UNIVERSITI PUTRA MALAYSIA

A HIGH-ORDER COMPACT FINITE DIFFERENCE SOLVER FOR THE TWO-DIMENSIONAL EULER AND NAVIER-STROKER EQUATIONS

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By

MAHMOOD KHALID MAWLOOD

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

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To my mother, my wife and our children:
Zainab, Zubaida, Batool, Mohammed and Hajar

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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The objective of this study was to develop a high-order compact (HOC) finite difference solver for the two-dimensional Euler and Navier-Stokes equations. Before developing the solver, a detailed investigation was conducted for assessing the performance of the basic fourth-order compact central discretization schemes that are known as Hermitian or Padé schemes. Exact solutions of simple scalar model problems, including the one-dimensional viscous Burgers equation and two-dimensional convection-diffusion equation were used to quantitatively establish the spatial convergence rate of these schemes. Examples of two-dimensional incompressible flow including the driven cavity and the flow past a backward facing step were used for qualitatively evaluating the accuracy of the discretizations. Resolution properties of the HOC and conventional schemes were demonstrated through Fourier analysis. Stability criteria for explicit integration of the convection-diffusion equation were derived using the von-Neumann method and validated.

Due to aliasing errors associated with the central HOC schemes investigated, these were only used for the discretization of the viscous terms of the Navier-Stokes equations in developing the aimed solver. Dealiasing HOC methods were developed for the discretization of the Euler equations and the convective terms of the Navier-Stokes equations.

Spatial discretization of the Euler equations was based on flux-vector splitting. A fifth-order compact upwind method with consistent boundary closures was developed for the Euler equations. Shock-capturing properties of the method were based on the idea of total variation diminishing (TVD). The accuracy, stability and shock capturing issues of the developed method were investigated through the solution of one-dimensional scalar conservation laws.

Discretization of the convective flux terms of the Navier-Stokes equations was based on a hybrid flux-vector splitting, known as the advection upstream splitting method (AUSM), which combines the accuracy of flux-difference splitting and the robustness of flux-vector splitting. High-order accurate approximation to the derivatives was obtained by a fourth-order cell-centered compact scheme. The mid-point values of the staggered mesh were constructed using a fourth-order MUSCL (monotone upstream-centered scheme for conservation law) polynomial.

Two temporal discretization methods were built into the developed solver. Explicit integration was performed using a multistage strong stability preserving (SSP) Runge-Kutta method for unsteady time-accurate flow problems. For steady state flows an implicit method using the lower-upper (LU) factorization scheme with local time stepping convergence accelerator was employed.

An advanced two-equation turbulence model, known as k-ω shear-stress-transport (SST), model has also been incorporated in the solver for computing turbulent flows.

A wide variety of test problems in unsteady and steady state were solved to demonstrate the accuracy, robustness and the capability to preserve positivity of the developed solver. Although the main solver was developed for two-dimensional problems, a one-dimensional version of it has been used to solve some interesting and challenging one-dimensional test problems as well. The test problems considered contain various types of discontinuities such as shock waves, rarefaction waves and contact surfaces and complicated wave interaction phenomena. Quantitative and qualitative comparisons with exact solutions, other numerical results or experimental data, whichever is available, are presented.

The tests and comparisons conducted have shown that the developed HOC methods and the solver are high-order accurate and reliable as an application CFD code for two-dimensional compressible flows and conducting further research. A number of avenues for further research are identified and proposed for future extension and improvement of the solver.

Abstrak tesis dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

PEMBENTUKAN MODEL PENYELESAIAN BEZAAN PADAT TERHINGGA PERINGKAT TINGGI UNTUK PERSAMAAN-PERSAMAAN EULER DAN NAVIER-STOKES

Oleh

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Objektif kajian ini ialah untuk membentuk model penyelesaian perbezaan terhingga peringkat tinggi padat (high order compact) bagi persamaan Euler dan Navier Stokes dua dimensi. Satu penyiasatan rapi telah dijalankan bagi menilai keupayaan skema pendiskretan padat peringkat keempat asas pusat atau dikenali sebagai skema Hermitian atau Pade' sebelum penyelesai dibina. Penyelesaian tepat bagi permasalahan model skala ringkas, termasuk persamaan satu dimensi kelikatan Burgers dan persamaan dua dimensi resapan olakan telah digunakan secara kualitatif untuk membentuk kadar ruang tumpu skema-skema ini. Contoh-contoh bagi aliran tidak boleh mampat dua dimensi termasuk ruang terpandu dan aliran melepasi langkah undur digunakan untuk menilai secara kualitatif ketepatan pendiskretan. Peleraian sifat-sifat HOC dan skema-skema konvensional diperlihatkan melalui analisis Fourier. Kriteria stabiliti untuk intergrasi tak tersirat persamaan resapan olakan diterbitkan dan disahkan menggunakan kaedah von-Neumann.

Disebabkan kesilapan penamaan berkaitan skema pusat HOC yang disiasat, perkara ini hanya digunakan untuk pendiskretan istilah likat dalam persamaan Navier Stokes semasa membentuk penyelesai yang dicari. Kaedah HOC untuk membetulkan kesalahan penamaan dibentuk untuk pendiskretan persamaan Euler dan istilah konvektif persamaan- persamaan Navier Stokes.

Pendiskretan ruang untuk persamaan Euler adalah berasaskan pemisahan fluks-vektor. Kaedah arah angin padat peringkat kelima dengan penutupan sempadan secara konsisten dibentuk untuk persamaan Euler. Sifat tahan gegaran kaedah ini adalah berasaskan idea pengurangan variasi secara total (TVD). Isu-isu ketepatan, kestabilan dan ketahanan terhadap gegaran kaedah yang telah dibentuk telah disiasat melalui penyelesaian hukum-hukum pemulihan skalar satu dimensi.

Istilah pendiskretan fluks berolak persamaan Navier-Stokes adalah berasaskan pemecahan vektor-fluks hybrid yang dikenali sebagai kaedah pemecahan 'advection' hulu yang menggabungkan ketepatan pemecahan perbezaan fluks dan kekuatan pemecahan vektor-fluks. Anggaran tepat peringkat tinggi untuk hasil terbitan diperolehi oleh skema padat berpusatkan sel peringkat keempat. Nilai-nilai pertengahan jaringan berperingkat dibentuk menggunakan polynomial (MUSCL) peringkat keempat.

Dua kaedah pendiskretan temporal dimasukkan ke dalam penyelesai yang telah dibentuk. Intergrasi tak tersirat dilakukan dengan menggunakan kaedah pengawetan kestabilan kuat pelbagai peringkat Runge-Kutta untuk masalah aliran masa tepat yang tidak stabil. Untuk aliran stabil, kaedah tersirat menggunakan skema pemfaktoran bawah-atas dengan meninggikan pecutan penumpuan waktu tempatan digunakan. Model dua persamaan gelora maju dikenali sebagai k-ω pengangkutan tegasan ricih juga telah dimasukkan ke dalam penyelesai untuk membetulkan aliran bergelora.

Penyelesai telah diuji dengan pelbagai masalah pengujian dalam bentuk stabil dan tidak stabil, bagi membuktikan ketepatan, kekuatan dan keupayaan mengekalkan nilai positif penyelesai yang dibentuk itu. Masalah pengujian yang telah diambil kira untuk menguji penyelesai antaranya ialah pelbagai jenis ketidaksinambungan seperti gelombang kejutan, gelombang kumpulan nadir dan permukaan sentuh serta fenomena interaksi gelombang rumit. Perlu dinyatakan bahawa walaupun penyelesai utama dibina untuk menyelesaikan masalah – masalah dua dimensi, versi penyelesai satu dimensi juga telah diguna untuk menyelesaikan masalah – masalah pengujian satu dimensi yang menarik serta mencabar. Setelah selesai pengujian perbandingan kuantitatif serta kualitatif dengan penyelesaian yang tepat, keputusan numerikal yang lain serta mana – mana data eksperimental yang diperolehi telah diperlihatkan.

Ujian-ujian dan perbandingan yang telah dijalankan menunjukkan bahawa kaedah HOC dan penyelesai yang dibina adalah agak tepat dan boleh digunapakai sebagai aplikasi kod CFD untuk aliran bolehmampat dan juga untuk menjalankan kajian-kajian seterusnya. Beberapa ruang untuk kajian seterusnya telah dikenalpasti dan pengembangan serta perbaikan penyelesai juga dicadangkan.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

a, b, c	Coefficients in high-order compact schemes
a	Cross-sectional area of nozzle
a_1	Turbulence closure constant = 0.31
$A^{\mp}, B^{\mp}$	Flux Jacobian matrices
A, B	Discretization coefficient matrices
b, b_1	Flux limiting parameters in MUSCL scheme
c	Sonic speed
c_f	Corrected sonic speed
c_p	Specific heat at constant pressure
C	Chord length, wave speed
C_f	Coefficient of friction
C_L	Coefficient of lift
C_D	Coefficient of drag
C_D, C_k	Constants in turbulence closure
C_p	Coefficient of pressure
C_x, C_y	Courant number
$d\hat{f}$	Flux difference
D	Diffusion number
D_k, D_w	Decay terms in k-ω SST equations
e	Total energy
E	Expansion ratio of backward facing step

E, E_v	Flux vectors
$\widetilde{E}, \widetilde{E}_v$	Flux vectors in transformed coordinates
$\widetilde{E}_t, \widetilde{F}_t$	Flux vectors of turbulence equations
$\widetilde{E}', \widetilde{E}'_v$	First derivatives of flux vectors
f	Scalar flux
$\hat{f}$	Flux function
F_1, F_2	Turbulence model functions
$F_i, \overline{F}_i, F_i^*$	High-order approximation to first derivative
F, F_v	Flux vectors
$\widetilde{F}, \widetilde{F}_v$	Flux vectors in transformed coordinates
$\widetilde{F}', \widetilde{F}'_v$	First derivatives of flux vectors
$FX\psi$	First derivative of ψ with respect to x
$FY\psi$	First derivative of ψ with respect to y
g	Vector defining boundary dependence
G	Amplification factor
h	Mesh size
H	Total enthalpy, channel height
$\widetilde{H}$	Source vector in turbulence equations
I	Unit (identity) matrix
I	$\sqrt{-1}$
J	Jacobian of transformation
k	Turbulent kinetic energy
K	Thermal conductivity

l	Turbulence length scale, also mixing length
L	Characteristic length
L, U	Lower-upper factorization matrices
M	Mach number
$M^{\mp}$	Split mach number
n	Normal direction
n_1	Normal distance between the surface and first grid point
N	Maximum number of grid points
p	Pressure
$p^{\mp}$	Split pressure terms
P	Matrix
P_k, P_w	Production terms in turbulence model
Pr	Prandtl number
q	Heat flux vector
Q	Conservative variable vector
$\tilde{Q}$	Conservative variable vector $= Q / J$
$\tilde{Q}_t$	Conservative variable vector of turbulence equations
$R, \bar{R}$	Residual
R	Specific gas constant
$R^{\mp}$	Characteristic variables
$\tilde{R}, \tilde{S}$	Flux vectors of the k-ω equations
Re	Reynolds number
s	Source term