UNIVERSITI PUTRA MALAYSIA

SIMILARITY SOLUTIONS FOR MATHEMATICAL MODELLING OF BOUNDARY LAYER FLOW AND HEAT TRANSFER IN VISCOUS FLUIDS AND NANOFLUID

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By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

May 2015

DEDICATIONS

To my husband, mom, children and sibling

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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May 2015

Chair: Norihan Md. Arifin, PhD

Faculty: Institute for Mathematical Research

In this thesis, similarity solutions of boundary layer flow and heat transfer in viscous fluid and nanofluid are considered. The objectives of the thesis are to mathematically model heat and mass transfer problems and to obtain the numerical results of each problem. The scope of this study is restricted to two-dimensional, steady, incompressible, laminar boundary layer flows in viscous fluid or nanofluid. Two problems are considered in viscous fluid and three problems are considered in nanofluid which related to the Marangoni boundary layer flow, boundary layer stagnation point flow and boundary layer flow with the effect of radiation and stretching/shrinking sheet or cylinder. The radiation effects have important applications in physics and engineering particularly in space technology and high temperature processes. On the other hand, slip flow and permeable surface have also been considered. The governing nonlinear partial differential equations are transformed into a system of nonlinear ordinary differential equations using similarity transformation which is then solved numerically using a shooting function in Maple software to get the similarity solutions. Results and discussion which comprise the analysis of skin friction, temperature gradient, velocity and temperature profiles for some values of the governing parameters are presented in tabular and graphical form. In order to validate the numerical results obtained in this thesis, comparisons with known results from the previous literature have been made and show very good agreements. All the governing parameters influence the flow and heat transfer characteristics. For example, the heat transfer rate at the surface decreases as the radiation parameter increases. Besides, it was also shown that the imposition of suction was to decrease the heat transfer rate at the surface whereas injection showed the opposite effects. Furthermore, dual solutions exist for a certain range of the governing parameters and nanofluid can increase the heat transfer rate.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

PENYELESAIAN KESERUPAAN UNTUK PEMODELAN MATEMATIK BAGI ALIRAN LAPISAN SEMPADAN DAN PEMINDAHAN HABA DALAM BENDALIR LIKAT DAN NANOBENDALIR

Oleh

NOR AZIAN AINI BINTI MAT

Mei 2015

Pengerusi: Norihan Md. Arifin, PhD

Fakulti: Institut Penyelidikan Matematik

Di dalam tesis ini, penyelesaian keserupaan bagi aliran lapisan sempadan dan pemindahan haba dipertimbangkan dalam bendalir likat dan nanobendalir. Objektif tesis adalah untuk memodelkan secara matematik bagi masalah pemindahan haba dan jisim serta untuk mendapatkan penyelesaian berangka bagi setiap masalah. Skop kajian berkisar pada dua dimensi, tetap, tak termampat, aliran lapisan sempadan berlamina dalam bendalir likat atau nanobendalir. Dua masalah dipertimbangkan dalam bendalir likat dan tiga masalah dipertimbangkan dalam nanobendalir yang berkaitan dengan aliran lapisan sempadan Marangoni, lapisan sempadan aliran titik genangan dan aliran lapisan sempadan bebas dengan kesan radiasi dan permukaan serta silinder meregang/mengecut. Kesan radiasi mempunyai kegunaan dalam fizik dan kejuruteraan terutamanya dalam teknologi angkasa dan proses suhu tinggi. Di samping itu, aliran gelincir dan permukaan telap juga dipertimbangkan. Persamaaan terbitan separa tak linear dijelmakan kepada sistem persamaan terbitan biasa tak linear menggunakan penjelmaan keserupaan yang seterusnya diselesaikan secara berangka dengan menggunakan kaedah tembakan dalam perisian Maple untuk mendapatkan penyelesaian keserupaan. Keputusan dan perbincangan yang terbina oleh analisis pekali geseran kulit, nombor Nusselt setempat yang mewakili kadar pemindahan haba pada permukaan, profil halaju dan suhu untuk beberapa nilai parameter dipersembahkan dalam bentuk jadual dan graf. Perbandingan dengan keputusan kajian lepas telah dibuat untuk mengesahkan keputusan berangka yang diperolehi di dalam tesis dan menunjukkan perbandingan adalah sangat baik. Kesemua parameter menakluk mempengaruhi sifat aliran dan pemindahan haba. Contohnya, kadar pemindahan haba pada permukaan menyusut apabila parameter radiasi meningkat. Di samping itu, turut ditunjukkan bahawa kesan sedutan menyusutkan kadar pemindahan haba pada permukaan, manakala semburan bertindak sebaliknya. Tambahan lagi, penyelesaian dual wujud bagi julat parameter yang tertentu dan nanobendalir meningkatkan kadar pemindahan haba.

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I certify that a Thesis Examination Committee has met on 21 May 2015 to conduct the final examination of Nor Azian Aini binti Mat on her thesis entitled "Similarity Solutions for Mathematical Modelling of Boundary Layer Flow and Heat Transfer in Viscous Fluids and Nanofluid" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U. (A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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$a\,(>0)$ straining rate parameter
c shrinking/stretching rate (of the surface)
C nanoparticle fraction
C_f skin friction coefficient
$C_f \, \mathrm{Re}_x^{1/2} = f''(0)$ reduced skin friction coefficient (shear stress at the surface)
C_p specific heat of the fluid at a constant pressure
D thermal slip factor
D_B Brownian diffusion coefficient
D_T thermophoresis diffusion coefficient
f_0, f_w suction/injection parameter
$f'(0)$ interface velocity
k thermal conductivity
k_f thermal conductivity of the fluid
k_{nf} effective thermal conductivity of the nanofluid
k_s thermal conductivity of the solid
l_0 constant scale factor
L velocity slip factor
L_0 origin of the curvilinear abscissa x
L extension of the relevant interface S
Le Lewis number
m constant exponent
Nr thermal radiation parameter
$Nu_x = -\theta'(0)$ surface temperature gradient or local Nusselt number (rate of heat transfer at the surface)
Nur= $Nu_x \, \mathrm{Re}_x^{-1/2}$ reduced Nusselt number
Nb Brownian motion number
Nt thermophoresis number
Pr Prandtl number
q_w local heat flux
q_r radiative heat flux
Q_0 heat generation/absorption constant
Q heat generation parameter
Re Reynolds number
Re_x local Reynolds number
R radius
r coordinates measured along the surface of the radial direction
Sh Sherwood number
Shr reduced Sherwood number
t_0 constant scale factor
t_s interface temperature
T fluid temperature
ΔT is the positive increments of temperature linked to the temperature gradients imposed on the interface

$T_S(x)$ interface temperature distribution
T_m constant temperature of the external flow
u x-components of velocity
u_0 constant scale factor
U_c reference velocity
u_e, U_∞ external/ambient velocity
U_w velocity of a stretching/shrinking sheet
v y-components of velocity
v_w transpiration (suction or injection) velocity
x, y Cartesian coordinate along the plate and normal to it, respectively

Greek symbols

α thermal diffusivity
α_{nf} thermal diffusivity of the nanofluid
α_R Rosseland mean absorption coefficient
η similarity variable
ε value of velocity ratio parameter
λ velocity slip parameter in Chapter 8
μ dynamic viscosity
μ_{nf} effective viscosity of the nanofluid
μ_f viscosity of the fluid fraction
ν kinematic viscosity
$\phi(\eta)$ dimensionless nanoparticle fraction
ρ fluid density
$(\rho C_p)_{nf}$ heat capacitance of the nanofluid
ρ_{nf} effective density of the nanofluid
ρ_f reference density of the fluid fraction
ρ_s reference density of the solid fraction
ρ_p density of the nanoparticles
σ surface tension
σ_0 constants surface tension at the origin
$\theta(\eta)$ temperature in the similarity plane
φ solid volume fraction of the nanofluid
ψ stream function
τ_w wall shear stress

CHAPTER 1

INTRODUCTION

1.1 Convection

Convection is heat transfer by the movement of molecules within fluids such as air or water when heated fluid is caused to move away from the source of heat, carrying energy with it. Convection above a hot surface occurs because hot fluid expands, becomes less dense, and rises. Convection are generally described as one of either natural or forced, although other mechanism also exist. In natural or free convection, the fluid motion occurs by natural means such as buoyancy. Since the buoyancy force is proportional to the density difference, the larger the temperature difference between the fluid and the body, the larger the buoyancy force will be. At heating the density change in the boundary layer will cause the fluid to rise and be replaced by cooler fluid that also will heat and rise.

Forced convection occurs when a fluid flow is induced by an external force where the fluid has a nonzero streaming motion in the far field away from the body surface, caused perhaps by a pump or fan or other driving force independent of the presence of the body. Forced convection also occurs as a product to other processes, such as the action of a propeller in a fluid or aerodynamic heating and also it is often encountered by engineers designing or analysing pipe flow, flow over a plate, heat exchanger and so on.

Convection can also be induced by surface-tension forces provided it is a function of temperature called Marangoni convection. Surface tension can exchange because of inhomogeneous composition of the substances, and/or the temperature-dependence of surface tension forces. Marangoni convection is usually undesirable in material processing applications, the crystal growth melts and other processes (such as welding, balance the soap films and drying silicon wafers) with liquid-liquid or liquid-gas interfaces.

1.2 Boundary Layer

A boundary layer is a layer of fluid in the immediate region of a bounding surface. This thin layer has the velocity of the fluid that increases from zero at the wall (no slip) to its full value which corresponds to external frictionless flow, and the concept is due to Prandtl in 1904 (Schlichting, 1979). Figure 1.1 represents diagrammatically the velocity distribution in such a boundary layer at the plate, with the dimensions across it considerably exaggerated. In front of the leading edge of the plate the velocity distribution is uniform. With increasing distance from the leading edge in the downstream direction the thickness, δ, of the retarded layer increases continuously, as increasing quantities of fluid become affected. Evidently the thickness of the boundary

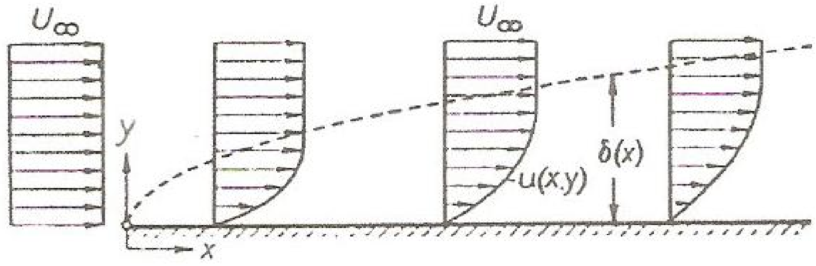

Figure 1.1. Boundary layer on a flat plate in parallel flow at zero incidence (Schlichting, 1979)

layer decreases with decreasing viscosity. Even with very small viscosities (large Reynolds numbers) the frictional shearing stresses $\tau = \mu \partial u / \partial y$ in the boundary layer are considerable because of the large velocity gradient across the flow, whereas outside the boundary layer they are very small.

This physical picture suggests that the field of flow in the case of fluids of small viscosity can be divided, for the purpose of mathematical analysis, into two regions: the thin boundary layer near the wall, in which friction must be taken into account, and the region outside the boundary layer, where the forces due to friction are small and may be neglected, and where, therefore, the perfect-fluid theory offers a very good approximation. Such a division of the field of flow, brings about a considerable simplification of the mathematical theory of the motion of fluids of low viscosity. In fact, the theoretical study of such motions was only made possible by Prandtl when he introduced this concept (Schlichting, 1979).

1.3 Boundary Layer Stagnation Point Flow

A stagnation point is a point in a fluid flow where the flow has come to rest (speed is equal to zero adjacent to some solid body immersed in the fluid flow). Stagnation-point flow describing the fluid motion near the stagnation region of a circular body, exist for both the cases of a fixed or moving body in a fluid (Nandy and Mahapatra, 2013). Figure 1.2 is the simple example of this type of flow, is that leading to a stagnation point in plane that is two-dimensional flow. The velocity distribution in frictionless potential flow in the neighbourhood of the stagnation point at $x = y = 0$ is given by $U = ax$ and $V = -ay$ where a denotes a constant. The figure below is an example of a plane potential flow which arrives from the y-axis and impinges on a flat wall place at $y = 0$, divides into two streams on the wall and leaves in both directions. The viscous flow must adhere to the wall, whereas the potential flow slides along it (Schlichting, 1979).

The stagnation region encounters the highest pressure, the highest rate of heat transfer and the highest rate of mass decomposition (Nandy and Mahapatra, 2013). The forced convection flow at a stagnation point (region) is a classical problem and it has been

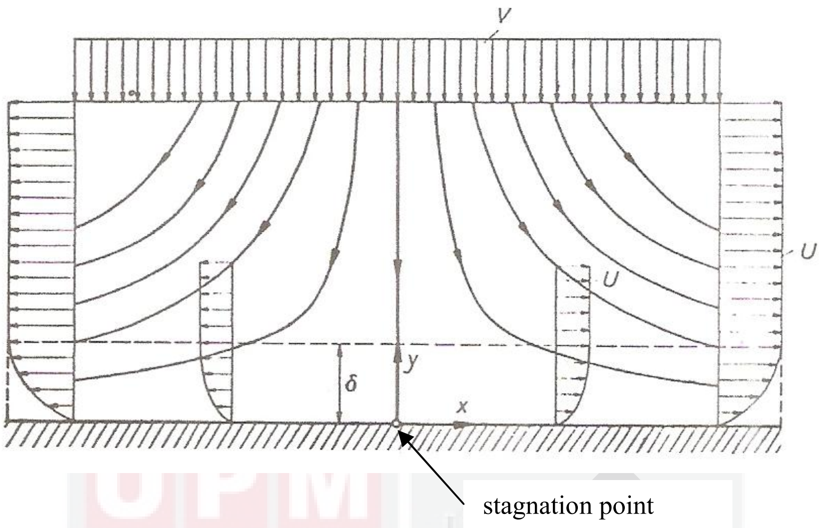

Figure 1.2. Stagnation in plane flow (Schlichting, 1979)

studied by many authors (Lok and Pop, 2011). In recent years, considerable amount of interest has been given to the stagnation point flows of viscous fluids because of their great importance in both theoretical and practical point of views. From the theoretical point of view, such kind of flow is fundamental in fluid mechanics and forced convective heat transfer. From the practical point of view, these flows have applications in forced convection cooling processes where a coolant is impinged on continuously moving surfaces.

1.4 Viscous Fluid

The word of viscous came from the Latin word, 'viscum' which means glue. Viscous fluid has an ability to cling at the solid's surface. This is one of the most important boundary condition, that is in mechanic of viscous fluid. Fluid friction was invented for the first time by Mariotte, 1620-1684 (Darus, 1989). It had been realized even before Prandtl that the discrepancies between the results of classical hydrodynamics and experiment were, in very many cases, due to the fact that the theory neglected fluid friction (Schlichting, 1979).

A Newtonian fluid is a viscous fluid for which the shear stress is proportional to the velocity gradient (i.e. to the time-rate of strain), $\tau = \mu \partial u/\partial y$; τ is the shear stress, μ is the constant dynamic or absolute viscosity of the fluid and $\partial u/\partial y$ is the velocity gradient perpendicular to the direction of shear. While a kinematic viscosity is $v = \mu/\rho$. For a non-Newtonian fluid, the viscosity changes with the applied strain rate (velocity gradient). As a result, non-Newtonian fluids may not have a well-defined viscosity. The Reynolds number is a dimensionless parameter defined as $Re = UL/v$ where U denotes a typical flow speed, L is a characteristic length scale of the flow and v is the kinematic viscosity of the fluid. The Reynolds number gives a rough indication of the relative amplitudes of two key terms in the equations of motion. For the high Reynolds number flow, *Re*>>1, means a motion of a fluid of small viscosity. So the viscous effects can be on the whole negligible. While for the low Reynolds number flow, *Re*<<1 means a very viscous flow.

1.5 Nanofluid

Nanofluids is a fluid by dispersing solid nanoparticles in base fluid such as water and oil. Nanofluids are used to increase thermal conductivity, which goes up with increasing volumetric fraction of nanoparticles and it is concept to describe a fluid in which nanometer-sized particles are suspended in conventional heat transfer basic fluids. The nanofluid concept which was firstly introduced by Choi and Eastman (1995), have remarkable properties that make them have many practical applications in heat transfer, including microelectronics, fuel cells, pharmaceutical processes, and hybrid-powered engines. There have been published several recent papers in nanofluids.

Convectional heat transfer fluids, including oil, water, and ethylene glycol mixture are poor heat transfer fluids, since the thermal conductivity of these fluids plays an important role in determining the coefficient of heat transfer between the heat transfer medium and the heat transfer surface. Therefore, numerous methods have been used to improve the thermal conductivity of these fluids by suspending nanometer/micrometer-sized particle materials in liquids (Hamad et al., 2011). There are two models used by researchers; Tiwari and Das model and Buongiorno model. Buongiorno model used for nanofluid incorporated the effets of Brownian motion and thermophoresis and this model depended on seven slip mechanisms: inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity settling. Buongiorno (2006) proceeded to write down conservation equations based on these two effects (Kuznetsov and Nield, 2010). His analysis however did not consider the influence of local velocity on the diffusion coefficients. Tiwari and Das (2007) have proposed a theoretical model to analyze the behaviour of nanofluids considering the solid volume fraction. It is found that both the Richardson number and the direction of the moving walls affect the fluid flow and heat transfer in the cavity.

1.6 Objectives and Scope

The objectives of the thesis are to model, analyse and to obtain the numerical results of the following five problems:

1. Marangoni boundary layer flows due to an imposed temperature gradient with radiation where it can be formed along the interface of two immiscible fluids when the wall is permeable, where there is suction or injection effect is considered.
2. Marangoni boundary layer flow in nanofluids with thermal radiation where the nanofluids were made by dispersion of Cu, Al_2O_3, and TiO_2 in a water-based fluid.

3. Similarity solutions for the stagnation point flow and heat transfer over a nonlinear stretching/shrinking sheet in a nanofluid with various constant exponent m.
4. The steady boundary layer stagnation-point flow and heat transfer towards a shrinking/stretching cylinder in the presence of velocity and thermal slips over a permeable surface.
5. Forced convection boundary layer flow of nanofluid past a stretching surface due to presence of heat generation with effects of velocity and thermal slips condition.

The scope of this study is restricted to two-dimensional, steady incompressible laminar boundary layer flows in viscous fluids or nanofluid, with the effect of radiation or with the effect of stretching or shrinking. Problem 1 and 4 are related with viscous fluid. Problem 2, 3 and 5 are related with nanofluids where Problem 2 and 3 are using model of Tiwari and Das (2007) while Problem 5 is using model of Buongiorno (2006).

1.7 Outline of the Thesis

This thesis is comprised of nine chapters. In Chapter 1, a brief introduction of convection and the types of fluids have been deliberated. Meanwhile, objectives, scope and the thesis outline which gives the overview of thesis have been highlighted in this chapter.

The literature reviews are discussed in Chapter 2 which related to the problems of studies that are Marangoni boundary layer flow, boundary layer stagnation point flow and forced convection boundary layer flow. Summary from books and journals have also been discussed.

In Chapter 3, the derivation of mathematical formulation and numerical method are given. The similarity transformation is used to transformed the nonlinear governing equations into similarity equations are then solved numerically using shooting method.

In each of Chapter 4 to Chapter 8, the chapter is divided into four main sections and each chapter started with the introduction. Then, followed by the mathematical formulation of the problem and the method of solution are given in second section. Numerical results and discussion which comprise the analysis of shear stress or skin friction, temperature gradient, velocity and temperature profiles are presented in the third section and finally followed by the conclusions.

In Chapter 4, we study the first problem in viscous fluid i.e. Marangoni boundary layer flow over a permeable surface in the presence of radiation, while in Chapter 5, we consider the above problem in nanofluid. Then, we investigated the similarity solutions for the stagnation point flow and heat transfer over a nonlinear stretching/shrinking sheet in a nanofluid, in Chapter 6. In Chapter 7, we consider the permeable surface effect on boundary layer stagnation point flow and heat transfer over a shrinking/stretching cylinder with slip and finally in Chapter 8, we investigated the forced convection slip flow and heat transfer of nanofluids past stretching sheet with heat generation. The summary of the whole thesis, conclusion and future study this research and also some future research are presented in Chapter 9.

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