An estimation of the p-adic sizes of common zeros of partial derivative polynomials of degree six

ABSTRACT

Let $x=(x_1,x_2,\ldots,x_n)$ be a vector in $\mathbb{Z}^n$ with $\mathbb{Z}$ ring of integers and $q$ be a positive integer, $f$ a polynomial in $x$ with coefficient in $\mathbb{Z}$. The exponential sum associated with $f$ is defined as $S(f;q)\equiv \sum_{x \mod q} e((2\pi i f(x))/q)$ where the sum is taken over a complete set of residues modulo $q$. The value of $S(f;q)$ depends on the estimate of cardinality $|V|$, the number of elements contained in the set $V=\{x \mod q| f(x) \equiv 0 \mod q\}$ where $f_x$ is the partial derivatives of $f$ with respect to $x$. To determine the cardinality of $V$, the p-adic sizes of common zeros of the partial derivative polynomials need to be obtained. In this paper, we estimate the p-adic sizes of common zeros of partial derivative polynomials of $f(x,y)$ in $\mathbb{Z}_p[x,y]$ with a sixth degree form by using Newton polyhedron technique. The polynomial is of the form $f(x,y)=ax^6+bx^5y+cx^4y^2+sx+ty+k$.

Keyword: Cardinality; Exponential sums; Newton polyhedron; p-adic sizes