On the cardinality of twelfth degree polynomial

ABSTRACT

Let $p$ be a prime and $f(x, y)$ be a polynomial in $\mathbb{Z}_p[x, y]$. It is defined that the exponential sums associated with $f$ modulo a prime $p$ is $S(f; q) = \sum e^{2\pi i f(x)q}$ for $\alpha > 1$, where $f(x)$ is in $\mathbb{Z}[x]$ and the sum is taken over a complete set of residues $x$ modulo positive integer $q$. Previous studies has shown that estimation of $S(f; p^\alpha)$ is depends on the cardinality of the set of solutions to congruence equation associated with the polynomial. In order to estimate the cardinality, we need to have the value of $p$-adic sizes of common zeros of partial derivative polynomials associated with polynomial. Hence, $p$-adic method and newton polyhedron technique will be applied to this approach. After that, indicator diagram will be constructed and analyzed. The cardinality will in turn be used to estimate the exponential sums of the polynomials. This paper concentrates on the cardinality of the set of solutions to congruence equation associated with polynomial in the form of $f(x, y) = ax^{12} + bx^{11}y + cx^{10}y^2 + sx + ty + k$.

Keyword: Cardinality; Polynomial