production or strategic resources such as water. Thus, pay an increasingly higher water table, so that people who use more water have to pay an increasingly higher percentage of the water consumption tax. In ancient times, a salt tax was common in many countries as salt was considered a scarce commodity.

In recent articles in *The Straits Times*, the cost of electricity has gone up at 0.046 to 0.123 (26% of new S\$1). There is 80¢ more for the same use of 100 kWh per month (100.00 per unit) compared with 100.00 per unit. Electricity prices by SPP. Procter when the cost might be the same.

Just the houses to get the lowest collection. You may think the publisher's policy.

### The water rates

**Domestic**

Consumption (per unit)	1997	1998	1999	2000
1st 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00

**Commercial**

Consumption (per unit)	1997	1998	1999	2000
1st 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00
Next 100 units	100.00	100.00	100.00	100.00

Let the Internet to get the conversion exchange rates for the given countries.

One place to look for such information would be the homepage of a bank such as the IMF, then and visit its "home-exchange-rate" site.

Also try the online homepage and check "currency converter".

**NET**

The bank will buy from customers at a lower rate and sell to customers at a higher rate to make a profit.

When I want to exchange Singapore dollars for Malaysian ringgit, the bank will sell me Malaysian ringgit for Singapore dollars, the bank will buy Malaysian ringgit from the tourist.

For example, the bank will buy RM100 from a tourist and pay the tourist S\$46.50. However the bank will sell RM100 for S\$47.50. Therefore, the bank makes a profit of S\$1.00 (S\$47.50 - S\$46.50 = S\$1.00).

Money changers normally quote one unit or 100 units of local currency to foreign currencies. The following table illustrates the exchange rate quoted by a particular money changer on a certain day.

## 1.4 Money Exchange

The value of the Singapore dollar (S\$) in terms of other currencies varies from day to day. These exchange rates are often published in newspapers, and displayed in banks and on the Internet. The tables below show the exchange rates of a bank on a certain day.

### Singapore dollars to one unit of foreign currency

	Buying	Selling
Sterling pound (£)	2.552 0	2.578 0
Australian dollar (AUD)	0.875 0	0.887 0
Canadian dollar (Cdn\$)	1.129 0	1.153 0
New Zealand dollar (NZ\$)	0.719 0	0.734 0
US dollar (US\$)	1.789 0	1.801 0

### Singapore dollars to 100 units of foreign currency

	Buying	Selling
Malaysian ringgit (RM)	46.50	47.30
Philippine peso	3.500	3.750
Thai baht	3.970	4.080
Hong Kong dollar (HK\$)	22.80	23.20
New Taiwan dollar (NT\$)	5.320	5.540
Japanese yen (¥)	1.446 0	1.460 0
Suisse franc (F)	103.50	104.70

Let the Internet to get the conversion exchange rates for the given countries.

One place to look for such information would be the homepage of a bank such as the IMF, then and visit its "home-exchange-rate" site.

Also try the online homepage and check "currency converter".

**NET**

The bank will buy from customers at a lower rate and sell to customers at a higher rate to make a profit.

When I want to exchange Singapore dollars for Malaysian ringgit, the bank will sell me Malaysian ringgit for Singapore dollars, the bank will buy Malaysian ringgit from the tourist.

For example, the bank will buy RM100 from a tourist and pay the tourist S\$46.50. However the bank will sell RM100 for S\$47.50. Therefore, the bank makes a profit of S\$1.00 (S\$47.50 - S\$46.50 = S\$1.00).

Money changers normally quote one unit or 100 units of local currency to foreign currencies. The following table illustrates the exchange rate quoted by a particular money changer on a certain day.

Source: The Straits Times, 13 June 1997

## Simple Financial Transactions: Compound Interest

In Example 40, compound interest is lower than simple interest as the principal is decreasing (due to monthly repayment of loan). As the interest charged is calculated on a monthly basis the calculation will be quite tedious. Thus banks provide a table of monthly repayment per unit loan for each specified percentage interest as shown in Table 1.

A computerized spreadsheet program will be able to produce such a table.

### TABLE 1

Fluores per unit of \$1000

Year	4.00%	4.25%	4.50%	4.75%	5.00%	5.25%	5.50%	5.75%	6.00%
1	85.6496	85.2647	84.8792	84.4937	84.1082	83.7227	83.3372	82.9517	82.5662
2	43.4292	43.3623	43.2954	43.2285	43.1616	43.0947	43.0278	42.9609	42.8940
3	28.2239	28.1935	28.1631	28.1327	28.1023	28.0719	28.0415	28.0111	27.9807
4	22.7768	22.7493	22.7218	22.6943	22.6668	22.6393	22.6118	22.5843	22.5568
5	18.4162	18.3916	18.3670	18.3424	18.3178	18.2932	18.2686	18.2440	18.2194
6	15.0418	15.0191	15.0000	14.9809	14.9618	14.9427	14.9236	14.9045	14.8854
7	13.0881	13.0670	13.0459	13.0248	13.0037	12.9826	12.9615	12.9404	12.9193
8	11.8216	11.8025	11.7834	11.7643	11.7452	11.7261	11.7070	11.6879	11.6688
9	10.8987	10.8815	10.8643	10.8471	10.8299	10.8127	10.7955	10.7783	10.7611
10	10.2141	10.2087	10.2034	10.1980	10.1927	10.1874	10.1820	10.1767	10.1714
11	9.7567	9.7523	9.7479	9.7435	9.7391	9.7347	9.7303	9.7259	9.7215
12	9.4528	9.4494	9.4460	9.4426	9.4392	9.4358	9.4324	9.4290	9.4256
13	9.2416	9.2390	9.2364	9.2338	9.2312	9.2286	9.2260	9.2234	9.2208
14	9.0886	9.0868	9.0850	9.0832	9.0814	9.0796	9.0778	9.0760	9.0742
15	8.9658	8.9648	8.9638	8.9628	8.9618	8.9608	8.9598	8.9588	8.9578
16	8.8654	8.8652	8.8650	8.8648	8.8646	8.8644	8.8642	8.8640	8.8638
17	8.7836	8.7834	8.7832	8.7830	8.7828	8.7826	8.7824	8.7822	8.7820
18	8.7158	8.7156	8.7154	8.7152	8.7150	8.7148	8.7146	8.7144	8.7142
19	8.6586	8.6584	8.6582	8.6580	8.6578	8.6576	8.6574	8.6572	8.6570
20	8.6100	8.6098	8.6096	8.6094	8.6092	8.6090	8.6088	8.6086	8.6084
21	8.5686	8.5684	8.5682	8.5680	8.5678	8.5676	8.5674	8.5672	8.5670
22	8.5330	8.5328	8.5326	8.5324	8.5322	8.5320	8.5318	8.5316	8.5314
23	8.5018	8.5016	8.5014	8.5012	8.5010	8.5008	8.5006	8.5004	8.5002
24	8.4736	8.4734	8.4732	8.4730	8.4728	8.4726	8.4724	8.4722	8.4720
25	8.4470	8.4468	8.4466	8.4464	8.4462	8.4460	8.4458	8.4456	8.4454
26	8.4216	8.4214	8.4212	8.4210	8.4208	8.4206	8.4204	8.4202	8.4200
27	8.3970	8.3968	8.3966	8.3964	8.3962	8.3960	8.3958	8.3956	8.3954
28	8.3730	8.3728	8.3726	8.3724	8.3722	8.3720	8.3718	8.3716	8.3714
29	8.3496	8.3494	8.3492	8.3490	8.3488	8.3486	8.3484	8.3482	8.3480
30	8.3266	8.3264	8.3262	8.3260	8.3258	8.3256	8.3254	8.3252	8.3250
31	8.3040	8.3038	8.3036	8.3034	8.3032	8.3030	8.3028	8.3026	8.3024
32	8.2818	8.2816	8.2814	8.2812	8.2810	8.2808	8.2806	8.2804	8.2802
33	8.2600	8.2598	8.2596	8.2594	8.2592	8.2590	8.2588	8.2586	8.2584
34	8.2386	8.2384	8.2382	8.2380	8.2378	8.2376	8.2374	8.2372	8.2370
35	8.2176	8.2174	8.2172	8.2170	8.2168	8.2166	8.2164	8.2162	8.2160
36	8.1970	8.1968	8.1966	8.1964	8.1962	8.1960	8.1958	8.1956	8.1954
37	8.1768	8.1766	8.1764	8.1762	8.1760	8.1758	8.1756	8.1754	8.1752
38	8.1570	8.1568	8.1566	8.1564	8.1562	8.1560	8.1558	8.1556	8.1554
39	8.1376	8.1374	8.1372	8.1370	8.1368	8.1366	8.1364	8.1362	8.1360
40	8.1186	8.1184	8.1182	8.1180	8.1178	8.1176	8.1174	8.1172	8.1170
41	8.1000	8.0998	8.0996	8.0994	8.0992	8.0990	8.0988	8.0986	8.0984
42	8.0818	8.0816	8.0814	8.0812	8.0810	8.0808	8.0806	8.0804	8.0802
43	8.0640	8.0638	8.0636	8.0634	8.0632	8.0630	8.0628	8.0626	8.0624
44	8.0466	8.0464	8.0462	8.0460	8.0458	8.0456	8.0454	8.0452	8.0450
45	8.0296	8.0294	8.0292	8.0290	8.0288	8.0286	8.0284	8.0282	8.0280
46	8.0130	8.0128	8.0126	8.0124	8.0122	8.0120	8.0118	8.0116	8.0114
47	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000

- (b) The total amount to be repaid.
- (c) The amount of each repayment if the total money is to be repaid in 60 equal instalments.
5. Aasen bought a computer set under a hire purchase scheme with a monthly payment of \$199 for 12 months with no down payment. If the cash price of the computer set is \$2 000, find how much more Aasen has to pay?
6. A refrigerator is advertised at \$1 500. Cash is \$150 per month for one year.
- Simple Financial Transactions: Taxation and Service Charge
- (a) How much would you pay for the refrigerator if you paid the monthly instalment?
- (b) How much more would you pay than if you paid cash? (This interest.)
- (c) What percentage of the cash price have you paid so far?
7. Zee Company charges 6% p.a. simple interest for hire purchase goods. The cash price of a multi-gramm set is \$1 500. If the down payment of \$300 is required, calculate the monthly instalment given that its payment has to be completed in 2 years' time.

### 1.6.7 Taxation and Service Charge

In order to pay for the many services provided by the government, it is necessary to collect money from the population in the form of taxes. One major type of tax is the Income Tax (see page 44). The government has also introduced a wider form of tax known as Goods and Services Tax (GST). Another form of tax, known as road tax, charges car owners according to the engine capacity of the vehicles (see Example 44).

Similarly, many restaurants charge customers a certain percentage of the bill for service. This is called the service charge.

#### KYE TOA PATON 178

GET PRICES AND AMOUNTS FOR A TABLE. Bill 100.00

10% TOTAL 110.00

10% SERVICE CHARGE 11.00

12% GST 13.20

10% TOTAL 144.20

10% TOTAL 158.64

10% TOTAL 174.50

10% TOTAL 191.95

10% TOTAL 210.95

10% TOTAL 231.45

10% TOTAL 253.39

10% TOTAL 276.73

10% TOTAL 301.41

10% TOTAL 327.35

10% TOTAL 354.59

10% TOTAL 383.05

10% TOTAL 412.77

10% TOTAL 443.69

10% TOTAL 475.83

10% TOTAL 509.13

10% TOTAL 543.63

10% TOTAL 579.37

10% TOTAL 616.39

10% TOTAL 654.71

10% TOTAL 694.37

10% TOTAL 735.41

10% TOTAL 777.85

10% TOTAL 821.73

10% TOTAL 867.99

10% TOTAL 915.67

10% TOTAL 964.81

10% TOTAL 1015.45

10% TOTAL 1067.83

10% TOTAL 1121.89

10% TOTAL 1178.67

10% TOTAL 1237.21

10% TOTAL 1297.55

10% TOTAL 1359.73

10% TOTAL 1423.79

10% TOTAL 1489.77

10% TOTAL 1557.71

10% TOTAL 1627.65

10% TOTAL 1699.53

10% TOTAL 1773.41

10% TOTAL 1849.33

10% TOTAL 1927.33

10% TOTAL 2007.45

10% TOTAL 2089.73

10% TOTAL 2174.21

10% TOTAL 2260.93

10% TOTAL 2349.93

10% TOTAL 2441.25

10% TOTAL 2533.93

10% TOTAL 2628.01

10% TOTAL 2724.43

10% TOTAL 2823.25

10% TOTAL 2924.51

10% TOTAL 3028.17

10% TOTAL 3134.27

10% TOTAL 3242.85

10% TOTAL 3353.95

10% TOTAL 3467.61

10% TOTAL 3583.87

10% TOTAL 3702.77

10% TOTAL 3824.35

10% TOTAL 3948.65

10% TOTAL 4075.61

10% TOTAL 4205.27

10% TOTAL 4338.67

10% TOTAL 4474.85

10% TOTAL 4613.75

10% TOTAL 4755.41

10% TOTAL 4900.00

#### BUNIT MERAH RESTAURANT

GET PRICES AND AMOUNTS FOR A TABLE. Bill 100.00

10% TOTAL 110.00

10% SERVICE CHARGE 11.00

12% GST 13.20

10% TOTAL 144.20

10% TOTAL 158.64

10% TOTAL 174.50

10% TOTAL 191.95

10% TOTAL 210.95

10% TOTAL 231.45

10% TOTAL 253.39

10% TOTAL 276.73

10% TOTAL 301.41

10% TOTAL 327.35

10% TOTAL 354.59

10% TOTAL 383.05

10% TOTAL 412.77

10% TOTAL 443.69

10% TOTAL 475.83

10% TOTAL 509.13

10% TOTAL 543.63

10% TOTAL 579.37

10% TOTAL 616.39

10% TOTAL 654.71

10% TOTAL 694.37

10% TOTAL 735.41

10% TOTAL 777.85

10% TOTAL 821.73

10% TOTAL 867.99

10% TOTAL 915.67

10% TOTAL 964.81

10% TOTAL 1015.45

10% TOTAL 1067.83

10% TOTAL 1121.89

10% TOTAL 1178.67

10% TOTAL 1237.21

10% TOTAL 1297.55

10% TOTAL 1359.73

10% TOTAL 1423.79

10% TOTAL 1489.77

10% TOTAL 1557.71

10% TOTAL 1627.65

10% TOTAL 1699.53

10% TOTAL 1773.41

10% TOTAL 1849.33

10% TOTAL 1927.33

10% TOTAL 2007.45

10% TOTAL 2089.73

10% TOTAL 2174.21

10% TOTAL 2260.93

10% TOTAL 2349.93

10% TOTAL 2441.25

10% TOTAL 2533.93

10% TOTAL 2628.01

10% TOTAL 2724.43

10% TOTAL 2823.25

10% TOTAL 2924.51

10% TOTAL 3028.17

10% TOTAL

**1.6.6 Income Tax**

The tax charged on salaries received from employment is known as **income tax**. Everyone is allowed to earn a certain amount of money which is exempted from tax. This amount is called tax relief. If a person earns above this amount, he has to pay a percentage of the excess as income tax. Take the example of a man earning \$25 000 per year and contributing \$5 000 to the CPF Central Provident Fund. If he is allowed tax relief on the first \$3 000 of his income, he will be taxed on:

$\$25\ 000 - \$5\ 000 - \$3\ 000 = \$17\ 000$ .

This \$17 000 is his **chargeable (or taxable) income**.

The amount of a person's tax relief depends on his personal circumstances. For the year of assessment 2001, the assessment of tax on income earned in 2000, a married man (with a non-working wife) below the age of 55 years was given a relief of \$6 000 while a single person below 55 years was given a relief of \$4 000.

The amount of tax due can be split into 12 instalments per year, in one instalment each month. This instalment can be deducted from his bank account using the Auto-Debit Reduction Scheme.

The rates of tax on chargeable income in the case of an individual resident in the year of assessment 2001 were as follows:

Chargeable Income \$	Rate (%)	Gross Tax payable \$
On the first 7 500	2	150.00
On the next 12 500	5	625.00
On the first 20 000		775.00
On the next 15 000	8	1 200.00
On the first 35 000		1 975.00
On the next 15 000	12	1 800.00
On the first 50 000		3 775.00
On the next 25 000	16	4 000.00
On the first 75 000		7 775.00
On the next 100 000	20	11 000.00
On the first 150 000		12 775.00
On the next 50 000	23	11 000.00
On the first 200 000		23 775.00
On the next 200 000	26	52 000.00
On the first 400 000		87 775.00
Above 400 000		

A CPF Related Tax Rebate not exceeding \$300 will be given for the Year of Assessment 2001.

**Investigating Maths**

The Central Provident Fund (CPF) has evolved from a simple old-age saving plan (1955) into a comprehensive social security savings scheme which provides many working Singaporeans with a sense of security for their old age. Today, the CPF encompasses a whole lifetime of benefits, from meeting statutory expenses to receiving an annuity income for life.

The money in the CPF is divided into three accounts: Ordinary Account, Special Account and Medisave Account.

Study the excerpt from *The Money Matters*, dated 30 August 2000. Which word indicates that the CPF contribution was reduced? Which contribution was reduced?

**Other workers to get more in special retirement account; Medisave savings to go up by one point five**

Great Prudence Fund members will be able to pay every one their special retirement account with progressively higher returns in their given order.

The new long-term savings rate is 8 per cent of salary, instead of the current 7 per cent for members aged 55 and below.

It goes up to 9 per cent for those aged 55 to 60 and 10 per cent for those aged 61 and above.

There will also be a one-percentage-point increase in contributions to the Medisave account.

The increase, to keep pace with greater medical expenditures and medical costs, will raise the rate to between 7 and 10 per cent, depending on the member's age group.

The contribution rate to the ordinary account, used mainly for housing, will now be adjusted over tables.

The new Medisave rate will be slightly raised, but the member's contribution has been reduced in fact, to 20 per cent.

Age 35 years and below	Ordinary account		Special account		Medisave account		Total	
	Current %	Target %	Current %	Target %	Current %	Target %		
24	25	2	7	8	7	23	48	
25-29	25	2	7	8	8	23	48	
30-34	22	5	7	8	9	32	48	
Above 35 years	9	11	9	10	8	9	17	28
Above 40-49 years	3	2.5	9	10	8	9	19	11.5
Above 50 years	0	0	9	10	8	9	17.5	9

From 1955 to 2000, the CPF contributions were reduced twice. For each reduction, find out:

- when it was reduced,
- why it was reduced,
- by how much it was reduced.

Source: Hong, T. C., Riddington, M. & Grier, M. (2001). *New Mathematics Counts for Secondary Normal (Academic) 2*. Singapore: Time Publishing Group.

There is a great difference between the level of the content in Malaysian mathematics textbooks as compared to the Singaporean textbooks. The contexts provided in the Malaysian textbooks, in the chapter on percentage, revolve around discounts in purchasing some goods in departmental stores and calculation of a percentage of an amount. Compare this to what was mentioned earlier on the Singaporean textbook contents.

## THE MATHEMATICS ASSESSMENT

To reduce the examination oriented culture that Malaysia has been practicing for years, school based assessment was introduced in 2012, in which, one of the components is assessment using performance standards. This was believed to be a transformation in school assessments. However, due to the over zealous reactions of the public following the PT3 results and the implications it brings,

actual implementation will probably require a few more years than was initially intended.

Malaysian examination questions look quite challenging on the surface, but the recycling of examination questions over the years has made the questions too predictable. There are an abundance of workbooks that provide PMR or SPM clone questions, thus the ability to answer those questions may no longer reflect true ability and the achievement of some level of mathematical thinking among the students.

The choices given for some sections in the examination have also spurred negative consequences. For instance, as the topic on Earth as a Sphere is rather challenging, teachers tend to exclude this topic altogether, in their teaching because students can choose questions from other sections. The 'teaching for examination' culture is strong for multiple reasons. Firstly, to ensure school performance and, secondly, to ensure that students get good grades.

## CONCLUSION

Malaysia should keep on participating in international assessments such as TIMSS and PISA. Without comparisons such as these, we may get too complacent about our students' mathematics performance. Although only a small percentage of schools are involved, the exposure to such assessment standards can enlighten our educators on the standards and benchmarks for mathematical thinking and competence that we need to aim for to ensure Malaysian students' competitiveness at the international level. There is also a need to review the examination formats for SPM, UPSR and the assessment conducted at the form 3 level. These assessments will be key in determining standards because classroom instruction will always be geared towards preparing students for the type of



assessment administered. The last few years, with the criticisms on our students' mathematics performance in TIMSS, PISA and PT3, has been a wakeup call for us to transform the way we determine intended learning outcomes, teach and assess our students. We must ensure that we continuously promote the thinking culture amongst our students

## REFERENCES

- Ahmad Nurulazam, Aida Suraya Md. Yunus, Abdul Ghani Kanesan Abdullah, Othman Lebar, Nor Shafrin Ahmad & Aziah Ismail. (2015). *Laporan Kajian Pelaksanaan Pentaksiran Tingkatan 3 (Pt3): Isu Dan Penambahbaikan*. Penang: Institut Penyelidikan Pendidikan Tinggi Negara.
- Aida Suraya Md. Yunus. (2001a). Levels of reasoning in mathematical concepts among preservice mathematics teachers. *2nd APED International Conference in Education Research Understanding Educational Issues in the Asia Pacific Region Universals, Uniqueness and Cooperation*. Seoul, Korea, 52-61.
- Aida Suraya Md. Yunus. (2001b). Algoritma Pendaraban Nombor Perpuluhan dari Perspektif Pelajar Tingkatan Satu. *Pertanika Journal of Social Science and Humanities*, 9(1), 21 – 33.
- Aida Suraya Md. Yunus. (2003). Reasoning in school mathematical tasks among pre-service mathematics teachers: A case study. *Journal of Science and Mathematics Education in South East Asia*, 26(10), 114-128.
- Aida Suraya Md. Yunus. (2008). Pembentukan individu yang berpemikiran matematik. In Aida Suraya Md. Yunus. Ahmad Fauzi Mohd Ayub & Othman Talib (ed), *Amalan dalam Pengajaran dan Pembelajaran Sains, Matematik dan Pembelajaran berasaskan ICT*, pp. 57 -75. Serdang: Penerbit Universiti Putra Malaysia.
- Aida Suraya Md. Yunus, Sharifah Mohd Nor & Habsah Ismail. (1994). Analisis Kesilapan Masalah-Masalah Berkaitan Nombor Perpuluhan dan Pecahan Bagi Pelajar-Pelajar Tahun Lima Sekolah Rendah. *Jurnal Pendidik dan Pendidikan*, 12, 15 – 32.

- Aida Suraya Md. Yunus & Wan Zah Wan Ali. (2008). Metacognition and motivation in mathematical problem solving. *The International Journal of Learning*, 15(3), 121-131.
- Breyfogle, M. L. & Courtney, M. L. (2010). van Hiele Revisited. *Mathematics Teaching in the Middle School*, 16(4), 232 – 238.
- Burill, G. Allison, J., Breaux, G., Kastberg, S., Leatham, K. & Sanchez, W. (2002). *Handheld Graphing Technology in Secondary Mathematics: Research Findings And Implications For Classroom Practice*. Dallas, TX: Texas Instrument Corp.
- Byrnes, J. P. & Wasik, B. A. (1991). Role of conceptual knowledge in mathematical procedural learning. *Developmental Psychology*, 27, 777 – 786.
- Cai, J. & Cifarelli, V. (2004). Thinking mathematically by Chinese learners: A cross-national comparative perspective. In F. Lianghuo, N. Y. Wong, J. Cai. & S. Li (eds), *How Chinese Learn Mathematics: Perspectives from Insiders*, pp. 71-106). London: World Scientific.
- de Castro, B. (2004). Pre-Service Teachers' Mathematical Reasoning as an Imperative for Codified Conceptual Pedagogy in Algebra: A case Study in Teacher Education. *Asia Pacific Education Review*, 5(2), 157-166.
- Devlin, K. (2012). *Introduction to Mathematical Thinking (2nd edition)*. Petaluma, CA: Keith Devlin.
- Ding, L., & Jones, K. (2006). Teaching geometry in lower secondary school in Shanghai, China. In *Proceedings of the British Society for Research into Learning Mathematics*, 26(1), 41 - 46.
- Eby, J. W., Herrell, A. L., & Jordan, M. L. (2005). *Teaching K-12 schools: A reflective action approach*. Englewood Cliffs, N.J.: Prentice Hall.
- Flavell, J. H. (1976). Metacognitive aspects of problem solving. In L. B. Resnick, *The Nature of Intelligence*, 12, 231-23 Hillsdale, NJ: Lawrence Erlbaum Associates.
- Gersten, R. & Chard D. (1999). Number Sense: Rethinking Arithmetic Instruction for Students with Mathematical Disabilities. *Journal of Special Education*, 33, 18-28.

- Hanley, S. (1994). *On Constructivism*. Available WWW: T. Cey EdCmm 802.6 Page #31 [file:///Macintosh%20HD/Desktop%20Folder/paper%20work/Constructivism.txt]
- Harel, G. & Sowder, L. (2005). Advanced Mathematical-Thinking at Any Age: Its Nature and Its Development. *Mathematical Thinking and Learning*, 7(1), 27–50.
- Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics*, pp. 1- 27. Hillsdale, NJ: Lawrence Erlbaum Associates.
- Hong, T. C., Riddington, M. & Grier, M. (2001). *New Mathematics Counts for Secondary Normal (Academic) 2*. Singapore: Time Publishing Group.
- Hwa, T. Y. & Stephens, M. (2011). Implementing a mathematical thinking assessment framework: Cross cultural perspectives. *Proceedings of Mathematics Education Research Group of Australia (MERGA)*.
- Kamariah Abu Bakar, Rohani Ahmad Tarmizi, Ahmad Fauzi Mohd Ayub & Aida Suraya Md. Yunus. (2009). Effect of utilizing Geometer's Sketchpad on performance and mathematical thinking of secondary learners: An initial exploration. *International Journal of Education and Information Technologies*, 1(3), 20 – 27.
- Kang Soon Chen, 2013. *Poor show in PISA rankings*. The Star, Dec 8, 2013.
- Katz, J. (2014). *Developing mathematical thinking: A guide to rethinking the mathematics classroom*. Lanham, MD: Rowman & Littlefield Publishers.
- Kieran, T. & Pirie, S. B. (1991). Recursion and the mathematical experience. In L. P. Steffe (Ed.), *Epistemological Foundations of Mathematical Experience*, pp. 78 – 101. New York: Springer-Verlag.
- Klieme, E. & Vieluf, S. (2009). Teaching Practices, Teachers' Beliefs and Attitudes. In Indicators and Analysis Division of the OECD Directorate for Education. OECD. *Creating Effective Teaching and Learning Environments: First Results from TALIS*, pp 87 – 135. Accessed May 1<sup>st</sup> from <http://www.oecd.org/edu/school/43023606.pdf>

Developing Students' Mathematical Thinking: How Far Have We Come?

- Lessani, A. (2015). Implementation of Secondary School Mathematics Curriculum by Teacher in Two Malaysian Schools. Unpublished doctoral thesis, Universiti Putra Malaysia.
- Livingston, J. A. (2003). Metacognition: An Overview. Accessed April 17 from <http://files.eric.ed.gov/fulltext/ED474273.pdf>
- Mason, M. (1998). The van Hiele levels of geometric understanding. In L. McDougal (Ed.), *The professional handbook for teachers: Geometry* (pp. 4-8). Boston, MA: McDougal-Littell/Houghton-Mifflin.
- Mason, J., Burton, L., & Stacey, K. (1982). *Thinking mathematically*. London: Pearson.
- Ministry of Education, Malaysia. (1989). Kurikulum Bersepadu Sekolah Menengah: Huraian Sukatan Pelajaran Matematik Tingkatan 1. Kuala Lumpur: Ministry of Education.
- Ministry of Education, Malaysia. (2006a). Integrated Curriculum for Secondary School: Curriculum Specifications for Mathematics Form Five. Putrajaya: Ministry of Education.
- Ministry of Education, Malaysia. (2006b). Integrated Curriculum for Primary School: Curriculum Specifications for Mathematics Year Six. Putrajaya: Ministry of Education.
- Ministry of Education Singapore. (2006). *Secondary Mathematics Syllabus*. Singapore: Curriculum Planning and Development Division, Ministry of Education Singapore.
- Mullis, I. V. S., Martin M. O., Gonzalez, E. J., & Chrostowski, S. J. (2004). TIMSS 2003 international mathematics report: Findings from IEA's Trends in International Mathematics and Science Study at the fourth and eighth grades. Boston College: TIMSS & PIRLS International Study Centre, Lynch School of Education.
- Mullis, I. V. S., Martin, M. O., & Foy, P. (2008). TIMSS 2007 international mathematics report: Findings from IEA's Trends in International Mathematics and Science Study at the fourth and eighth grades. Chestnut Hill, MA: TIMSS & PIRLS International Study Centre, Boston College.

- Mullis, I. V. S., Martin, M. O., Foy, P., & Arora, A. (2012). TIMSS 2011 international mathematics report: Findings from IEA's Trends in International Mathematics and Science Study at the fourth and eighth grades. Chestnut Hill, MA: TIMSS & PIRLS International Study Centre, Boston College.
- National Centre for Education Statistics (NCES). (2013). *Highlights for TIMSS 2011*, p. 1 – 56. NCES, IES: Department of Education.
- National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.
- Ng, E. (2014). Fewer As in PT3 show ministry 'manipulated' PMR results, says education group. Accessed on April 2, 2015 from <http://www.themalaysianinsider.com/malaysia/article/fewer-as-in-pt3-show-ministry-manipulated-pmr-results-says-education>
- Noraini, I. (2007). The effect of Geometer's Sketchpad on the performance in geometry of Malaysian students ' achievement and van Hiele geometric thinking. *Malaysian Journal of Mathematical Sciences*, 1(2), 169 - 180.
- PISA (Programme for International Student Assessment). (2006). *Assessing Scientific, Reading and Mathematical Literacy. A Framework for PISA 2006*. Paris: Organisation for Economic Cooperation and Development (OECD).
- Polya, G. (1965). *Mathematical Discovery: On understanding, learning and teaching problem solving*. John Wiley and Sons.
- Prystay, C. (December 13, 2004). As math skills slip, U.S. schools seek answers from Asia. *The Wall Street Journal*.
- Rohani Ahmad Tarmizi, Ahmad Fauzi Mohd Ayub, Kamariah Abu Bakar & Aida Suraya Md. Yunus . (2008). Instructional efficiency of utilization of autograph technology vs handheld graphing calculator for learning algebra. *International Journal of Education and Information Technologies*, 3(2), 184 – 191.
- Rohani Ahmad Tarmizi, Aida Suraya Md. Yunus, Ahmad Fauzi Mohd Ayub & Kamariah Abu Bakar. (2009). Integration of Autograph Technology for Learning Algebra. *European Journal of Social Sciences*, 9(1), 129-146.

Developing Students' Mathematical Thinking: How Far Have We Come?

- Sanchez-Alonso, S., & Vovides, Y. (2007). Integration of metacognitive skills in the design of learning objects. *Computer in Human Behavior*, 23, 2585-2595.
- Schafer, M. (2003). *The impact of learners' spatial capacity and worldviews on their spatial conceptualisation: A case study*. Unpublished doctoral thesis, Curtin University of Technology, Perth.
- Schoenfeld, A. H. (1992). Learning to thinking mathematically: Problem solving, metacognition and sense-making in mathematics. In D. Grouws (Eds), *Handbook of research on mathematics teaching and learning*, 334 - 370. New York: MacMillan.
- Scusa, T. (2008). *Five Processes of Mathematical Thinking*. Summative Projects for MA Degree. Paper 38. Accessed on 23 Dec 2014 from <http://digitalcommons.unl.edu/mathmidsummative/38>
- Schraw, G. & Dennison, R.S. (1994). Assessing metacognitive awareness. *Contemporary Educational Psychology*, 19,460- 475.
- Sheffield, J. L., & Cruikshanks, D. (2005). *Teaching and Learning Mathematics: Pre-Kindergarten Through Middle School*. (5th ed). New York, NY: John Wiley and Sons. Inc.
- Singh, P. & White, A. (2006). Unpacking first year university students' mathematical content knowledge through problem solving. *Asian Journal of University Education*, 2(1), 33-56.
- Steen, L. A. (1999). Twenty questions about mathematical reasoning. In L. V. Stiff & F. R. Curcio (eds), *Developing mathematical reasoning in grades K-12*, pp. 270 - 285. Reston, VA: National Council of Teachers of Mathematics (NCTM).
- Sternberg, R. J. (1984). What should intelligence tests test? Implications for a triarchic theory of intelligence for intelligence testing. *Educational Researcher*, 13(1), 5-15.
- Sternberg, R. J. (1986). *Intelligence applied*. New York: Harcourt Brace Jovanovich Publishers.
- Sternberg, R. J. (1991). Cognitive theory and psychometrics. In R. K. Hambleton & J. N. Zaal (eds), *Advances in educational and psychological testing: Theory and application*, pp. 367-393. Boston, MA: Kluwer Academic Publishers.

- Sternberg, R. J. (1999). The nature of mathematical reasoning. In L. V. Stiff & F. R. Curcio (eds), *Developing mathematical reasoning in grades K-12*, pp. 37 - 44. Reston, VA: National Council of Teachers of Mathematics (NCTM).
- Tan Tong Hock, Rohani Ahmad Tarmizi, Aida Suraya Md. Yunus & Ahmad Fauzi Ayub. (2015). Understanding the primary school students' van Hiele levels of geometry thinking in learning shapes and spaces: A Q-methodology. *Eurasia Journal of Mathematics, Science & Technology Education*, 11(4), 505 – 514.
- Usiskin, Z. (1982). van Hiele levels and achievement in secondary school geometry: Cognitive development and achievement in a secondary school geometry project. Chicago: University of Chicago Press.
- van Hiele, P. M. (1986). *Structure and insight: A theory of Mathematics Education*. Orlando, FL: Academic Press.
- Wong, K. Y. & Lee, N. H. (2009). Singapore education and mathematics curriculum. In Wong K. Y., Yoong, L. P., Lee, P. Y., Kaur, B., Foong, P. Y. & Ng, S. W. *Mathematics Education: The Singapore Journey 2*, pp. 13 – 47. Singapore: World Scientific Publishing.
- Wu, D. B. & Ma, H. L. (2009). An Application of GM (O.N) on Analyzing the First van Hiele Geometrical Thinking Level. *Journal of Grey System*, 12(4), 161-168.

## BIOGRAPHY

**Dr. Aida Suraya Md. Yunus** is a Professor at the Department of Science and Technical Education, Faculty of Educational Studies, Universiti Putra Malaysia (UPM). Currently, she is the Director of the Centre for Academic Development (CADE) at UPM. She is also the Chairman for the Council for Directors of Teaching Learning Centres of Malaysian Public Universities. Dr Aida Suraya is a research fellow with the National Higher Education Research Institute (IPPTN) and associate researcher with the Institute of Mathematical Research (INSPEM). She has held several administrative posts including head of research laboratory, head of department, deputy director and director.

Born in Bahau, Negeri Sembilan in 1960, Dr Aida Suraya completed her primary education at St. Aidan Primary School and was fortunate to be selected to continue her secondary schooling at the MARA Junior Science College (MRSM) Seremban, the Class of 1977. With the sponsorship of MARA, in 1978, she pursued undergraduate study in Ohio University and graduate study in West Virginia University, in the field of mathematics. Upon graduation, Dr Aida Suraya was appointed as a lecturer in UPM and to date, she has loyally served UPM for 31 years. She pursued her Ph.D in Mathematics Education in University Malaya and graduated in 1996, and was promoted to associate professor in the year 2001 and full professor in 2010.

Dr. Aida Suraya's research interests are primarily in the teaching and learning of mathematics and issues pertaining to higher education. She has conducted 31 researches where she was the lead researcher for 14 of the projects and has received 15 research awards. She has graduated nine doctoral and 24 master students. She had also examined 15 theses of graduate students from Universiti Kebangsaan Malaysia, University Malaya, Universiti Sains Malaysia and Universiti Teknologi Malaysia.



## Developing Students' Mathematical Thinking: How Far Have We Come?

Dr. Aida Suraya has authored and co-authored 60 journal articles, two books, 12 international book chapters, 29 Malaysian book chapters and 91 conference papers. She is also the editor for seven other books. Most of Dr. Aida Suraya's journal publications are in international journals. The book chapters that she authored and co-authored have been published by reputable publishers, including Springer, IGI Global, UNESCO ERI-Net, Hiroshima University, University of Kassel, Universiti Putra Malaysia Press, Universiti Sains Malaysia Press and International Islamic University Malaysia Press.

Apart from these publications, Dr Aida Suraya has also contributed modules for the National Higher Education Leadership Academy (AKEPT), Open University Malaysia, Multimedia Development Corporation (MDeC) and the Curriculum Development Center. She is also one of the authors of the Guidelines for Good Practices: Assessment, a project carried out for the Malaysian Qualifications Agency (MQA).

Dr Aida Suraya has participated in conferences in many parts of the world including the United States of America, Canada, Jamaica, Finland, Spain, Iran, Korea, Japan, China, Australia and many more. She has also been acknowledged and invited to give the keynote address in Iran, Thailand and Malaysia and to speak at conferences in Hong Kong, China and Japan. Dr Aida Suraya had participated in the Asia Pacific Higher Education Research partnership (APHERP) Senior Seminar. Her most extensive collaboration is with the Research Institute for Higher Education (RIHE) of Hiroshima University and UNESCO Education Research Institutes Network (ERI-Net) in the Asia-Pacific. The research work on the Changing Academic Profession in Asia with RIHE began in 2011, and the outcomes were presented in three meetings in Japan and one in Taiwan. The collaboration with UNESCO ERI-Net

started in 2013 and to date Dr Aida Suraya and two other colleagues have contributed two chapters in UNESCO ERI-Net publications.

One of Dr Aida Suraya's strengths is in giving training. She has conducted hundreds of training sessions for UPM, AKePT, MQA, Malaysian public and private universities, polytechnics, teacher training institutes, Multimedia Development Corporation (MDeC), and schools. She has also participated in giving training to academics from several foreign universities. With her vast knowledge on Malaysian higher education, she has also been recognized as one of AKePT's master trainers for Teaching and Learning in Higher Education.

Dr Aida Suraya's expertise has been sought by several other agencies. She has been a member of the jury panel for National Academic Awards (AAN) for the category of journal publication over the last three years. She has also been appointed as a working committee member for the National Committee for MOOC Development in Higher Education Institutions, National Committee for Critical Agenda Project (CAP) Teaching and Learning, resource person for MQA for Assessment in Teaching in Learning and had also contributed to the formulation of CAP Academia and CAP Life Long Learning. She has also been appointed as assessor for promotions to associate professor and professor status by several Malaysian universities. With her commitment and passion for the profession, Dr Aida Suraya will continue to contribute significantly to UPM and Malaysian higher education.

## ACKNOWLEDGEMENT

My utmost gratitude goes to all the people who helped me grow and become the person that I am today, a Professor at Universiti Putra Malaysia (UPM).

My special thanks to Prof Dato' Dr. Mohd Fauzi Hj Ramlan, Vice Chancellor of UPM, for his continuous support and to Prof Tan Si Dato' Dr Nik Mustapha R. Abdullah the previous Vice Chancellor of UPM, for acknowledging my talent and potential. My gratitude also to the Deputy Vice Chancellor (Academic and International), Prof Datuk Dr. Mad Nasir Shamsudin and the Dean of the Faculty of Educational Studies (FPP), Prof Dr. Ab Rahim Bakar, for supporting and encouraging me to give this inaugural lecture. My sincere thanks to former deans of FPP, Prof Dr. Nazaruddin Md Jali who saw the potential in the young, pretty and vibrant lady, Prof Dr. Abdul Rahman Aroff for always providing me a chance, Prof Dr. Kamariah Abu Bakar for her inspiration and for her high expectations that we should produce the best, and Prof Dato Dr. Zakaria Kasa and Associate Prof. Md Shah Lassim for their unflinching support.

My thanks also for the friendly and professional experiences that I have enjoyed throughout numerous research endeavours, especially to Prof Dr. Wan Zah Wan Ali, Associate Prof. Dr Rohani Ahmad Tarmizi, Prof Dr. Kamariah Abu Bakar, Associate Prof. Dr Ahmad Fauzi Ayub, Associate Prof. Datin Dr Ramlah Hamzah, Associate Prof. Dr Habsah Ismail and Associate Prof. Dr Rosini Abu, and to my mentor in doing research in higher education, Prof Dato Dr. Morshidi Sirat, the former Director General of Higher Education. Special thanks to the Director of the National Higher Education Research Institute (IPPTN), Prof Dr Ahmad Nurulazam, and my dear colleagues from IPPTN, Prof Dato Dr Norzaini Azman, Prof Dato Dr Ibrahim Che Omar, Prof Dr. Vincent Pang, Associate

Prof. Dr Shukran Abdul Rahman, Associate Prof. Dr. Munir Shuib and Prof Dr Rosni Bakar, Ms Noraini Yusof and the many others who have helped me throughout my professional journey.

This lecture took time to concretize, with ideas coming at odd times of the day, especially after midnight. Shaping of the lecture took place while driving, shopping and relaxing. Thank you so much to Prof Dr. Wan Zah Wan Ali for going through the first draft of this lecture, Prof Dr Jayakaran Mukundan for sharing his experience and Associate Prof Datin Dr. Mardziah Hayati Abdullah for giving this piece an attractive and appealing title.

I wholeheartedly register my sincere thanks to all the staff members of FPP and the Centre for Academic Development (CADE) for their kindness, support and assistance. Not forgetting also my doctoral students, Mr Abdolreza Lessani, Mr Tan Tong Hock, and Mr Yeoh Hong Beng whose work I have cited in this paper and to students in Mathematics Education at UPM, you provide me the drive to keep on researching.

I would like to express my sincere gratitude to the staff of FPP, CADE and MARCOMM for assisting with the lecture and organizing the event and many thanks to UPM Press for their assistance with the publication. Last but not least, my thanks to the Ministry of Education (MOE) and Ministry of Science, Technology and Innovation (MOSTI) for providing funds for academics to do research.

This lecture is dedicated to

- .... my late parents, Md. Yunus Hj Ali and Zabedah Kidam who had great belief in my potential but never had the chance to share my ultimate success.
- .... my life partner, Zainal Abidin Hashim, for the endless support, guidance, advice, trust and the confidence that I can reach the sky.
- .... My children, Yuza Iskandar, Aiza Nur Izdihar, Zharif Iskandar and Aiza Nur Batrisyia for their tolerance in keeping up with a super mum.
- .... My son-in-law, Mohd Shazwan Ab Halim for his objective and sincere criticism and comments.
- .... My darlings, Ayyash Mohd Shazwan and Asma' Mohd Shazwan – hope they will learn to love mathematics.
- .... My siblings, Sanusi, Mastura and Rozainor for their endless love.

*You don't learn any mathematics unless you can attach meaning to the procedures that you do.*

- Aida Suraya Md Yunus -

*Mathematical thinking is a whole way of looking at things, of stripping them down to their numerical, structural, or logical essentials, and of analyzing the underlying patterns. Moreover, it involves adopting the identity of a mathematical thinker.*

-Keith Devlin -

## LIST OF INAUGURAL LECTURES

1. Prof. Dr. Sulaiman M. Yassin  
*The Challenge to Communication  
Research in Extension*  
22 July 1989
2. Prof. Ir. Abang Abdullah Abang Ali  
*Indigenous Materials and Technology  
for Low Cost Housing*  
30 August 1990
3. Prof. Dr. Abdul Rahman Abdul Razak  
*Plant Parasitic Nematodes, Lesser  
Known Pests of Agricultural Crops*  
30 January 1993
4. Prof. Dr. Mohamed Suleiman  
*Numerical Solution of Ordinary  
Differential Equations: A Historical  
Perspective*  
11 December 1993
5. Prof. Dr. Mohd. Ariff Hussein  
*Changing Roles of Agricultural  
Economics*  
5 March 1994
6. Prof. Dr. Mohd. Ismail Ahmad  
*Marketing Management: Prospects  
and Challenges for Agriculture*  
6 April 1994
7. Prof. Dr. Mohamed Mahyuddin Mohd.  
Dahan  
*The Changing Demand for Livestock  
Products*  
20 April 1994
8. Prof. Dr. Ruth Kiew  
*Plant Taxonomy, Biodiversity and  
Conservation*  
11 May 1994
9. Prof. Ir. Dr. Mohd. Zohadie Baraie  
*Engineering Technological  
Developments Propelling Agriculture  
into the 21st Century*  
28 May 1994
10. Prof. Dr. Shamsuddin Jusop  
*Rock, Mineral and Soil*  
18 June 1994
11. Prof. Dr. Abdul Salam Abdullah  
*Natural Toxicants Affecting Animal  
Health and Production*  
29 June 1994
12. Prof. Dr. Mohd. Yusof Hussein  
*Pest Control: A Challenge in Applied  
Ecology*  
9 July 1994
13. Prof. Dr. Kapt. Mohd. Ibrahim Haji  
Mohamed  
*Managing Challenges in Fisheries  
Development through Science and  
Technology*  
23 July 1994
14. Prof. Dr. Hj. Amat Juhari Moan  
*Sejarah Keagungan Bahasa Melayu*  
6 August 1994
15. Prof. Dr. Law Ah Theem  
*Oil Pollution in the Malaysian Seas*  
24 September 1994
16. Prof. Dr. Md. Nordin Hj. Lajis  
*Fine Chemicals from Biological  
Resources: The Wealth from Nature*  
21 January 1995
17. Prof. Dr. Sheikh Omar Abdul Rahman  
*Health, Disease and Death in  
Creatures Great and Small*  
25 February 1995

## Developing Students' Mathematical Thinking: How Far Have We Come?

18. Prof. Dr. Mohamed Shariff Mohamed Din  
*Fish Health: An Odyssey through the Asia - Pacific Region*  
25 March 1995
19. Prof. Dr. Tengku Azmi Tengku Ibrahim  
*Chromosome Distribution and Production Performance of Water Buffaloes*  
6 May 1995
20. Prof. Dr. Abdul Hamid Mahmood  
*Bahasa Melayu sebagai Bahasa Ilmu-Cabaran dan Harapan*  
10 June 1995
21. Prof. Dr. Rahim Md. Sail  
*Extension Education for Industrialising Malaysia: Trends, Priorities and Emerging Issues*  
22 July 1995
22. Prof. Dr. Nik Muhammad Nik Abd. Majid  
*The Diminishing Tropical Rain Forest: Causes, Symptoms and Cure*  
19 August 1995
23. Prof. Dr. Ang Kok Jee  
*The Evolution of an Environmentally Friendly Hatchery Technology for Udang Galah, the King of Freshwater Prawns and a Glimpse into the Future of Aquaculture in the 21st Century*  
14 October 1995
24. Prof. Dr. Sharifuddin Haji Abdul Hamid  
*Management of Highly Weathered Acid Soils for Sustainable Crop Production*  
28 October 1995
25. Prof. Dr. Yu Swee Yean  
*Fish Processing and Preservation: Recent Advances and Future Directions*  
9 December 1995
26. Prof. Dr. Rosli Mohamad  
*Pesticide Usage: Concern and Options*  
10 February 1996
27. Prof. Dr. Mohamed Ismail Abdul Karim  
*Microbial Fermentation and Utilization of Agricultural Bioresources and Wastes in Malaysia*  
2 March 1996
28. Prof. Dr. Wan Sulaiman Wan Harun  
*Soil Physics: From Glass Beads to Precision Agriculture*  
16 March 1996
29. Prof. Dr. Abdul Aziz Abdul Rahman  
*Sustained Growth and Sustainable Development: Is there a Trade-Off 1 or Malaysia*  
13 April 1996
30. Prof. Dr. Chew Tek Ann  
*Sharecropping in Perfectly Competitive Markets: A Contradiction in Terms*  
27 April 1996
31. Prof. Dr. Mohd. Yusuf Sulaiman  
*Back to the Future with the Sun*  
18 May 1996
32. Prof. Dr. Abu Bakar Salleh  
*Enzyme Technology: The Basis for Biotechnological Development*  
8 June 1996
33. Prof. Dr. Kamel Ariffin Mohd. Atan  
*The Fascinating Numbers*  
29 June 1996
34. Prof. Dr. Ho Yin Wan  
*Fungi: Friends or Foes*  
27 July 1996
35. Prof. Dr. Tan Soon Guan  
*Genetic Diversity of Some Southeast Asian Animals: Of Buffaloes and Goats and Fishes Too*  
10 August 1996

Aida Suraya Md. Yunus

36. Prof. Dr. Nazaruddin Mohd. Jali  
*Will Rural Sociology Remain Relevant  
in the 21st Century?*  
21 September 1996
37. Prof. Dr. Abdul Rani Bahaman  
*Leptospirosis-A Model for  
Epidemiology, Diagnosis and Control  
of Infectious Diseases*  
16 November 1996
38. Prof. Dr. Marziah Mahmood  
*Plant Biotechnology - Strategies for  
Commercialization*  
21 December 1996
39. Prof. Dr. Ishak Hj. Omar  
*Market Relationships in the Malaysian  
Fish Trade: Theory and Application*  
22 March 1997
40. Prof. Dr. Suhaila Mohamad  
*Food and Its Healing Power*  
12 April 1997
41. Prof. Dr. Malay Raj Mukerjee  
*A Distributed Collaborative  
Environment for Distance Learning  
Applications*  
17 June 1998
42. Prof. Dr. Wong Kai Choo  
*Advancing the Fruit Industry in  
Malaysia: A Need to Shift Research  
Emphasis*  
15 May 1999
43. Prof. Dr. Aimi Ideris  
*Avian Respiratory and  
Immunosuppressive Diseases- A Fatal  
Attraction*  
10 July 1999
44. Prof. Dr. Sariah Meon  
*Biological Control of Plant Pathogens:  
Harnessing the Richness of Microbial  
Diversity*  
14 August 1999
45. Prof. Dr. Azizah Hashim  
*The Endomycorrhiza: A Futile  
Investment?*  
23 October 1999
46. Prof. Dr. Noraini Abdul Samad  
*Molecular Plant Virology: The Way  
Forward*  
2 February 2000
47. Prof. Dr. Muhamad Awang  
*Do We Have Enough Clean Air to  
Breathe?*  
7 April 2000
48. Prof. Dr. Lee Chnoong Kheng  
*Green Environment, Clean Power*  
24 June 2000
49. Prof. Dr. Mohd. Ghazali Mohayidin  
*Managing Change in the Agriculture  
Sector: The Need for Innovative  
Educational Initiatives*  
12 January 2002
50. Prof. Dr. Fatimah Mohd. Arshad  
*Analisis Pemasaran Pertanian  
di Malaysia: Keperluan Agenda  
Pembaharuan*  
26 January 2002
51. Prof. Dr. Nik Mustapha R. Abdullah  
*Fisheries Co-Management: An  
Institutional Innovation Towards  
Sustainable Fisheries Industry*  
28 February 2002
52. Prof. Dr. Gulam Rusul Rahmat Ali  
*Food Safety: Perspectives and  
Challenges*  
23 March 2002
53. Prof. Dr. Zaharah A. Rahman  
*Nutrient Management Strategies for  
Sustainable Crop Production in Acid  
Soils: The Role of Research Using  
Isotopes*  
13 April 2002



## Developing Students' Mathematical Thinking: How Far Have We Come?

54. Prof. Dr. Maisom Abdullah  
*Productivity Driven Growth: Problems & Possibilities*  
27 April 2002
55. Prof. Dr. Wan Omar Abdullah  
*Immunodiagnosis and Vaccination for Brugian Filariasis: Direct Rewards from Research Investments*  
6 June 2002
56. Prof. Dr. Syed Tajuddin Syed Hassan  
*Agro-ento Bioinformation: Towards the Edge of Reality*  
22 June 2002
57. Prof. Dr. Dahlan Ismail  
*Sustainability of Tropical Animal-Agricultural Production Systems: Integration of Dynamic Complex Systems*  
27 June 2002
58. Prof. Dr. Ahmad Zubaidi Baharumshah  
*The Economics of Exchange Rates in the East Asian Countries*  
26 October 2002
59. Prof. Dr. Shaik Md. Noor Alam S.M. Hussain  
*Contractual Justice in Asean: A Comparative View of Coercion*  
31 October 2002
60. Prof. Dr. Wan Md. Zin Wan Yunus  
*Chemical Modification of Polymers: Current and Future Routes for Synthesizing New Polymeric Compounds*  
9 November 2002
61. Prof. Dr. Annuar Md. Nassir  
*Is the KLSE Efficient? Efficient Market Hypothesis vs Behavioural Finance*  
23 November 2002
62. Prof. Ir. Dr. Radin Umar Radin Sohadi  
*Road Safety Interventions in Malaysia: How Effective Are They?*  
21 February 2003
63. Prof. Dr. Shamsher Mohamad  
*The New Shares Market: Regulatory Intervention, Forecast Errors and Challenges*  
26 April 2003
64. Prof. Dr. Han Chun Kwong  
*Blueprint for Transformation or Business as Usual? A Structural Perspective of the Knowledge-Based Economy in Malaysia*  
31 May 2003
65. Prof. Dr. Mawardi Rahmani  
*Chemical Diversity of Malaysian Flora: Potential Source of Rich Therapeutic Chemicals*  
26 July 2003
66. Prof. Dr. Fatimah Md. Yusoff  
*An Ecological Approach: A Viable Option for Aquaculture Industry in Malaysia*  
9 August 2003
67. Prof. Dr. Mohamed Ali Rajion  
*The Essential Fatty Acids-Revisited*  
23 August 2003
68. Prof. Dr. Azhar Md. Zain  
*Psychotherapy for Rural Malays - Does it Work?*  
13 September 2003
69. Prof. Dr. Mohd. Zamri Saad  
*Respiratory Tract Infection: Establishment and Control*  
27 September 2003
70. Prof. Dr. Jinap Selamat  
*Cocoa-Wonders for Chocolate Lovers*  
14 February 2004

Aida Suraya Md. Yunus

71. Prof. Dr. Abdul Halim Shaari  
*High Temperature Superconductivity:  
Puzzle & Promises*  
13 March 2004
72. Prof. Dr. Yaakob Che Man  
*Oils and Fats Analysis - Recent  
Advances and Future Prospects*  
27 March 2004
73. Prof. Dr. Kaida Khalid  
*Microwave Aquametry: A Growing  
Technology*  
24 April 2004
74. Prof. Dr. Hasanah Mohd. Ghazali  
*Tapping the Power of Enzymes-  
Greening the Food Industry*  
11 May 2004
75. Prof. Dr. Yusof Ibrahim  
*The Spider Mite Saga: Quest for  
Biorational Management Strategies*  
22 May 2004
76. Prof. Datin Dr. Sharifah Md. Nor  
*The Education of At-Risk Children:  
The Challenges Ahead*  
26 June 2004
77. Prof. Dr. Ir. Wan Ishak Wan Ismail  
*Agricultural Robot: A New Technology  
Development for Agro-Based Industry*  
14 August 2004
78. Prof. Dr. Ahmad Said Sajap  
*Insect Diseases: Resources for  
Biopesticide Development*  
28 August 2004
79. Prof. Dr. Aminah Ahmad  
*The Interface of Work and Family  
Roles: A Quest for Balanced Lives*  
11 March 2005
80. Prof. Dr. Abdul Razak Alimon  
*Challenges in Feeding Livestock:  
From Wastes to Feed*  
23 April 2005
81. Prof. Dr. Haji Azimi Hj. Hamzah  
*Helping Malaysian Youth Move  
Forward: Unleashing the Prime  
Enablers*  
29 April 2005
82. Prof. Dr. Rasedee Abdullah  
*In Search of An Early Indicator of  
Kidney Disease*  
27 May 2005
83. Prof. Dr. Zulkifli Hj. Shamsuddin  
*Smart Partnership: Plant-  
Rhizobacteria Associations*  
17 June 2005
84. Prof. Dr. Mohd Khanif Yusop  
*From the Soil to the Table*  
1 July 2005
85. Prof. Dr. Annuar Kassim  
*Materials Science and Technology:  
Past, Present and the Future*  
8 July 2005
86. Prof. Dr. Othman Mohamed  
*Enhancing Career Development  
Counselling and the Beauty of Career  
Games*  
12 August 2005
87. Prof. Ir. Dr. Mohd Amin Mohd Soom  
*Engineering Agricultural Water  
Management Towards Precision  
Framing*  
26 August 2005
88. Prof. Dr. Mohd Arif Syed  
*Bioremediation-A Hope Yet for the  
Environment?*  
9 September 2005
89. Prof. Dr. Abdul Hamid Abdul Rashid  
*The Wonder of Our Neuromotor  
System and the Technological  
Challenges They Pose*  
23 December 2005

## Developing Students' Mathematical Thinking: How Far Have We Come?

90. Prof. Dr. Norhani Abdullah  
*Rumen Microbes and Some of Their Biotechnological Applications*  
27 January 2006
91. Prof. Dr. Abdul Aziz Saharee  
*Haemorrhagic Septicaemia in Cattle and Buffaloes: Are We Ready for Freedom?*  
24 February 2006
92. Prof. Dr. Kamariah Abu Bakar  
*Activating Teachers' Knowledge and Lifelong Journey in Their Professional Development*  
3 March 2006
93. Prof. Dr. Borhanuddin Mohd. Ali  
*Internet Unwired*  
24 March 2006
94. Prof. Dr. Sundararajan Thilagar  
*Development and Innovation in the Fracture Management of Animals*  
31 March 2006
95. Prof. Dr. Zainal Aznam Md. Jelan  
*Strategic Feeding for a Sustainable Ruminant Farming*  
19 May 2006
96. Prof. Dr. Mahiran Basri  
*Green Organic Chemistry: Enzyme at Work*  
14 July 2006
97. Prof. Dr. Malik Hj. Abu Hassan  
*Towards Large Scale Unconstrained Optimization*  
20 April 2007
98. Prof. Dr. Khalid Abdul Rahim  
*Trade and Sustainable Development: Lessons from Malaysia's Experience*  
22 June 2007
99. Prof. Dr. Mad Nasir Shamsudin  
*Econometric Modelling for Agricultural Policy Analysis and Forecasting: Between Theory and Reality*  
13 July 2007
100. Prof. Dr. Zainal Abidin Mohamed  
*Managing Change - The Fads and The Realities: A Look at Process Reengineering, Knowledge Management and Blue Ocean Strategy*  
9 November 2007
101. Prof. Ir. Dr. Mohamed Daud  
*Expert Systems for Environmental Impacts and Ecotourism Assessments*  
23 November 2007
102. Prof. Dr. Saleha Abdul Aziz  
*Pathogens and Residues; How Safe is Our Meat?*  
30 November 2007
103. Prof. Dr. Jayum A. Jawan  
*Hubungan Sesama Manusia*  
7 December 2007
104. Prof. Dr. Zakariah Abdul Rashid  
*Planning for Equal Income Distribution in Malaysia: A General Equilibrium Approach*  
28 December 2007
105. Prof. Datin Paduka Dr. Khatijah Yusoff  
*Newcastle Disease virus: A Journey from Poultry to Cancer*  
11 January 2008
106. Prof. Dr. Dzulkefly Kuang Abdullah  
*Palm Oil: Still the Best Choice*  
1 February 2008
107. Prof. Dr. Elias Saion  
*Probing the Microscopic Worlds by Ionizing Radiation*  
22 February 2008

Aida Suraya Md. Yunus

108. Prof. Dr. Mohd Ali Hassan  
*Waste-to-Wealth Through  
Biotechnology: For Profit, People  
and Planet*  
28 March 2008
109. Prof. Dr. Mohd Maarof H. A. Moksin  
*Metrology at Nanoscale: Thermal  
Wave Probe Made It Simple*  
11 April 2008
110. Prof. Dr. Dzolkhifli Omar  
*The Future of Pesticides Technology  
in Agriculture: Maximum Target Kill  
with Minimum Collateral Damage*  
25 April 2008
111. Prof. Dr. Mohd. Yazid Abd. Manap  
*Probiotics: Your Friendly Gut  
Bacteria*  
9 May 2008
112. Prof. Dr. Hamami Sahri  
*Sustainable Supply of Wood and  
Fibre: Does Malaysia have Enough?*  
23 May 2008
113. Prof. Dato' Dr. Makhdzir Mardan  
*Connecting the Bee Dots*  
20 June 2008
114. Prof. Dr. Maimunah Ismail  
*Gender & Career: Realities and  
Challenges*  
25 July 2008
115. Prof. Dr. Nor Aripin Shamaan  
*Biochemistry of Xenobiotics:  
Towards a Healthy Lifestyle and Safe  
Environment*  
1 August 2008
116. Prof. Dr. Mohd Yunus Abdullah  
*Penjagaan Kesihatan Primer di  
Malaysia: Cabaran Prospek dan  
Implikasi dalam Latihan dan  
Penyelidikan Perubahan serta  
Sains Kesihatan di Universiti Putra  
Malaysia*  
8 August 2008
117. Prof. Dr. Musa Abu Hassan  
*Memfaatkan Teknologi Maklumat  
& Komunikasi ICT untuk Semua*  
15 August 2008
118. Prof. Dr. Md. Salleh Hj. Hassan  
*Role of Media in Development:  
Strategies, Issues & Challenges*  
22 August 2008
119. Prof. Dr. Jariah Masud  
*Gender in Everyday Life*  
10 October 2008
120. Prof. Dr. Mohd Shahwahid Haji  
Othman  
*Mainstreaming Environment:  
Incorporating Economic Valuation  
and Market-Based Instruments in  
Decision Making*  
24 October 2008
121. Prof. Dr. Son Radu  
*Big Questions Small Worlds:  
Following Diverse Vistas*  
31 October 2008
122. Prof. Dr. Russly Abdul Rahman  
*Responding to Changing Lifestyles:  
Engineering the Convenience Foods*  
28 November 2008
123. Prof. Dr. Mustafa Kamal Mohd  
Shariff  
*Aesthetics in the Environment an  
Exploration of Environmental:  
Perception Through Landscape  
Preference*  
9 January 2009
124. Prof. Dr. Abu Daud Silong  
*Leadership Theories, Research  
& Practices: Farming Future  
Leadership Thinking*  
16 January 2009

## Developing Students' Mathematical Thinking: How Far Have We Come?

125. Prof. Dr. Azni Idris  
*Waste Management, What is the Choice: Land Disposal or Biofuel?*  
23 January 2009
126. Prof. Dr. Jamilah Bakar  
*Freshwater Fish: The Overlooked Alternative*  
30 January 2009
127. Prof. Dr. Mohd. Zobir Hussein  
*The Chemistry of Nanomaterial and Nanobiomaterial*  
6 February 2009
128. Prof. Ir. Dr. Lee Teang Shui  
*Engineering Agricultural: Water Resources*  
20 February 2009
129. Prof. Dr. Ghizan Saleh  
*Crop Breeding: Exploiting Genes for Food and Feed*  
6 March 2009
130. Prof. Dr. Muzafar Shah Habibullah  
*Money Demand*  
27 March 2009
131. Prof. Dr. Karen Anne Crouse  
*In Search of Small Active Molecules*  
3 April 2009
132. Prof. Dr. Turiman Suandi  
*Volunteerism: Expanding the Frontiers of Youth Development*  
17 April 2009
133. Prof. Dr. Arbakariya Ariff  
*Industrializing Biotechnology: Roles of Fermentation and Bioprocess Technology*  
8 May 2009
134. Prof. Ir. Dr. Desa Ahmad  
*Mechanics of Tillage Implements*  
12 June 2009
135. Prof. Dr. W. Mahmood Mat Yunus  
*Photothermal and Photoacoustic: From Basic Research to Industrial Applications*  
10 July 2009
136. Prof. Dr. Taufiq Yap Yun Hin  
*Catalysis for a Sustainable World*  
7 August 2009
137. Prof. Dr. Raja Noor Zaliha Raja Abd. Rahman  
*Microbial Enzymes: From Earth to Space*  
9 October 2009
138. Prof. Ir. Dr. Barkawi Sahari  
*Materials, Energy and CNGDI Vehicle Engineering*  
6 November 2009
139. Prof. Dr. Zulkifli Idrus  
*Poultry Welfare in Modern Agriculture: Opportunity or Threat?*  
13 November 2009
140. Prof. Dr. Mohamed Hanafi Musa  
*Managing Phosphorus: Under Acid Soils Environment*  
8 January 2010
141. Prof. Dr. Abdul Manan Mat Jais  
*Haruan Channa striatus a Drug Discovery in an Agro-Industry Setting*  
12 March 2010
142. Prof. Dr. Bujang bin Kim Huat  
*Problematic Soils: In Search for Solution*  
19 March 2010
143. Prof. Dr. Samsinar Md Sidin  
*Family Purchase Decision Making: Current Issues & Future Challenges*  
16 April 2010

Aida Suraya Md. Yunus

144. Prof. Dr. Mohd Adzir Mahdi  
*Lightspeed: Catch Me If You Can*  
4 June 2010
145. Prof. Dr. Raha Hj. Abdul Rahim  
*Designer Genes: Fashioning Mission  
Purposed Microbes*  
18 June 2010
146. Prof. Dr. Hj. Hamidon Hj. Basri  
*A Stroke of Hope, A New Beginning*  
2 July 2010
147. Prof. Dr. Hj. Kamaruzaman Jusoff  
*Going Hyperspectral: The "Unseen"  
Captured?*  
16 July 2010
148. Prof. Dr. Mohd Sapuan Salit  
*Concurrent Engineering for  
Composites*  
30 July 2010
149. Prof. Dr. Shattri Mansor  
*Google the Earth: What's Next?*  
15 October 2010
150. Prof. Dr. Mohd Basyaruddin Abdul  
Rahman  
*Haute Couture: Molecules &  
Biocatalysts*  
29 October 2010
151. Prof. Dr. Mohd. Hair Bejo  
*Poultry Vaccines: An Innovation for  
Food Safety and Security*  
12 November 2010
152. Prof. Dr. Umi Kalsom Yusuf  
*Fern of Malaysian Rain Forest*  
3 December 2010
153. Prof. Dr. Ab. Rahim Bakar  
*Preparing Malaysian Youths for The  
World of Work: Roles of Technical  
and Vocational Education and  
Training (TVET)*  
14 January 2011
154. Prof. Dr. Seow Heng Fong  
*Are there "Magic Bullets" for  
Cancer Therapy?*  
11 February 2011
155. Prof. Dr. Mohd Azmi Mohd Lila  
*Biopharmaceuticals: Protection,  
Cure and the Real Winner*  
18 February 2011
156. Prof. Dr. Siti Shapor Siraj  
*Genetic Manipulation in Farmed  
Fish: Enhancing Aquaculture  
Production*  
25 March 2011
157. Prof. Dr. Ahmad Ismail  
*Coastal Biodiversity and Pollution:  
A Continuous Conflict*  
22 April 2011
158. Prof. Ir. Dr. Norman Mariun  
*Energy Crisis 2050? Global  
Scenario and Way Forward for  
Malaysia*  
10 June 2011
159. Prof. Dr. Mohd Razi Ismail  
*Managing Plant Under Stress: A  
Challenge for Food Security*  
15 July 2011
160. Prof. Dr. Patimah Ismail  
*Does Genetic Polymorphisms Affect  
Health?*  
23 September 2011
161. Prof. Dr. Sidek Ab. Aziz  
*Wonders of Glass: Synthesis,  
Elasticity and Application*  
7 October 2011
162. Prof. Dr. Azizah Osman  
*Fruits: Nutritious, Colourful, Yet  
Fragile Gifts of Nature*  
14 October 2011

## Developing Students' Mathematical Thinking: How Far Have We Come?

163. Prof. Dr. Mohd. Fauzi Ramlan  
*Climate Change: Crop Performance and Potential*  
11 November 2011
164. Prof. Dr. Adem Kiliçman  
*Mathematical Modeling with Generalized Function*  
25 November 2011
165. Prof. Dr. Fauziah Othman  
*My Small World: In Biomedical Research*  
23 December 2011
166. Prof. Dr. Japar Sidik Bujang  
*The Marine Angiosperms, Seagrass*  
23 March 2012
167. Prof. Dr. Zailina Hashim  
*Air Quality and Children's Environmental Health: Is Our Future Generation at Risk?*  
30 March 2012
168. Prof. Dr. Zainal Abidin Mohamed  
*Where is the Beef? Vantage Point form the Livestock Supply Chain*  
27 April 2012
169. Prof. Dr. Jothi Malar Panandam  
*Genetic Characterisation of Animal Genetic Resources for Sustainable Utilisation and Development*  
30 November 2012
170. Prof. Dr. Fatimah Abu Bakar  
*The Good The Bad & Ugly of Food Safety: From Molecules to Microbes*  
7 December 2012
171. Prof. Dr. Abdul Jalil Nordin  
*My Colourful Sketches from Scratch: Molecular Imaging*  
5 April 2013
172. Prof. Dr. Norlijah Othman  
*Lower Respiratory Infections in Children: New Pathogens, Old Pathogens and The Way Forward*  
19 April 2013
173. Prof. Dr. Jayakaran Mukundan  
*Steroid-like Prescriptions English Language Teaching Can Ill-afford*  
26 April 2013
174. Prof. Dr. Azmi Zakaria  
*Photothermals Affect Our Lives*  
7 June 2013
175. Prof. Dr. Rahinah Ibrahim  
*Design Informatics*  
21 June 2013
176. Prof. Dr. Gwendoline Ee Cheng  
*Natural Products from Malaysian Rainforests*  
1 November 2013
177. Prof. Dr. Noor Akma Ibrahim  
*The Many Facets of Statistical Modeling*  
22 November 2013
178. Prof. Dr. Paridah Md. Tahir  
*Bonding with Natural Fibres*  
6 December 2013
179. Prof. Dr. Abd. Wahid Haron  
*Livestock Breeding: The Past, The Present and The Future*  
9 December 2013
180. Prof. Dr. Aziz Arshad  
*Exploring Biodiversity & Fisheries Biology: A Fundamental Knowledge for Sustainable Fish Production*  
24 January 2014
181. Prof. Dr. Mohd Mansor Ismail  
*Competitiveness of Beekeeping Industry in Malaysia*  
21 March 2014

Aida Suraya Md. Yunus

182. Prof. Dato' Dr. Tai Shzee Yew  
*Food and Wealth from the Seas: Health Check for the Marine Fisheries of Malaysia*  
25 April 2014
183. Prof. Datin Dr. Rosenani Abu Bakar  
*Waste to Health: Organic Waste Management for Sustainable Soil Management and Crop Production*  
9 May 2014
184. Prof. Dr. Abdul Rahman Omar  
*Poultry Viruses: From Threat to Therapy*  
23 May 2014
185. Prof. Dr. Mohamad Pauzi Zakaria  
*Tracing the Untraceable: Fingerprinting Pollutants through Environmental Forensics*  
13 June 2014
186. Prof. Dr. -Ing. Ir. Renuganth Varatharajoo  
*Space System Trade-offs: Towards Spacecraft Synergisms*  
15 August 2014
187. Prof. Dr. Latiffah A. Latiff  
*Transformasi Kesihatan Wanita ke Arah Kesejahteraan Komuniti*  
7 November 2014
188. Prof. Dr. Tan Chin Ping  
*Fat and Oils for a Healthier Future: Makro, Micro and Nanoscales*  
21 November 2014
189. Prof. Dr. Suraini Abd. Aziz  
*Lignocellulosic Biofuel: A Way Forward*  
28 November 2014
190. Prof. Dr. Robiah Yunus  
*Biobased Lubricants: Harnessing the Richness of Agriculture Resources*  
30 January 2015
190. Prof. Dr. Khozirah Shaari  
*Discovering Future Cures from Phytochemistry to Metabolomics*  
13 February 2015
191. Prof. Dr. Tengku Aizan Tengku Abdul Hamid  
*Population Ageing in Malaysia: A Mosaic of Issues, Challenges and Prospects*  
13 March 2015
192. Prof. Datin Dr. Faridah Hanum Ibrahim  
*Forest Biodiversity: Importance of Species Composition Studies*  
27 March 2015
192. Prof. Dr. Mohd Salleh Kamarudin  
*Feeding & Nutritional Requirements of Young Fish*  
10 April 2015
193. Prof. Dato' Dr. Mohammad Shatar Sabran  
*Money Boy: Masalah Sosial Era Generasi Y*  
8 Mei 2015



→ **THE** INAUGURAL LECTURES are given by honored faculty members within the University who have obtained the rank of full professor. This event gives the honoree the opportunity to deliver a lecture to fellow faculty and other university guests concerning their work and research interests.

The context of the lecture itself typically includes a summary of the evolution and nature of the honoree's specialized field, highlights of some of the general issues of that particular field, and a description of how the honoree situates his/her work within their field.

UPM conducts this event to highlight and bring attention to the scholarly work that is being done by its distinguished faculty and to illustrate how the work contributes to mankind as a whole.