UNIVERSITI PUTRA MALAYSIA

SIMPLE MOTION PURSUIT DIFFERENTIAL GAME OF MANY PURSUERS AND ONE EVADER ON CONVEX COMPACT SET

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By

RAJA NOORSURIA BINTI RAJA RAMLI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Master of Science

May 2016

DEDICATIONS

Special dedicated to;
Ma & Ku
Beloved lecturers
Siblings & Friends
and to those who knowing me

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the Degree of Master of Science

SIMPLE MOTION PURSUIT DIFFERENTIAL GAME OF MANY PURSUERS AND ONE EVADER ON CONVEX COMPACT SET

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RAJA NOORSURIA BINTI RAJA RAMLI

May 2016

Chairman : Gafurjan Ibragimov, PhD
Institute : Mathematical Research

A pursuit differential game of m pursuers and single evader in nonempty closed bounded convex subset of $\mathbb{R}^n$ is studied. At this juncture, all players must not leave the given set A which is subset of $\mathbb{R}^n$ and control parameters of all players are subjected to geometric constraints. All players move with speeds less than or equal to 1. We say that pursuit is completed if geometric position of at least one pursuer coincides with that of the evader. In this game, pursuers try to complete the pursuit.

The problem of this study is to obtain the estimation for guaranteed pursuit time (GPT). To solve this problem, first, we construct strategies for the pursuers in n-dimensional cube. Then, reduce the problem to the game in the cube and apply the method of fictitious pursuers. In this thesis, we improve the estimation of GPT from third degree polynomial $O(n^3)$ to second degree polynomial $O(n^2)$.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk Ijazah Master Sains

GERAKAN MUDAH PERMAINAN PEMBEZAAN PEMBURUAN BAGI RAMAI PEMANGSA DAN SATU MANGSA DALAM SET KONVEKS KOMPEK

Oleh

RAJA NOORSURIA BINTI RAJA RAMLI

Mei 2016

Pengerusi : Gafurjan Ibragimov, PhD
Institut : Penyelidikan Matematik

Kami mengkaji satu permainan pembezaan yang melibatkan m pemangsa dan satu mangsa dalam subset konveks dibatasi tertutup $\mathbb{R}^n$ yang bukan set kosong. Semasa permainan, semua pemain tidak boleh meninggalkan set A, iaitu suatu subset $\mathbb{R}^n$, yang telah ditetapkan dan parameter kawalan semua pemain adalah tertakluk kepada kekangan geometri. Semua pemain bergerak dengan kelajuan kurang atau bersamaan dengan 1. Pemburuan dikatakan selesai jika kedudukan geometri sekurang-kurangnya seorang pemangsa sekena dengan kedudukan geometri mangsa. Pemangsa berusaha menyelesaikan pemburuan dalam permainan ini.

Masalah kajian ini adalah untuk mendapatkan anggaran jaminan masa pemburuan. Untuk menyelesaikan masalah ini, pertama sekali, kami merangka strategi bagi pemangsa di dalam kubus berdimensi n. Kemudian, kami selesaikan masalah kajian ini kepada permainan di dalam kubus tersebut dengan mengaplikasikan kaedah pemangsa rekaan. Dalam tesis ini, kami memperbaiki anggaran jaminan masa pemburuan (GPT) daripada polinomial darjah tiga $O(n^3)$ kepada polinomial darjah dua $O(n^2)$.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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This is to confirm that:

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LIST OF ABBREVIATIONS

$\mathbb{R}^d$	the d-dimensional Euclidean space with the standard basis
$int conv A$	the point interior to the convex hull of a set A
$A\langle, B\rangle$	A (respectively B)
$P\langle, E\rangle$	the pursuer (the evader)
$x\langle, y\rangle$	the radius-vector of $P\langle, E\rangle$
$u\langle, v\rangle$	the parameter controlled by $P\langle, E\rangle$
$u(\cdot)\langle, v(\cdot)\rangle$	the admissible control functions for $P\langle, E\rangle$
$\|u\|\langle, \|v\|\rangle$	the velocities of $P\langle, E\rangle$
$O(n^3)\langle, O(n^2)\rangle$	the third degree polynomial (the second degree polynomial)
$H(0,1)$	the ball of a radius 1 in $\mathbb{R}^d$ and centered at origin $(0,0)$
AC	absolutely continuous
GPT	guaranteed pursuit time
INSPEM	Institut Penyelidikan Matematik

CHAPTER 1

INTRODUCTION

1.1 Summary of the chapter

This chapter is divided into four sections. First section simply explains the definition of differential game, the types of differential games and the constraints that used in differential games. Then, the second section describes in brief the application of differential games in navigation problem. The third and fourth sections discuss the pursuit and evasion differential game of lion and man problem. The last section gives in summary the outline of each chapter in this thesis.

1.2 Introduction to Differential Game

Hajek (2008) divides game into two different parts, which are stochastic game and deterministic game. Differential game is a branch of deterministic game and it is defined as a group of problems that relates to the modeling and analysis of conflict in the context of a dynamical system. The problem usually consists of two kind of players, which are pursuer and evader, with conflicting goals. The degree of differential game can be either two-player zero-sum differential game or multi-objectives differential game. In this research, we are focusing on two-player zero-sum differential game.

The trajectories of the players are modeled by any systems of differential equations such as simple differential equations, linear differential equations and etc. For our research, the trajectories of all players are described by a simple system of differential equations. As for the control functions of all players, they can be subjected to any constraints like geometric constraints, integral constraints, coordinate-wise integral constraints, complex constraints and state constraints.

In our study, the control functions of all players are subjected to geometric constraints and the movements of all players are under the state constraints. Plainly, geometric constraints mean the restriction on the vector for velocity of all players. Normally, the vector for velocity of pursuer and evader are denoted by u and v respectively, and in this thesis they are defined by the Definitions 2.5.1 and 2.5.2. Besides, the movements of pursuer and evader are showed by the changing of the state variables x and y respectively. So, in general, the state constraints mean the limitation on the movements of all players in a given set.

There are three type of differential games, which are:

1. Pursuit differential game. In this game, we find the conditions of completion of pursuit. Then, we construct the strategy of pursuer and find GPT. Our research

is considered this type of game.

2. Evasion differential game. In this game, one finds conditions that guarantee evader to avoid from being captured by the pursuer at all time $t \geq 0$. Then, the strategy of evader is constructed.

3. Pursuit-evasion differential game. In this game, we consider separately completion of pursuit and possibility of evasion of the players involved in the game. Thus strategy of pursuer and evader are be constructed for the pursuit and evasion game respectively.

In this thesis, we study a pursuit differential game problem that involve many pursuers and one evader under geometric and state constraints.

1.3 The Real Life Application of Differential Game

The study of this field yields a great deal of useful applications in real life such as in artificial intelligence, economic branches, military, surveillance security and many more. One of the application that was discussed by Hajek (2008) in his book entitled ``Pursuit Game´´ is a Navigation Problem. The Navigation Problem is about an airplane whose speed is relatively constant to the moving air mass in unbounded plane, $\mathbb{R}^2$, and the wind was given as a vector valued function of position and time (Hajek, 2008; Zemerlo, 2013).

The question that arise in this problem is how the airplane should be steered so as to arrive a given targeting point from a given starting point in shortest time. Zemerlo (2013) has solved this problem by using the calculus of variation. Meanwhile, Hajek (2008) did some modifications before solving this problem in differential game. The Navigation problem can be illustrated as below.

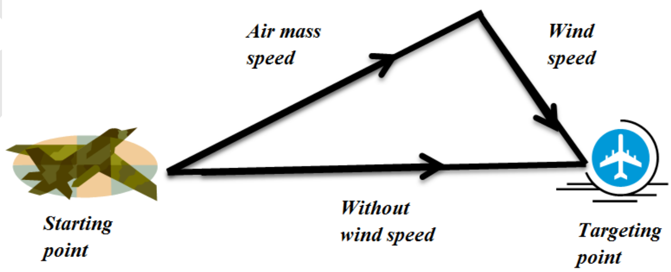

Figure 1.1: Illustration of the Navigation problem.

1.4 Lion and Man game

Lion and Man game was one of the early and interesting example of differential game problems that involve one pursuer and one evader with geometric constraints under a closed circular arena as the state constraint. This problem was firstly proposed by Rado (1953) and later was extended by many researchers from past and present such as Besicovitch (1953), Croft (1964), Flyn (1973, 1974), Lewin (1986) and recently, by Azamov and Kuchkarov (2009).

In this game, lion and man play roles of pursuer and evader, respectively. The lion aims to catch the man while the man aims to avoid from being captured by the lion. Both lion and man cannot leave the given closed circular arena. The lion and man have similar motion capabilities and move with speeds at most 1. In addition, the lion and man have the informations about each other position. Lion and Man game is example of pursuit-evasion differential game where we consider the pursuit and evasion problem separately.

1.4.1 Pursuit in Lion and Man Game by Rado (1953)

In this game, it is assumed that the man always moves on circumference of the circular arena whereas the lion moves in the circular arena. The strategy for the lion to capture the man is first, by moving to the center of the circular arena. Thus, the initial position of lion is assumed to be at the center. The lion then will ensure that the man position are always on the same radius at all time until the man is captured. This strategy is called Radial Strategy, introduced by Rado (1953).

The Figure 1.2 illustrates the movements of lion and man. In this illustration, $x(t)$ and $y(t)$ are the distances of lion and man at time t from the center of circular arena, O, respectively. Note that $x(t) < y(t)$ and $y(t) = R$ where R is the radius of the circular arena.

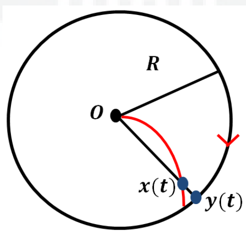

Figure 1.2: Illustration of lion and man game.

Let $e(t)$ be a unit vector directed to position of the pursuer $P(t)$, $e^{\perp}(t)$ be orthonormal vector to $e(t)$, and $u(t) = u_n(t)e(t) + u_{\tau}(t)e^{\perp}(t)$, and $v(t) = v_n(t)e(t) + v_{\tau}(t)e^{\perp}(t)$ be the decompositions of $u(t)$ and $v(t)$. Then, in general, the radial strategy can be illustrated as in the Figure 1.3 below.

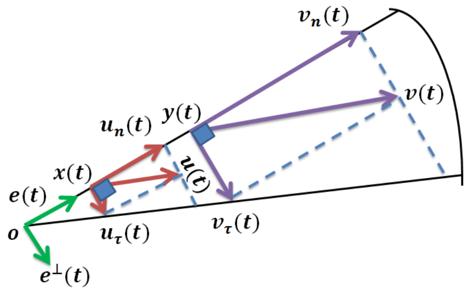

Figure 1.3: The Radial strategy.

According to the paper Azamov and Kuchkarov (2009), the angular speeds of lion and man are equal, described by the equation

$$\frac{u_\tau(t)}{x(t)} = \frac{v_\tau(t)}{y(t)}. \tag{1.1}$$

By equation (1.1), the radial strategy is defined as follows:

$$u(t) = \xi(t)v_\tau(t)e^\perp(t) + e(t)\sqrt{1-\xi^2(t)v_\tau^2(t)}, \tag{1.2}$$

where $\xi(t) = \dfrac{x(t)}{y(t)} = \dfrac{x(t)}{R}$.

Now, we discuss of Rado's theorem.

Theorem 1.1 *(Rado, 1953). Pursuit can be completed for the time* $t = \dfrac{\pi R}{2}$.

Proof:
We have

$$\dot{x}(t) = u_n(t), \;\; x(0) = 0, \tag{1.3}$$

where $x(0)$ is the initial point of the pursuer at the center of the circular arena.

Referring to the radial strategy in (1.2), it is clear that

$$u_\tau(t) = \xi(t)v_\tau(t), \;\; u_n(t) = \sqrt{1-\xi^2(t)v_\tau^2(t)}. \tag{1.4}$$

Substitute the equation of (1.4) into the equation of (1.3), and since $v_\tau^2 \leq 1$ we get

$$\begin{aligned} \dot{x}(t) &= \sqrt{1-\xi^2(t)v_\tau^2(t)} \\ &\geq \sqrt{1-\xi^2(t)}. \end{aligned} \tag{1.5}$$

As $\xi(t) = \dfrac{x(t)}{R}$, the inequality (1.5) can be written as

$$\int_0^t \frac{dx}{\sqrt{1-\frac{x^2}{R^2}}} \geq \int_0^t ds. \tag{1.6}$$

Hence, in view of $x(0) = 0$, we obtain

$$R\arcsin\left(\frac{x(t)}{R}\right) \geq t. \tag{1.7}$$

Since $\arcsin\left(\dfrac{x(t)}{R}\right) \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$, then $\arcsin\left(\dfrac{x(\tau)}{R}\right) = \dfrac{\pi}{2}$ at some time $t = \tau \in \left[0, \dfrac{R\pi}{2}\right]$ which implies that $\dfrac{x(\tau)}{R} = 1$, and so $x(\tau) = R$ since $x(t)$ and $y(t)$ belong to one radius for all $t \geq 0$, therefore $x(\tau) = y(\tau)$. Proof of Theorem 1.1 is complete. ■

1.4.2 Evasion in Lion and Man Game by Besicovitch (1953)

Besicovitch (1953) proposed a theorem that evasion is possible for lion and man game. Here we will discuss the proof of Besicovitch′s theorem with some modifications.

Theorem 1.2 *(Besicovitch, 1953). Evasion is possible in the game of Lion and Man.*

Proof:
1. Construction the strategy of the man.

The Figure 1.4 illustrates the positions of lion and man in the circular arena. We denote lion and man by P and E respectively. We assume that the movement of E is always inside the circular arena and behavior of P is any.

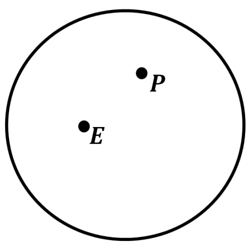

Figure 1.4: The positions of all players inside the circle.

Let t_i be a time at which the distance of E from the circumference is equal to $\frac{r}{i+1}$, where r is a distance of E from the circumference at $t_0 = 0$ and $i = 0, 1, 2, 3, \ldots$. Let l_i be the line passing through the center of the circle, O, and the position of E at time t_i, denoted by E_i, and P_i be the position of P at time t_i. The three possible movements of E are:

1. If P_i is to the right of l_i, then E moves to the left from E_i perpendicularly to l_i.
2. If P_i is to the left of l_i, then E moves to the right from E_i perpendicularly to l_i.
3. If P_i is lie on l_i, then E moves either to the left or to the right from E_i perpendicularly to l_i.

The Figure 1.5 illustrates the possible movements of E.

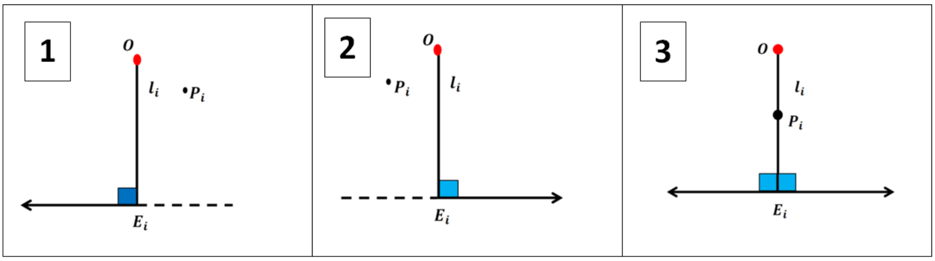

Figure 1.5: The possible movements of E.

Since all three possible movements of E are identical to each other, thus, without any loss of generality, we assume that P_i is always to the right of l_i or lie on l_i. Then, E moves to the left from E_i perpendicularly to l_i (see picture no. 1 in Figure 1.5). Thus, the trajectory of E and P can be illustrated as the figure below.

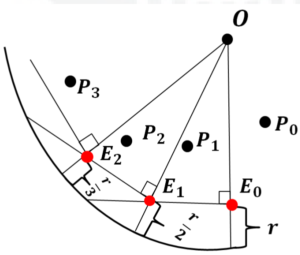

Figure 1.6: The trajectory of P and E inside the circular arena.

2. Evasion is possible on each section E_iE_{i+1}.

Here, we prove that evasion is possible for each section E_iE_{i+1}. The section E_iE_{i+1} is illustrated as follow:

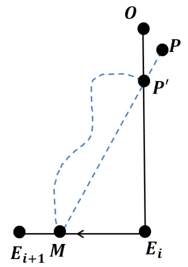

Figure 1.7: The trajectory of E on section E_iE_{i+1}.

To prove that E can avoid from being captured by P in each section E_iE_{i+1}, we assume the contrary. Let pursuit can be completed at a point $M \in E_iE_{i+1}$ at some time $\tau \in [t_i, t_{i+1}]$. Mathematically, we write

$$P(\tau) = E(\tau) = M$$

We let P moves with speed $\alpha(t)$, where $\alpha(t) \leq 1$. As all players move with the maximum speed 1, so the time taken of P to reach the point M is calculated as follow,

$$PM = \int_{t_i}^{\tau} \alpha(s)\, ds \leq \int_{t_i}^{\tau} 1\, ds = \tau - t_i.$$

Meanwhile, the time taken of E from point E_i to point M is exactly $\tau - t_i$.

Hence, we can see that

$$\tau - t_i = E_iM < P'M \leq PM \leq \tau - t_i = E_iM.$$

The above relations show contradiction where $E_iM < E_iM$. So, we can conclude that evasion is possible on each section E_iE_{i+1}.

3. Estimation of the total time.

To complete the proof of this theorem, we need to show that the total time is infinite (see Figure 1.8).

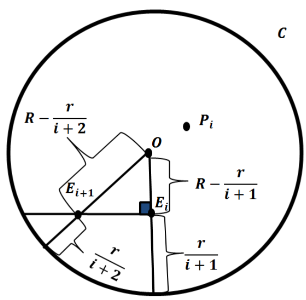

Figure 1.8: Figure of estimation total time.

We know that the time taken of E to move from E_i to E_{i+1} is

$$t_i = \frac{E_iE_{i+1}}{1} = E_iE_{i+1} = \sqrt{\left(R - \frac{r}{i+2}\right)^2 - \left(R - \frac{r}{i+1}\right)^2} \tag{1.8}$$

where R be the radius of the circle and $i = 0, 1, 2, 3, ..., n$. Thus, we will get

$$t_0 = E_0E_1 = \sqrt{\left(R - \frac{r}{2}\right)^2 - \left(R - r\right)^2}$$

$$t_1 = E_1E_2 = \sqrt{\left(R - \frac{r}{3}\right)^2 - \left(R - \frac{r}{2}\right)^2}$$

$$\vdots$$

$$t_n = E_nE_{n+1} = \sqrt{\left(R - \frac{r}{n+2}\right)^2 - \left(R - \frac{r}{n+1}\right)^2}.$$

We can show that expression (1.8) is greater than or equal to $\frac{r}{i+2}$, that is

$$\sqrt{\left(R - \frac{r}{i+2}\right)^2 - \left(R - \frac{r}{i+1}\right)^2} \geq \frac{r}{i+2}. \tag{1.9}$$

Hence, the total time can be estimated by the following series,

$$\sum_{i=1}^{\infty} t_i \geq \sum_{i=1}^{\infty} \frac{r}{i+2} = r \cdot \sum_{i=1}^{\infty} \frac{1}{i+2} = \infty.$$

Therefore evasion is possible in the Lion and Man game. ∎

1.5 Outline of thesis

This thesis covers six chapters with the following contents:

Chapter 1 describes briefly about the differential games and their applications. The pursuit and evasion differential game of lion and man problem by Rado (1953) and Besicovitch (1953) is also discussed here. The outline of the thesis is included in this chapter.

Chapter 2 focuses on the previous work done by many researchers. At the beginning, this chapter introduces a famous and early pioneer that started studies this field systematically Isaacs (1965) and describes generally the first differential game problem that analyzed by that pioneer. Then, this chapter reviews the solutions and results of the previous works on the problem of simple motion differential game of many pursuers versus one or several evaders with geometric constraints. The scope of the study, the problem statement, the definitions and the objectives of the study are also included in this chapter.

Chapter 3 discusses the P-strategy which includes definition of P-strategy, and construction of P-strategy in special and general cases. This chapter also explains paper of Pshenichnii (1976) in details.

Chapter 4 introduces the differential game of many pursuers versus one evader with geometric constraints under specified state constraint.

Chapter 5 discusses the solution of the main problem of thesis. For the first section in this chapter, the problem is studied in the n-dimensional cube and it was obtained second degree polynomial for guaranteed pursuit time (GPT), denoted by T. Then, the main problem is reduced to the game in the cube and method of fictitious pursuers is used.

Chapter 6 gives brief and precise conclusion on this thesis about.

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