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DIAGONALLY IMPLICIT MULTISTEP BLOCK METHODS FOR SOLVING FIRST ORDER ORDINARY AND FUZZY DIFFERENTIAL EQUATIONS

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By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Master of Science

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I dedicate this humble endeavor to Allah.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Master of Science

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AZIZAH BINTI RAMLI

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Chairperson: Zanariah binti Abdul Majid, PhD
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In this study, two-point diagonally implicit multistep block methods are proposed for solving single first order ordinary and fuzzy differential equations. The methods are based on the diagonally implicit multistep block methods. It approximates two points simultaneously at y_{n+1} and y_{n+2} in a block along the interval. Subsequently, the methods of order three, four and five are implemented and numerically tested using constant step size.

The numerical results show that the two-point diagonally implicit multistep block methods could solve the ordinary differential equations without any difficulty. These methods are also able to reduce the number of steps and execution times even when the number of iterations is being increased.

Meanwhile, the first order fuzzy differential equations is interpreted based on Seikkala's derivative. By including characterization theorem, the fuzzy differential equations can be replaced by the equivalent system of ordinary differential equations. The numerical results show that the two-point diagonally implicit multistep block methods could solve the fuzzy differential equations. The accuracy of the approximate solutions is obtained by means of implementation of the method under the Seikkala's derivative interpretation. Nevertheless, these methods respectively have the advantage in terms of reducing the number of function evaluations, total steps and execution times.

In conclusion, the diagonally implicit multistep block methods are suitable for solving the single first order ordinary and fuzzy differential equations.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

KAEDAH-KAEDAH BLOK MULTILANGKAH PEPENJURU TERSIRAT UNTUK MENYELESAIKAN PERSAMAAN PEMBEZAAN BIASA DAN KABUR PERINGKAT PERTAMA

Oleh

AZIZAH BINTI RAMLI

Disember 2015

Pengerusi: Zanariah binti Abdul Majid, PhD
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Dalam kajian ini, kaedah-kaedah blok multilangkah pepenjuru tersirat dua titik dikemukakan untuk menyelesaikan persamaan pembezaan biasa dan kabur tunggal peringkat pertama. Kaedah-kaedah ini berasaskan kaedah blok multilangkah pepenjuru tersirat. Ia menghasilkan nilai hampir dua titik secara serentak pada y_{n+1} dan y_{n+2} dalam satu blok sepanjang selang. Kemudian, kaedah-kaedah pada peringkat ketiga, keempat dan kelima dilaksana dan diuji secara berangka menggunakan saiz langkah yang malar.

Keputusan berangka menunjukkan kaedah-kaedah blok multilangkah pepenjuru tersirat dua titik dapat menyelesaikan persamaan pembezaan biasa tanpa sebarang kesukaran. Kaedah-kaedah ini juga dapat mengurangkan bilangan langkah dan masa pelaksanaan walaupun bilangan lelaran meningkat.

Manakala persamaan pembezaan kabur peringkat pertama diterjemahkan berdasarkan terbitan Seikkala. Dengan teorem pencirian, persamaan pembezaan kabur dapat digantikan dengan sistem yang sama dengan persamaan pembezaan biasa. Keputusan berangka menunjukkan kaedah-kaedah blok multilangkah pepenjuru tersirat dua titik dapat menyelesaikan persamaan pembezaan kabur. Ketepatan bagi nilai hampir adalah berdasarkan pelaksanaan kaedah di bawah terjemahan terbitan Seikkala. Walau bagaimanapun, kaedah ini masing-masing mempunyai kelebihan dari segi mengurangkan jumlah penilaian fungsi, jumlah langkah dan masa pelaksanaan.

Kesimpulannya, kaedah-kaedah blok multilangkah pepenjuru tersirat dua langkah adalah sesuai untuk menyelesaikan persamaan pembezaan biasa dan kabur tunggal peringkat pertama.

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I certify that a Thesis Examination Committee has met on 15th December 2015 to conduct the final examination of Azizah binti Ramli on her thesis entitled "Diagonally Implicit Multistep Block Methods for Solving First Order Ordinary and Fuzzy Differential Equations" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Sciencein Computational Mathematics.

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LIST OF ABBREVIATIONS

DEs	:	Differential Equations
IVP	:	Initial Value Problem
LMM	:	Linear Multistep Method
RK	:	Runge-Kutta
2DIMB	:	Two-point Diagonally Implicit Multistep Block
2DIMB(3)	:	Two-point Diagonally Implicit Multistep Block Method of Order Three
2DIMB(4)	:	Two-point Diagonally Implicit Multistep Block Method of Order Four
2DIMB(5)	:	Two-point Diagonally Implicit Multistep Block Method of Order Five
RK(3)	:	Runge-Kutta Method of Order Three
RK(4)	:	Runge-Kutta Method of Order Four
RK(5)	:	Runge-Kutta Method of Order Five

CHAPTER 1

INTRODUCTION

1.1 Introduction

According to Atkinson (2015), numerical mathematics is a part of mathematics and computer science which creates, develops, analyzes and implements algorithms in solving numerically the problems of continuous mathematics. The real-world applications ranging from physical sciences and biological sciences to engineering, medicine, finance and business are actually initiated from the branch of mathematics. These problems can be formulated in mathematical terms of differential equations (DEs).

A DE is an equation that involves variables and their rates of change (i.e., derivatives). For example, the equation

$$y' = y \tag{1.1}$$+

relates the function $y = y(t)$ and its derivative $y' = \frac{dy(t)}{dt}$.

The DEs consists of two types which are ordinary DEs and partial DEs. The ordinary DEs is an equation of function with only one variable while the partial DEs is an equation of function with two or more variables. The ordinary DEs can be classified by their order. The order is considered by the highest derivatives that occur in the equation.

An initial-value problem (IVP) provides initial condition which is the solution to a DEs. The IVP for first order ordinary DEs is defined by

$$\begin{aligned} y' &= f(t, y), t \in [a, b], \\ y(a) &= \eta, \end{aligned} \tag{1.2}$$

where η is the given initial condition. The existence of a unique solution of the IVP can be known based from the theorem proved in Henrici (1962).

The real-world applications problems are complicated and hard to obtain by exact method. Hence, methods for approximating the solution are used, where the problems are solved by approximation. The numerical methods can be classified into two families which are Single-step methods and Linear Multistep Method (LMM).

However, the information of the problems arise from engineers (civil, chemical, biomedical), natural scientists (biology, chemistry, and physic), social scientists (economics and finance) and variety of field are frequently pervaded with uncertainty. The problems are lacking of information. Therefore, it is essential to have some mathematical tools and theory to describe uncertainty notions.

1.2 Linear multistep method

In Lambert (1973), multistep methods are methods that used the approximation at more than one previous point to determine the approximation at the next point. The general LMM can be written as

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h \sum_{j=0}^{k} \beta_j f_{n+j} \tag{1.3}$$

where

i. α_j and β_j are constants and

ii. assume $\alpha_k \neq 0$ and

iii. that not both α_0 and β_0 are zero.

Since (1.3) can be multiplied on both sides by the same constant without altering the relationship, the coefficients α_j and β_j are arbitrary to the extent of a constant multiplier. It can be simplified by taking $\alpha_k = 1$. The method (1.3) is explicit if $\beta_k = 0$, and implicit if $\beta_k \neq 0$.

The approximation produced by the explicit methods can be improved by the implicit multistep methods. The explicit method worked as a predictor where it predicts the approximation and the implicit method worked as a corrector, corrects the prediction. The combination of the explicit and implicit methods is called a predictor-corrector method. It is denoted by PEC mode or $PECE$ mode, where P indicates an application of the predictor, C is a single application of the corrector and E is an evaluation of f in terms of known values of its arguments. There are various types of calculation for predictor-corrector method. The calculation depends on m times the corrector is applied, similarly denoted by $P(EC)^m$ mode or $PE(CE)^m$ mode.

1.3 Fuzzy theory

The real problems are often lack of information and do not have a specific criteria of membership. For example, a “beautiful person” does not clearly determine who is beautiful or who is not beautiful. Since the criterion of “beautiful” is not clarified, the uncertainty lies in the meaning of the word.

Most modeling, reasoning and computing tools are crisp, deterministic, and precise in character. However, these tools are inconvenient on definition where the word of relationship is not clear or dichotomous. The degree of membership in a set is expressed by number 0 or 1 only. This rarely happen since the real problems cannot be described precisely.

Several theories exist in order to describe and illustrate this uncertainty. Zadeh (1965) propose fuzzy theory which is a mathematical theory. Fuzziness involves with uncertainty, vagueness and ambiguity. Fuzzy theory states fuzziness by means of the concept of sets. A classical (crisp) set is defined as a collection of elements or objects. For a fuzzy set, the characteristic function, in which 1 indicates membership and 0 is non-membership, allows various grades of membership for the elements of a given set. The fuzzy set theory generalizes the classical set theory.

1.4 Fuzzy initial value problem

Fuzzy DEs is a theory of DEs that involves with uncertainty. It consists of constants and the initial values are fuzzy numbers. The fuzzy number is characterized by an α - cut (also r -level set). The α - cut of a fuzzy set is a crisp set that contains all the elements of the universal set that have a membership grade in the interval. There are numerous membership functions such as triangular, trapezoidal, parallelogram, Gaussian, generalized bell and sigmoid. This leads to a fuzzy IVP. The fuzzy IVP is defined by

$$\begin{aligned} y'(t) &= f\left(t, y(t)\right), t \in [t_0, T] \\ y(t_0) &= y_0 \end{aligned} \tag{1.4}$$

where y denotes as a fuzzy function of t, y_0 is fuzzy number, $f(t, y)$ is a fuzzy function of crisp variable t and fuzzy variable of y, and y' is the fuzzy derivative of y. The fuzzy IVP has a unique solution. The proof can be found in Seikkala (1987).

1.5 Objective of the study

The aim of this study is to solve the single first order ordinary and fuzzy DEs by using the two-point diagonally implicit multistep block (2DIMB) method. The objectives are:

i. To extend the 2DIMB method derived by Majid (2004) and to include the 2DIMB methods of order three and order five.
ii. To investigate the stability analysis of the 2DIMB methods.
iii. To solve single first order ordinary DEs and fuzzy DEs based on Seikkala's derivative by using constant step size.

1.6 Scope and limitation of the study

The single first order ordinary and fuzzy DEs are solved by using 2DIMB methods. The 2DIMB methods are of order three, four and five. The solutions are approximated by moving two points in a block. Runge-Kutta (RK) method of order three and four are being chosen to compute the initial point for the 2DIMB methods of order three, four and five respectively. The stability analysis is being discussed. Constant step size of $h = 0.1$ and $h = 0.01$ are used.

The single first order fuzzy DEs is interpreted using Seikkala's derivative. The interpretation implies the future behaviour of the solutions. Based from Ahmadian et al. (2012), the Seikkala's derivative has certain defect in some problems. In this study, the triangular, trapezoidal and parallelogram membership functions are being used in order to validate the performances of the 2DIMB methods under the Seikkala's derivative. The convergences of 2DIMB methods based on fuzzy DEs are also presented.

1.7 Outline of the thesis

In this chapter, explanations on ordinary and fuzzy DEs are given. The objectives, scope and limitations of the study are included.

Chapter 2 introduces the literature review for the block method, ordinary and fuzzy DEs. Short reviews on the development of the 2DIMB methods are stated. This chapter also comprises some related notations, definitions and theorems.

In Chapter 3, the 2DIMB methods are presented. The formulations of the 2DIMB methods by using Lagrange interpolation polynomial are shown from order three to four and five. The stability analysis, stability region and algorithm of the 2DIMB methods are given. The 2DIMB methods are implemented to solve the single first order ordinary DEs. Numerical results for the 2DIMB methods are tabulated and presented.

Chapter 4 presents the fuzzy version of 2DIMB methods of order three, four and five. Under the fuzzy DEs setting, the convergences of the 2DIMB methods are shown. This chapter also includes some related theorems for fuzzy DEs.

In Chapter 5, the implementation and results of the 2DIMB methods are furnished. These methods are numerically experimented on fuzzy IVP at constants step size and the results are tabulated. The results are compared with RK methods. This chapter includes parts which analyzes and describes the performance of the method.

Finally, Chapter 6 summarizes the conclusions from this study. Recommendations and outlines for further study in this area are stated.

REFERENCES

Abbasbandy, S. and Viranloo, T. A. 2004. Numerical Solution of Fuzzy Differential Equation by Runge-Kutta Method from *Nonlinear Studies* 11(1): 117-129.

Ahmad, M. Z. and De Baets, B. 2009. A Predator-Prey Model with Fuzzy Initial Populations from *Proc. of the Joint 13th IFSA World Congress and 6th EUSFLAT Conference* 1311-1314.

Ahmadian, A., Suleiman, M., Ismail, F. and Salahshour, S. 2012. An Improved Runge-Kutta Method for Solving Fuzzy Differential Equations under Generalized Differentiability from *International Conference on Fundamental and Applied Sciences 2012, AIP Conf. Proc.* 1482: 325-330.

Allahviranloo, T., Ahmady, N. and Ahmady, E. 2007. Numerical Solution of Fuzzy Differential Equations by Predictor-Corrector Method from *Information Sciences* 177: 1633-1647.

Allahviranloo, T., Ahmady, N. and Ahmady, E. 2008. Erratum to "Numerical Solution of Fuzzy Differential Equations by Predictor-Corrector Method" [Inform. Sci. 177(7) (2007) 1633-1647] from *Information Sciences* 178: 1780-1782.

Allahviranloo, T., Abbasbandy, S., Ahmady, N. and Ahmady, E. 2009.Improved Predictor-Corrector Method for Solving Fuzzy Initial Value Problems from *Information Sciences*179: 945-955.

Atkinson, K.E. 2015. University of Iowa Resources in Numerical Analysis. http://homepage.math.uiowa.edu/~atkinson/na-resources.pdf. Retrieved 31 December 2015.

Aziz, N. H. A.,Majid, Z. A. and Ismail, F. 2014. Solving Delay Differential Equations of Small and Vanishing Lag using Multistep Block Method from *Hindawi Publishing Corporation*2014: 348912.

Bede, B., Rudas, I. J. and Bencsik, A. L. 2007. First Order Linear Fuzzy Differential Equations Under Generalized Differentiability from *Information Sciences* 177: 1648 – 1662.

Bede, B. 2008. Note On "Numerical Solutions of Fuzzy Differential Equations by Predictor-Corrector Method from *Information Sciences* 178: 1917-1922.

Buckley, J. J. and Feuring, T. 2000. Fuzzy Differential Equations from *Fuzzy Sets and Systems* 110: 43-54.

Burden, R. L and Faires, J. D. 2005.*Numerical Analysis.*United States of America: Thomson Brooks/Cole.

Chang, S. L. and Zadeh, L. A. 1972. On Fuzzy Mapping and Control from *IEEE Transactions on Systems Man and Cybernetics* 2: 30-34.

Chartier, P. 1994. L-Stable Parallel One-Block Methods for Ordinary Differential Equations from *SIAM Journal on Numerical Analysis* 31(2): 552-571.

Chapra, S. C. and Canale, R. P.2006.*Numerical Methods for Engineers , Fifth Edition*. New York: McGraw-Hill.

Chu, M. T. and Hamilton, H. 1987. Parallel Solution of ODE's by Multi-Block Methods from *SIAM Journal on Scientific and Statistical Computing* 8: 342-353.

Cong-Xin, W. and Ming, M. 1991. Embedding Problem of Fuzzy Number Space: Part 1 from *Fuzzy Sets and Systems* 44(1): 33-38.

Dahlquist, G. and Björck, Å. 1974. *Numerical Methods*. Englewood Cliffs: Prentice-Hall, Inc.

Dubois, D. and Prade, H. 1982. Towards Fuzzy Differential Calculus, Part 3: Diffentiation from *Fuzzy Sets and Systems* 8: 225-235.

Duraisamy, C. and Usha, B. 2010. Another Approach to Solution of Fuzzy Differential Equations from *Applied Mathematical Sciences* 4(16): 777-790.

Duraisamy, C. and Usha, B. 2011. Numerical Solution of Fuzzy Differential Equations by Taylor Method from *International Journal of Mathematical Archieve* 2(8): 1368-1375.

Fard, O. S. and Kamyad, A. V. 2011. Modified k-Step Method for Solving Fuzzy Initial Value Problems from *Iranian Journal of Fuzzy Systems* 8(1): 49-63.

Fatunla, S. O. 1991. Block Methods for Second Order ODEs from *Intern. J. Computer Math* 41: 55-63.

Ghanbari, M. 2009. Numerical Solution of Fuzzy Initial Value Problems under Generalized Differentiability by HPM from *Int. J. Industrial Mathematics* 1(1): 19-39. Guang-Quan, Z. 1991. Fuzzy Continuous Function and Its Properties from *Fuzzy Sets and Systems* 43: 159-171.

Ghazanfari, B. and Shakerami, A. 2011. Numerical Solutions of Fuzzy Differential Equations by Extended Runge-Kutta-like Formulae of Order 4 from *Fuzzy Sets and Systems* 189: 74-91.

Georgiou, D. N., Nieto, J. J. and Rodriguez-Lopez, R. 2005.Initial Value Problems for Higher Order Fuzzy Differential Equations from *Nonlinear Analysis* 63: 587-600.

Goetschel Jr, R. and Voxman, W. 1986. Elementary Fuzzy Calculus from *Fuzzy Sets and Systems* 18: 31-43.

Hasni, M. M., Majid, Z. A. and Senu, N. 2013. Numerical Solution of Linear Dirichlet Two-Point Boundary Value Problems using Block Method from *International Journal of Pure and Applied Mathematics* 85(3): 495-506.

Henrici, P. 1962. *Discrete variable Methods in Ordinary Differential Equations*. New York: John Wiley & Sons, Inc.

Ibrahim, Z. B., Suleiman, M. and Othman, K. I. 2007. Implicit r-Point Block Backward Differentiation Formula for Solving First-Order Stiff ODEs from Applied Mathematics and Computation 186: 558-565.

Ismail, F. Ken, Y. L. and Othman, M. 2009. Explicit and Implicit 3-Point Block Methods for Solving Special Second Order Ordinary Differential Equations Directly from *Int. Journal of Math. Analysis* 3(5): 239-254.

Jayakumar, T., Meshkumar, D. and Kanagarajan, K. 2012. Numerical Solution of Fuzzy Differential Equations by Runge-Kutta Method of Order Five from *Applied Mathematical Sciences* 6(60): 2989-3002.

Jayakumar, T., Raja, C. and Muthukumar, T. 2014. Numerical Solution of Fuzzy Differential Equations by Adams Fifth Order Predictor-Corrector Method from *International Journal of Mathematics Trends and Technology* X(Y).

Jameel, A. F., Ismail, A. I. M. and Sadeghi, A. 2012. Numerical Solution of Fuzzy IVP with Trapezoidal and Triangular Fuzzy Numbers by Using Fifth Order Runge-Kutta Method from *World Applied Sciences Journal* 17(12): 1667-1674.

Kaleva, O. 1987. Fuzzy Differential Equations from *Fuzzy Sets and Systems* 24: 301-317.

Kanagarajan, K. and Sambath, M. 2010. Runge-Kutta Nystrom Method of Order Three for Solving Fuzzy Differential Equations from *Computational Methods in Applied Mathematics* 10(2): 195-203.

Lambert, J. D. 1973.*Computational Methods in Ordinary Differential Equations*. New York: John Wiley & Sons, Inc.

Ma, M., Friedman, M. and Kandel, A. 1999. Numerical Solutions of Fuzzy Differential Equations from *Fuzzy Sets and Systems* 105: 133-138.

Majid, Z. A. 2004. *Parallel Block Methods for Solving Ordinary Differential Equations*, PhD Thesis, Universiti Putra Malaysia.

Majid, Z. A. and Suleiman, M. 2006. Performance of 4-Point Diagonally Implicit Block Method for Solving Ordinary Differential Equations from *MATEMATIKA* 22(2): 137-146.

Majid, Z. A. and Suleiman, M. 2007. Implementation of Four-Point Fully Implicit Block Method for Solving Ordinary Differential Equations from *Applied Mathematics and Computation* 184: 514-522.

Majid, Z. A., Azmi, N. A., Suleiman, M. and Ibrahim, Z. B. 2012.Solving Directly General Third Order Ordinary Differential Equations using Two-Point Four Step Block Method from *SainsMalaysiana* 41(5): 623-632.

Mehrkanoon, S., Suleiman, M. and Majid, Z. A. 2009. Block Method for Numerical Solution of Fuzzy Differential Equations from *International Mathematical Forum* 4: 2269-2280.

Milne, W. E. 1953. *Numerical Solution of Differential Equations.*New York: Dover Publications, Inc.

Musa, H., Suleiman, M. B. and Senu, N. 2012. Fully Implicit 3-Point Block Extended Backward Differentiation Formula for Stiff Initial Value Problems from *Applied Mathematical Sciences* 6(25): 4211-4228.

Nieto, J. J. 1999. The Cauchy Problem for Continuous Fuzzy Differential Equations from *Fuzzy Sets and Systems* 102: 259-262.

Omar, Z. and Suleiman, M. 2005. Solving Higher Order Ordinary Differential Equations using Parallel 2-Point Explicit Block Method from *MATEMATIKA* 21(1): 15-23.

Palligkinis, S. Ch., Papageorgiou, G. and Famelis, I. Th. 2009.Runge-Kutta Methods for Fuzzy Differential Equations from *Applied Mathematics and Computation* 209: 97-105.

Phang, P. S., Majid, Z. A., Ismail, F., Othman, K. I. and Suleiman, M. 2013. New Algorithm of Two-Point Block Method for Solving Boundary Value Problem with Dirichlet and Neumann Boundary Conditions form *Mathematical Problems in Engineering* 2013: 917589.

Pulliam, T. H. and Chaussee, D. S. 1981. A Diogonal Form of An Implicit Approximate-Factorization Algorithm from *Journal of Computational Physics* 39: 347-363.

Puri, M. L. and Ralescu, D. A. 1983. Differentials of Fuzzy Functions from *Journal of Mathematical Analysis and Applications* 91: 552-558.

Radzi, H. M., Majid, Z. A., Ismail, F. and Suleiman, M. 2012. Two and Three Point One-Step Block Methods for Solving Delay Differential Equations from *Journal of Quality Measurement and Analysis* 8(1): 29-41.

Rosser, J. B. 1967. A Runge-Kutta for All Seasons from *SIAM REVIEW*9(3):417-452.

Seikkala, S. 1987. On the Fuzzy Initial Value Problem from *Fuzzy Sets and Systems* 24: 319-330.

Shampine, L. F. 1994. *Numerical Solution of Ordinary Differential Equations.Vol.4.*New York: Chapman & Hall.

Shampine, L. F. and Watts, H. A. 1969. Block Implicit One-Step Methods from *Math. Comp*. 23: 731-740.

Shang, D. and Guo, X. 2013. Adams Predictor-Corrector Systems for Solving Fuzzy Differential Equations from *Hindawi Publishing Corporation* 2013: 312328.

Sharmila, R. G. and Amirtharaj, E. C. H. 2013. Numerical Solution of Fuzzy Initial Value Problems by Fourth Order Runge-Kutta Method Based on Contraharmonic Mean from *Indian Journal of Applied Research* 3(4): 59-63.

Sharmila, R. G. and Amirtharaj, E. C. H. 2013. Numerical Solution of Nth-Order Fuzzy Initial Value Problems by Fourth Order Runge-Kutta Method Based on Centroidal Mean from *IOSR Journal of Mathematics* 6(3): 47-63.

Terano, T., Asai, K. and Sugeno, M. 1987.*Fuzzy Systems Theory and Its Applications.* San Diego: Academic Press, Inc.

Voss, D. and Abbas, S. 1997. Block Predictor-Corrector Schemes for the Parallel Solution of ODEs *from Computers Math Applic*. 33(6): 65-72.

Xu, J., Liao, Z. and Hu, Z. 2007. A Class of Linear Differential Dynamical Systems with Fuzzy Initial Condition from *Fuzzy Sets and Systems* 158:2339-2358.

Zadeh, L. A. 1965. Fuzzy Sets from *Information and Control* 8:338-353.

Zawawi, I. S. M., Ibrahim, Z. B. and Suleiman, M. 2013. Diagonally Implicit Block Backward Differentiation Formulas for Solving Fuzzy Differential Equations from *AIP Conference Proceeding* 1522: 681-687.

Zimmermann, H. J. 1985. *Fuzzy Set Theory-and Its Applications.* Boston: Kluwer-Nijhoff Publishing.

Zhang, D., Feng, W., Xie, Z. and Qiu, J. 1986. Solutions of First Order Fuzzy Differential Equations by A Characterization Theorem from *Journal of Mathematical Analysis and Applications* 114: 409-422.