



**UNIVERSITI PUTRA MALAYSIA**

***A Q-METHODOLOGY APPROACH IN DETERMINING PERSPECTIVES  
ON DIFFICULTIES IN MATHEMATICAL PROBLEM SOLVING AMONG  
MALAYSIAN MATRICULATION STUDENTS***

***RABIATUL ADAWIAH BINTI AYOP***

**IPM 2015 15**



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By

**RABIATUL ADAWIAH BINTI AYOP**

**Thesis Submitted to the School Graduate Studies, Universiti Putra Malaysia, in  
Fulfillment of the Requirements for the Degree of Master of Science**

**December 2015**

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the Degree of Master of Science

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**December 2015**

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Problem solving has been emphasized in the mathematics curriculum in Malaysia at all levels including matriculation programme. However, Malaysian students were reported to have difficulties in mathematics problem solving. Therefore, students' perspectives on the difficulties in solving mathematics problem can give some understanding on this matter. Furthermore, there is a research gap at this level as post-secondary level is less researched compared to those of the primary and secondary education.

Students' perspectives on the difficulties of mathematical problem solving among matriculation students were approached utilizing Q-methodology due to its appropriateness in studying perspectives. Ninety (90) matriculation students from two matriculation colleges were involved as participants. Each participants was required to rank 48 statements according to the order of agreement and disagreement for each statements about their personal views on the difficulties of mathematical problem solving. All the Q-methodology procedures: collecting concourse, developing Q-sample, selecting P-set, conducting Q-sort and analyzing data as well were done for this study.

Findings showed the difficulties of mathematical problem solving occurred in four main factors namely, (1) *Heuristics and Control*, (2) *Control and Beliefs*, (3) *Resources and Heuristics* and (4) *Resources and Beliefs*. In the domain of *resources*, conceptual knowledge such as understanding mathematical symbols and notation showed a major load from the students. In *heuristics* domain, students loaded difficulties at the first stage of problem-solving phase, that is, understanding the problem and the usage of heuristics skills such as drawing diagrams or pictures in understanding the problem. In the *control* domain, students were found, not realizing their mistakes in a solution until they got the wrong answers. While in the *beliefs* domain, students were found to fear the unexpected mathematics problem. The findings may provide some benefits for future researches to focus more on matriculation students' ability in solving mathematical problems with more rigorous analysis.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk Ijazah Master Sains

**PENDEKATAN KAEDAH-Q UNTUK MENENTUKAN PERSPEKTIF  
TERHADAP KESUKARAN PENYELESAIAN MASALAH MATEMATIK  
DALAM KALANGAN PELAJAR MATRIKULASI DI MALAYSIA**

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Penyelesaian masalah telah ditekankan dalam kurikulum matematik Malaysia di semua peringkat pengajian termasuk program matrikulasi. Walaubagaimanapun, pelajar Malaysia didapati menghadapi masalah dalam penyelesaian masalah matematik. Oleh itu, perspektif pelajar terhadap kesukaran dalam menyelesaikan masalah matematik boleh memberikan sedikit kefahaman tentang permasalahan ini. Tambahan pula, tidak banyak kajian dilakukan di peringkat ini berbanding peringkat menengah dan rendah.

Pandangan murid mengenai kesukaran menyelesaikan masalah matematik di kalangan pelajar matrikulasi telah menggunakan pendekatan kaedah-Q kerana kesesuaiannya dalam mengkaji perspektif. Sembilan puluh (90) pelajar matrikulasi dari dua kolej matrikulasi terlibat sebagai peserta. Setiap peserta dikehendaki menyusun 48 pernyataan berdasarkan pandangan peribadi mereka mengenai kesukaran penyelesaian masalah matematik. Semua prosedur Kaedah-Q: mengumpul '*concourse*', membangunkan '*Q-sample*', memilih '*P-set*', menjalankan '*Q-sort*' dan menganalisis data juga telah dilakukan bagi kajian ini.

Dapatan kajian menunjukkan kesukaran penyelesaian masalah matematik melibatkan empat faktor utama iaitu, (1) Heuristik dan Kawalan, (2) Kawalan dan Kepercayaan, (3) Pengetahuan dan Heuristik dan (4) Pengetahuan dan Kepercayaan. Dalam domain pengetahuan, pengetahuan konseptual seperti tidak memahami simbol dan notasi matematik menunjukkan kesukaran utama pelajar. Bagi domain heuristik, pelajar menghadapi kesukaran pada peringkat pertama fasa penyelesaian masalah, iaitu memahami masalah dan kesukaran dalam kemahiran seperti melakar rajah atau gambar dalam memahami masalah. Dalam domain kawalan, pelajar didapati tidak menyedari kesilapan yang dilakukan dalam penyelesaian masalah sehingga mereka mendapat jawapan yang salah. Manakala bagi domain kepercayaan, pelajar berasa takut untuk menyelesaikan masalah matematik yang tidak dijangka. Hasil kajian diharap dapat memberikan manfaat untuk kajian akan datang dengan menumpukan kebolehan penyelesaian masalah dalam kalangan pelajar matrikulasi menggunakan analisis yang lain.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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## LIST OF ABBREVIATIONS

CFA	Centroid Factor Analysis
GPMP	<i>Gred Purata Matapelajaran</i>
KPM	<i>Kementerian Pelajaran Malaysia</i>
MARA	<i>Majlis Amanah Rakyat</i>
MMP	Modern Mathematics Program
MOE	Minister of Education
NCTM	National Council of Teachers of Mathematics
OECD	Organisation for Economic Cooperation and Development
PISA	Programme for International Student Assessment
SE	Standard Error
SED	Standard Error for Differences
SPM	<i>Sijil Pelajaran Malaysia</i>
STPM	<i>Sijil Tinggi Pelajaran Malaysia</i>
TIMSS	Trends in International Mathematics and Science Studies

# CHAPTER 1

## INTRODUCTION

### 1.1 Background of the Study

Numerous documents have been written on mathematics, as it plays its roles in supporting many fields of study. Mathematics is valued not only in science and technology, but in everyday living as well as in the workplaces or businesses, and in making sense of everyday activities from a simple calculation to making decisions. Therefore an emphasis on mathematics education will ensure the development of a technologically and highly skilled scientifically based manpower to encounter the challenges of the 21st century, as this kind of manpower requires a strong grounding in mathematics (Martinez & Martinez, 2006). As a field of study, Mathematics trains the mind to think logically and systematically in solving problems and making decisions. This discipline encourages meaningful learning and challenges the mind, and hence contributes to the holistic development of the individual where it is said the most useful skill that mathematics students may take when they leave the university is problem solving skill (Rowlette, 2011). Many areas of employment will be tackling unfamiliar problems, hence graduates may have to use their mathematics skills in their jobs and be confident in problem solving (Badger, Hawkes & Sangwin, 2012).

Students develop numeracy, reasoning, thinking skills, and problem solving skills through the learning and application of mathematics. The curriculum places heavy emphases on relationships between mathematics and real life problems which are considered essential in helping students appreciate mathematics and in achieving goals for students who become mathematical problem solvers (NCTM, 2004). The importance of problem solving in mathematics learning can be seen in Secondary Mathematics Curriculum in Malaysia as well as in Additional Mathematics, which has been stated as one of the objectives in mathematics education, in which students should be able to acquire basic skills in mathematics such as solving problems and enhancing the skills (Curriculum Development Centre, 2013).

One of the aims of Malaysia mathematics' curriculum is to develop individuals who are able to apply mathematical knowledge effectively and responsibly in solving problems and making decisions, therefore teaching and learning process must include problem solving skills which are comprehensive and cover the whole curriculum (Curriculum Development Centre, 2013). Problem solving should be emphasized during the teaching and learning process in order to develop the problem solving skills and problem solving strategies among the students. These are clearly stated as the main elements to be focused in mathematics' curriculum as well as communication in mathematics, reasoning in mathematics, mathematical connections and application of technology (Curriculum Development Centre, 2013).

As problem solving is very important in mathematics, the emphasize not only at the secondary level, but at the matriculation level as well. Matriculation is a 1- or 2-year pre-



university programme conducted by the Malaysian Ministry of Education that qualifies students for placement in the public universities. Therefore, the mathematics syllabus was designed towards developing students' understanding of mathematical concepts and applications, and skills to interpret and solve problems so they have a complete and strong foundation to pursue programs in science, technology, social science and management (Matriculation Division, 2003; 2006).

However, problem solving has posed fear in students since ancient times where, additionally it is noted that there are also word problem solving difficulties among the poor mathematics learners as well as the existence of mathematics anxiety among undergraduate students (McKnight et al., 1987; Usop, Sam, Sabri, & Wah, 2009). Thus, problem solving remains a challenging task that demands instructional attention in our schools and matriculation programme as well.

### **1.1.1 Mathematics Learning in Matriculation**

Matriculation programme is a pre-university programme designed to prepare students for professional fields in institutions of higher learning (Ministry of Education Malaysia, 2013). Students who are selected for the one-year matriculation programme can either major in Science or Accountancy. All students have to take Mathematics subject that must be learned within two semester. Since Matriculation is a preparatory programme for tertiary education, students must possess a good mastery of mathematics that can be extensively used in the fields of science, technology and accounting at tertiary level.

The mathematics syllabus for matriculation programme contains ten chapters for Science students and nine chapters for Accounting students, which are taught in the first semester. While for the second semester, mathematics for Science students contains ten chapters and eleven chapters for Accounting students. Both mathematics syllabus was designed with objectives that matriculation students can formulate problems in mathematical forms and solve them; as well as can apply algebra, calculus and statistics in the field of social science, accounting and management (Matriculation Division, 2003; 2006).

Certainly, problem solving requires students to apply and to integrate many mathematical concepts and skills as well as making decisions (Badger, Sangwin, Hawkes, Burn, Mason, & Pope, 2012). One of the important mathematical concepts that needs to be learned in problem solving skills at matriculation level is algebra, such as simplifying and expanding algebraic expressions or equations (Matriculation Division, 2003; 2006). As a general overview, Semester One mathematics syllabuses for Accounting students are (1) Number System and Equations, (2) Inequalities and Absolute Values, (3) Sequences, (4) Matrices and Systems of Linear Equations, (5) Functions and Graphs, (6) Polynomials, (7) Limits, (8) Differentiation and (9) Applications of Differentiation. Semester One mathematics syllabuses for Science students are (1) Number System, (2) Equations, Inequalities and Absolute Values, (3) Sequences and Series, (4) Matrices and Systems of Linear Equations, (5) Functions and Graphs, (6) Polynomials, (7) Trigonometric Functions, (8) Limits and Continuity, (9) Differentiation and (10) Applications of Differentiation.

According to these syllabuses for both streams, content of algebra is devoted on algebraic expressions including concept of unknowns, algebraic terms and expressions. This is because algebraic problem solving skills are important to solve algebraic problems and other mathematical problems (Davrajoo, Tarmizi, Nawawi & Hassan, 2010). Furthermore, misconceptions on algebra skills such as algebraic expansions make the students fail to solve other mathematical problems such as differentiation (Nadirah, Yusof, Fatimah, Zabidi, Rahimah, & Ezrinda, 2012).

### **1.1.2 Problem Solving in Mathematics Education**

Mathematics education in Malaysia has undergone development in three distinct phases (Noor Azlan, 2000). In the first phase, the content emphasized mainly on basic skills which was the focus of the national syllabus at that time. In the late 70's during the second phase, modern mathematics programme (MMP) was introduced in schools. Underlying the theme of the syllabus was understanding the basic concepts rather than attaining computational efficiency. Finally, in the late 80's the mathematics curriculum was designed to strike a balance between skills and understanding (Noor Azlan, 2000).

The new Mathematics curriculum in Malaysia regards mathematics as a very powerful tool in solving problems experienced in our daily lives (Noor Azlan, 2000). Therefore, as problem solving becomes the focus in the curriculum, teachers are expected to intentionally teach students on the heuristics of problem solving (Noor Azlan, 2000). Mathematics' curriculum for secondary schools in Malaysia adopted the problem solving strategy from Polya (1945). The problem solving skills strategy has four stages. They are: 1) *Understanding the problem*: identifying the problem's known (given) and unknown and, if appropriate, using suitable notations, such as mathematical symbols to represent the problem, 2) *Devising a plan*: determining appropriate actions to solve the problem, 3) *Carrying out the plan*: executing the actions that have been determined to solve the problem and checking their effectiveness, 4) *Looking backward*: evaluating the overall effectiveness of the approach to the problem with the intention of learning on how similar problems may be solved in future occasions (Curriculum Development Centre, 2013).

Although problem solving has been emphasized in Malaysian Mathematics curriculum for a long time, Malaysian student's performance in problem solving were reported as the lowest amongst high achievement countries as well as in algebra performance (OECD, 2014). Based on Programme for International Student Assessment (PISA) 2012 results, where the focus was on performance in problem solving, Malaysia was ranked 39 with a mean score of 422 on creative problem solving, where the overall mean score for all countries was 500.

### **1.1.3 Students' Performance in Problem Solving**

PISA is administered by the Organisation for Economic Cooperation and Development (OECD) every three years on 15-year-olds in both OECD and non-OECD countries and offers students questions in the main language of instruction of their respective countries.

It is a widely recognized international assessment. Each round focuses on one area of either Reading, Mathematics or Science.

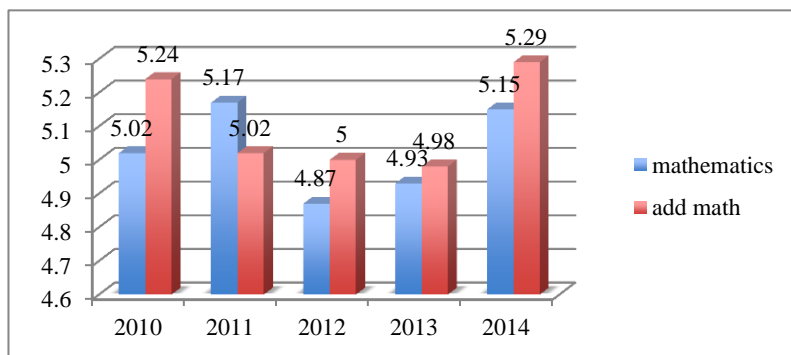
Trends in International Mathematics and Science Studies (TIMSS) on the other hand is another international assessment based on the Mathematics and Science curricula of schools around the world. It assesses students in Grades 4 (the Malaysian equivalent is Year 4) and 8 (the Malaysian equivalent is Form 2) in two aspects: content such as algebra and geometry, and cognitive skills, namely the thinking processes of knowing, applying, and reasoning (Ministry of Education Malaysia, 2013).

Malaysia was ranked at the bottom third in The Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Studies (TIMSS) for the 2012 results. In the survey, Malaysia was ranked 52 out of 65 countries. Malaysia scored 421 in Mathematics, 398 in Reading and 420 in Science respectively. The global average score was 494 in Mathematics, 496 in Reading and 501 in Science. While in 2009, Malaysia participated in the survey for the first time and scored 404 in Mathematics, 414 in Reading and 422 in Science (OECD, 2014).

When Malaysia first participated in TIMSS in 1999, its average student score was higher than the international average in both Mathematics and Science. However by 2011, the performance had slipped to below the international average in both Mathematics and Science with a commensurate drop in ranking. Critically, 35% and 38% of Malaysia's students failed to meet the minimum proficiency levels in Mathematics and Science in 2011, a two to fourfold drop from 7% and 13% respectively in 1999 (Ministry of Education Malaysia, 2013). These students were identified as possessing only limited mastery of basic mathematical and scientific concepts.

The first participation on PISA 2009 was also discouraging, with Malaysia ranking at the bottom third of 74 participating countries, below the international and OECD average. Almost 60% of the 15-year-old Malaysian students who participated in PISA failed to meet the minimum proficiency level in Mathematics, while 44% and 43% did not meet the minimum proficiency levels in Reading and Science respectively (Ministry of Education Malaysia, 2013).

For the national level, both Mathematics and Additional Mathematics' curriculum in Malaysia emphasizes on problem solving as the main element in teaching and learning. However, in the national examination for Mathematics and Additional Mathematics, students' performance in SPM (Malaysia Examination Certificate) started to show a declining trend in GPMP for last few years (2010 to 2014). Figure 1.1 shows, the average grade point (GPMP) for mathematics decreased from 4.93 in 2013 to 5.15 in 2014. For more details, actually, the declining trend started in 2011 where the GPMP dropped from 5.02 in 2010 to 5.17 in 2011, but started to increase in 2012 with GPMP at 4.87 but decreased again in 2013 with GPMP at 4.98 and from then continued to drop in 2014 with GPMP at 5.15.

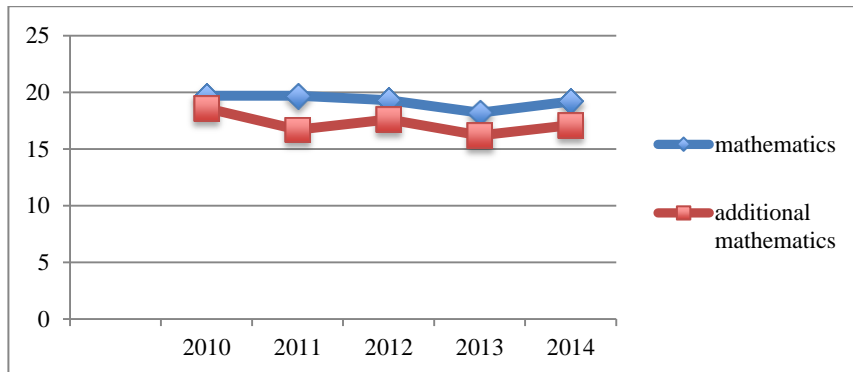


**Figure 1.1. GPMP for Mathematics and Additional Mathematics in SPM**  
(Source: *Lembaga Peperiksaan, MOE*)

As for Additional Mathematics, the declining trend started since 2014. In 2010, the GPMP was 5.24 and increased to 5.02 in 2011, continued to increase at 5.00 in 2012, and at 4.98 in 2013 as well. However it greatly dropped in 2014 with the GPMP at 5.29 in 2014. The declining trend for both Mathematics and Additional Mathematics followed by the inclining percentage in the number of failed students, are precisely shown in Table 1.1 and clearly seen in Figure 1.2.

**Table 1.1. SPM Performance**

Subjects	Year	Pass (%)	Fail (%)
Mathematics	2010	80.3	19.7
	2011	80.3	19.7
	2012	80.7	19.3
	2013	81.8	18.2
	2014	80.8	19.2
Additional Mathematics	2010	81.4	18.6
	2011	83.3	16.7
	2012	82.4	17.6
	2013	83.8	16.2
	2014	82.9	17.1



**Figure 1.2. Percentage of Failures in Mathematics and Additional Mathematics In SPM**

Therefore, from students' performance in SPM for Mathematics and Additional Mathematics as well as PISA performance of 15-year-old Malaysian students who are at the secondary education level, it becomes a wonder about the students' problem solving performance at the post secondary level such as those in the matriculation programme.

#### 1.1.4 Nature of Mathematical Problem Solving

There is ample evidence to show that the focus on problem solving has benefits in many areas. Problem solving supports students' mathematical conceptual and procedural knowledge, accurately reflects what it means to do mathematics, and plays a vital role in producing mathematicians because of the ability to solve problems in unfamiliar situations (Badger et.al, 2012; Otten, 2010). However, mathematics problem solving can be solved by some of the students easily while many others can't. So what are the factors involved that enable some students to solve difficult problems and what are the causes that make other students fail in their problem solving attempts? These are subjects to enquiry.

In problem solving, many mathematics skills are involved such as conceptual understanding and procedural knowledge (National Mathematics Advisory Panel, 2008). For algebra problem solving, students have to identify the unknowns, variables and relations among them, and express them symbolically in order to solve the problem (Martinez, 2002). Conceptual understanding and problem solving skills as well as computational fluency are essential and should be supported by cognitive systems that control focus and interference in information processing (Schoenfeld, 1992). According to Schoenfeld (1992), effective cognitive and metacognitive processes and strategies for mathematics problem solving are significant factors in solving problems.

Students should be taught on how to apply those processes and strategies when solving problems (Krawec, Huang, Montague, Kressler, & de Alba, 2012). In order to understand the success or failure of a problem solving attempt, teachers need to identify the students' knowledge base as resources, problem solving strategies, metacognitive



actions or controls, beliefs and practices (Schoenfeld, 1985). There are other variables suggested by previous researchers in explaining the success or failure in solving problems. The variables are knowledge, heuristics, metacognition and beliefs (Schoenfeld, 1985) as well as skills, meta-skills and will (Mayer, 1998).

According to Mayer (1998), skill is a domain specific knowledge relevant to the problem solving task, meta-skill is a strategy on how to use the knowledge in problem solving and will is the feelings and beliefs of one's interest and ability to solve problem. The problem solver's individual knowledge and skills help determine the difficulty or ease with which obstacles to solutions can be overcome.

Mathematical problem solving, referred to as thinking mathematically, will be the focus of this study. Think mathematically means (a) developing a mathematical point of view by ruminating the processes of mathematization and absorption and having the preference to apply them, and (b) developing capability with the tools of the skill and using those tools in the service of the goal of understanding structure (Schoenfeld, 1992). The difficulties of mathematics learning can be on content understanding or mathematical processes or both (Lithner, 2011). According to Lithner (2011), difficulties on content understanding means inherence of mathematical difficulties within specific mathematical while mathematical processes means steps shown on non-routine problem solving, proof and proving, reasoning, representing and modeling (Lithner, 2011; NCTM, 2000). From the students' perspectives, we then can get the understanding on how the students go through the process of solving mathematical problem.

## **1.2 Problem Statement**

The poor performance of Malaysian students in the latest TIMSS and PISA studies raises the concerns of many Malaysians educators. At the matriculation level, mathematics is a core subject taken by students majoring in accountancy, physical science, and life science. Students must possess a good mastery of mathematics at the matriculation level, as it is used extensively in the fields of science, technology, accounting, and economics at the university level.

As problem solving is an important concept in mathematics, the matriculation mathematics' curriculum was also designed to develop students' understanding of mathematical concepts and applications, and skills to interpret and solve problems so they can have a complete and strong foundation to pursue tertiary education in the field of science, technology, and accounting (Matriculation Division, 2003). In order to achieve this vision, students' difficulties should be identified especially when involving a core subject such as mathematics.

Therefore, students' difficulties in problem solving should be investigated at this level as the participants of PISA are the one that continues from secondary level to post-secondary education level. Students' perspectives on the important factors that influence problem solving attempts will largely highlight the understanding of the students' difficulties in mathematical problem solving.

Matriculation students are required to master in solving mathematical problems but according to Zakaria and Yusoff (2009), their algebra problem solving skill was just average. In another study, it was revealed that conceptual knowledge of matriculation accounting students was low when compared to physical and life science students (Zakaria, Yaakob, Maat & Adnan, 2010). A study by Ong and Lim (2014) showed that students from Penang Matriculation College have difficulties in understanding the mathematical symbols, which influence their abilities in solving mathematics problems.

On the other hand it was found that university students were not able to relate mathematics problems to their existing knowledge (Bayat & Tarmizi, 2010). Problem solving ability of Malaysian university students as well, was shown as poor or average or moderate (Yunus, Hamzah, Tarmizi, Abu, Nor, Ismail, Ali & Bakar, 2006). Therefore, this study is necessary in order to realize the vision of the Matriculation Division that to generate a quality bumiputera students for higher education institutions in science, technology and professional (“Ministry of Education Malaysia”, 2015).

Although the matriculation programme was introduced since 1998 by the Ministry of Education Malaysia, there are limited studies on this post-secondary education level especially on students’ difficulties in mathematical problem solving because mathematics achievement at the Matriculation level was considered an official secret and not for public view (Sam, Ngik & Usop, 2009). Therefore, it will be interesting to determine if the different kinds of difficulties in mathematical problem solving are relevant to Malaysia students at this level. Furthermore, this study hopes to provide insightful evidence on students’ difficulties in mathematical problem solving at the matriculation level.

### **1.3 Research Objectives**

The objectives of this research are as stated below:

1. To identify the students’ difficulties on mathematical problem solving.
2. To investigate the factors influencing students’ difficulties on mathematical problem solving from the students’ perspectives .

### **1.4 Research Questions**

This study aims to find out students' difficulties in solving mathematical problems from their perspectives. The research questions are as follows:

1. What are the students’ perspectives on the difficulties in mathematical problem solving?
2. What are the factors that contribute to the students’ difficulties in mathematical problem solving?
3. Which are the prominent factor that contributes to the students’ difficulties in mathematical problem solving?
4. What are the relationships between the identified factors?

## 1.5 Significance of Study

According to Mayer and Wittrock (1996), problem solving is “*cognitive processing directed at achieving a goal when no solution method is obvious to the problem solver*”. Problem solving is therefore an important, if not the most important, component of mathematics discipline and as mentioned by Schoenfeld (1985), the main aim of learning mathematics is to solve problems. Mathematical problem solving is important not just because it makes a person a better problem solver in general, but it is about the systematic exploration and investigation of mathematical objects (Schoenfeld, 2013). As the major goal of education is to help students learn in ways that enable them to use what they have learned to solve problems in new situations, hence, problem solving is fundamental to education because educators are interested in improving students' ability to solve non-routine problems (Mayer & Wittrock, 1996; 2006).

Therefore, this study will be a rich source of information for mathematics teaching and learning in general. The understanding of students' viewpoints will be valuable insights for those involved in teaching and learning in mathematics especially in determining the student's difficulties in mathematics problem solving. Hopefully, the findings will become a guideline in understanding the difficulties and to assist those students who have difficulties in problem solving.

Further, it hoped that the findings of this study, will lead to the utilization of suitable approaches that could facilitate students' understanding in problem solving. Therefore, the findings are crucial in ensuring a meaningful teaching and learning process as success in mathematics problem solving is highly correlated with overall mathematics achievement (Bryant, Bryant, & Hammill, 2000). The understanding of the difficulties faced by students in any phase of problem solving, could provide a greatly useful guideline for teachers as well as researchers to plan better effective teaching methods or modules.

Furthermore, there are numerous previous studies that discuss problem solving but this study specifically focuses on the difficulties of students in problem solving from the students' perspectives which involve students' cognitive domain. Thus, the findings will become an enriching information and valuable experience in mathematics education.

There are previous studies on students' difficulties in mathematical problem solving but very sparse on post-secondary education such matriculation students. Post-secondary level education in Malaysia especially the Matriculation Programme has been less researched as compared to primary and secondary as well as tertiary levels of education. This study will be a good start in understanding and determining students' difficulties in problem solving.

It is also hoped that through students' perspectives on their difficulties in problem solving, the poor performance of Malaysian students in PISA and TIMSS as well as the average performance of Malaysian university students in mathematical problem solving will be explained clearly by the students themselves which directly will help the Matriculation Division to make their vision comes true.



Lastly, there are limited number of researches that use Q-Methodology approach in Malaysia. In fact, this study will be an added research for mathematics education on the utilization of Q-Methodology. As this study is on students' difficulties in mathematics problem solving using Q-Methodology, it could help other researchers to go explore further on this methodology. It is good for Malaysia to research and to know further on Q-Methodology as the Q-methodology reveals subjectivity and allows inferences made on the basis of individual responses from their personal points of view (Previte, Pini, & Mckenzie, 2007).

## **1.6 Limitations of the Study**

This study is subjected to certain limitations with regard to studying the difficulties faced by students in mathematical problem solving. The limitations were related to the scope of this study which only a list of specific statements were utilized in the instrument to measure the variables in the study. The results obtained were dependent on the understanding, honesty and integrity of the participants when answering the item statements.

Only a selected few factors among the numerous factors that influenced the students' difficulties were taken into concern in this study basing on a chosen theory. The study only focused on Malaysian post-secondary education specifically the matriculation programme under the Ministry of Education Malaysia. Therefore, the findings should not be applied to other levels such as primary and tertiary.

The study involved only 102 matriculation students from two colleges of the Matriculation Division Ministry of Education Malaysia, with only two matriculation colleges were selected based on the permission from the colleges' director. Only Accounting students were selected based on lower requirements on their mathematics scores upon entry-level on the Malaysian Certificate of Education examination (Zakaria & Yusoff, 2009). The results of this study could not be generalized for all the students under this programme as only a small sample of students were chosen.

Content of knowledge that was emphasized in this study was only on certain parts of basic algebra for post-secondary education level especially from the matriculation syllabus of mathematics for accounting. Based on Malaysia performance related to algebra in TIMMS that was not perform well as reported by Lessani, Yunus, Tarmizi, and Mahmud (2014), performance of students at this level related to algebra is interesting to find out

## **1.7 Operational Definitions**

The following terms were used throughout the study, and clearly defined in order to get a correct understanding and to avoid any confusion on the terminologies used. They are as the following:

## **Mathematical Problem Solving**

Generally, mathematical problem solving context refers to the process of solving mathematics problem using mathematical knowledge. It consists of conceptual and procedural knowledge which are utilized when new information in a problem situation can be attempted until the goal is achieved. According to Lester (2013) problem solving is an activity requiring an individual (or group) to engage in a variety of cognitive actions, each of which requires some knowledge and skills, some of which are not routine. This process is considered as a thinking process which a student as a solver should try by applying the mathematical knowledge.

In this study, students' difficulties in problem solving were referred as the failure to solve mathematics problems. The failure can occurs at any phase in the process of solving because of the inadequate knowledge foundations in problem solving.

### **Categories of Knowledge in Problem Solving**

Major arguments on mathematical problem solving by Schoenfeld (1985) were based on the possible explanations of someone's success or failure in trying to solve problems on the four factors. Thus, main components of students' difficulties in mathematical problem solving are considered as difficulties in mathematics knowledge, difficulties in problem solving heuristics, difficulties in metacognition or control during problem solving and difficulties caused by individual beliefs.

Therefore, four categories of knowledge discussed in this study which influence the ability to solve problems are resources, heuristics, control and beliefs. Theoretically, resources are the students' foundation of basic mathematical knowledge, heuristics are the problem solving strategies and skills, control or meta-cognition is monitoring and self-regulate the resources, while beliefs are the students beliefs about themselves, about mathematics, about problem solving and their mathematics ability as well as motivation (Mayer, 1998; Schoenfeld, 1992).

**Resources:** According to Schoenfeld (1985), aspects of the knowledge base relevant for competent performance in a mathematical content which include, informal and intuitive knowledge; facts, definitions, and the like; algorithmic procedures; routine procedures; relevant competencies; and knowledge on the rules of discourse in mathematics. In this study, resources refer to student's foundation of basic mathematical knowledge which include all information and schemas that contained in the memory.

**Heuristics:** According to Tiong (2005), all heuristics have two characteristics; 1) heuristics that do not guarantee a solution, it is pointing towards possible ways in which problem solvers might be able to find the solution and 2) heuristics that do not come with specific procedures, it helps us to deal with difficult problems that are not familiar. While previously heuristics known as rules of the thumb in making progress on difficult problems (Polya, 1945). In this study, heuristics are suggestions on strategies or

techniques that are designed to help when solving problems. It can be heuristic methods, heuristics strategies or simple heuristics.

**Control:** Metacognitive is one of the metacognition component which means awareness and monitoring of one's own cognitive system and its functioning (Panaoura, 2009; Flavell, 1979). Metacognition is the ability to think about thinking which is complimentary with problem solving. It means self-awareness and the ability to monitor and control one's mental processing as well (Schoenfeld, 1992). It is a knowledge that allows students to better comprehend, monitor or assess conceptual and procedural knowledge associated to a domain (Panaoura, 2009). According to Gourgey (1998), metacognitive strategies enable one to monitor and improve one's progress; which means to evaluate understanding and apply knowledge to new situations. In this study, control means the student decisions made during problem solving that involve overall decisions regarding the selection and implementation of resources and strategies including planning, monitoring, decision-making and conscious metacognitive acts. It also means making use of what one already know (the resources).

**Beliefs:** According to Schoenfeld (1985), belief systems are one's mathematical world view, the perspective with which one approaches a mathematics problem and mathematical tasks. That work of Schoenfeld inspired Erik De Corte & Peter Op 't Eynde (2002) on their definition of beliefs. They defined that belief systems are constituted by beliefs about mathematics education, beliefs about the self, and beliefs about the class context. In this study, beliefs are focused on students' beliefs in themselves, beliefs in mathematics and as well as beliefs in problem solving. Specifically in this study, beliefs are to be interpreted as an student's understandings and engaging feelings that shape the ways that the student conceptualizes in mathematical behavior, how one decides to which techniques will be utilized or avoided, to what extent as well as how hard one will deal on it and so on.

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## APPENDICES

### APPENDIX A

#### Concourse

Collections of Concourse from verbal interviews, observations, research papers, articles, books, the Internet, and informal chats.

1. A letter in algebraic expression such as  $p$  or  $q$  represent a number.  
Example:  $3p$  does not mean 3 pupils but it is 3 times the number of pupil.
2. The meaning of  $3^n$  is a repeated multiplication  $3 \times 3 \times 3 \dots \dots \times 3$  for  $n$  times.
3. Solving algebraic expression such as  $a(x + y) = ax + ay$  or  $a(xy) = axy$  by using the three laws (commutative, associative and distributive).
4. Solving linear inequalities such  $6x - 5 > x + 20$ .
5. Solve multiplication of two expressions such as  $(A - B)(A + B) = A^2 - B^2$
6. Develop the algebraic equation from mathematical words problem to solve the given task.
7. Use of symbols, or letters to represent numbers (such as  $a$ ;  $x$ ; or  $y$  in the expression  
 $y = 3x + 5$ ;  $z = x - 3y$ ).
8. Solve simultaneous equation involving algebraic equations with 3 unknown.  
Example: Find the value of  $x$ ,  $y$  and  $z$  of the following equations
$$\begin{aligned}y - x - z &= 0 \\2x &= y + z \\x + y + z &= 4\end{aligned}$$
9. Though I know how to calculate, it is difficult to know the reasons for the calculation.
10. To read the problem more than once to understand the problem.
11. Put the problem into my own words.
12. Identify the information that was given in the problem.
13. To recall on a similar problem that I had worked before.
14. To apply the correct strategy for solving problem.
15. To identify mistake made during problem solving.
16. To look back over my solution method to check that I had done what the problem asked.
17. To look back to see if I did the correct procedures.
18. Draw diagram or picture to help me understand the problem.
19. To write down important information.
20. Use algebra to set up some equations to solve.
21. Feel confused and couldn't decide what to do.
22. To keep looking back at the problem after I did a step.
23. Thought to myself, do I understand what the problem is asking me?
24. Thought to myself about what information I needed to solve this problem.
25. To ask myself, is there information in this problem that I don't need?
26. Do checked my work step by step as I went through the problem.
27. To reread the problem to check that I was still on track.
28. To ask myself whether I was getting any closer to a solution.
29. Rethink my solution method and try a different approach.



30. Thought about all the steps as I worked the problem.
31. To stop and rethink a step I had already done.
32. Checked my calculations to make sure they were correct.
33. Asked myself whether my answer made sense.
34. Confidence in my ability to solve mathematics problems.
35. Enjoyment in solving mathematics problem.
36. Anxious when I am asked to solve mathematics problems.
37. Fear of unexpected mathematics problems.
38. The most important thing in mathematics is to get correct answers.
39. Give up fairly easily when the problem is difficult.
40. The math that I learn mostly facts and procedures that have to be memorized.
41. The math that I learn just a way of thinking about space, numbers, and problems.
42. Some people are good at math and some just aren't.
43. In mathematics something is either right or it's wrong.
44. In mathematics you can be creative and discover things by yourself.
45. Math problems can be done correctly in only one way.
46. Real math problems can be solved by common sense instead of the math rules that learn in school.
47. The best way to do well in math is to memorize all the formulas.
48. A wrong answer to a math problem is absolutely wrong, there's no room for argument.
49. A wrong answer only can be find out when it's different from the book's answer or when the teacher tells you.
50. Mathematics is very interesting to me.
51. When doing mathematics, my mind goes blank, and I am not able to think clearly.
52. Mathematics makes me feel uncomfortable and impatient.
53. I always enjoyed studying mathematics in school.
54. I am nervous in mathematics classes.
55. It makes me nervous to even think about having solving a mathematics problem.
56. Solving a problem that is different from the ones done in class makes me worry.
57. Most of the time, I need help from the teacher before I can solve a problem.
58. I have forgotten many of the mathematical concepts that I have learnt in previous mathematics classes.
59. I learn mathematics by understanding the main ideas, not by memorizing the rules and steps in a procedure.
60. If I cannot solve a mathematics problem, I just ignore it.
61. Successfully solving a problem on my own provides satisfaction similar to winning a game.
62. Mathematics classes provide the opportunity to learn skills that are useful in daily living.
63. Mathematics is not my strength and I avoid it whenever I can.
64. I don't think I could learn advanced mathematics, even if I really tried.
65. Doing mathematics encourages me to think creatively.
66. Mathematics is important for most jobs and careers.
67. Solving mathematics problems helps me learn to think and reason better.
68. To succeed in life you need to be able to do mathematics
69. I do not have to understand mathematics, I simply memorise the steps to solve a problem.
70. I learn mathematics well if I understand the reasons behind the methods used.
71. I work hard in Math (subject).
72. If I make mistakes, I work to correct them.

73. I like to invent new problems.
74. I learn mathematics quickly.
75. Mathematics allows us to understand the world we live in better.
76. When I am asked to solve math problems, I get nervous.
77. When I cannot solve a math problem quickly, I keep on trying.
78. I feel confident when I study or work on mathematics.
79. Everyone can learn mathematics.
80. I prefer challenging tasks in order to learn new things.
81. Mathematics should not place much importance on problem solving
82. I feel happy when I solve math problems.
83. Math consists of concepts and procedures that we have to memorize.



## BIODATA OF STUDENT

Rabiatul Adawiah binti Ayop was a mathematics lecturer at Matriculation College of Negeri Sembilan since 2003 to 2013 before continuing for her Master Degree in year 2013. She was first posting at Matriculation College of Negeri Sembilan after received her first degree in UKM in Bachelor Degree of Education with honors in Mathematics. Major in Mathematics and minor in Computer Literacy, she was posted as a mathematics lecturer for subject Mathematics for Accounting.

Since 2003 in Matriculation College of Negeri Sembilan, Rabiatul Adawiah was involved as an drafter and examiner for matriculation's internal examination. She was also a coordinator for Mathematics for Accounting which involved all the academic activities for Mathematics for Accounting students.

Serving as "*Pegawai Perkhidmatan Pendidikan Siswazah*" for mathematics subject, she has experience in teaching Mathematics for Science and Accounting at Matriculation College of Negeri Sembilan for 10 years (2003 to 2013) before continue with study leave under "*Hadiah Latihan Persekutuan*" sponsored by Scholarship Department, Ministry of Education Malaysia for 2 years (2013 to 2015).



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